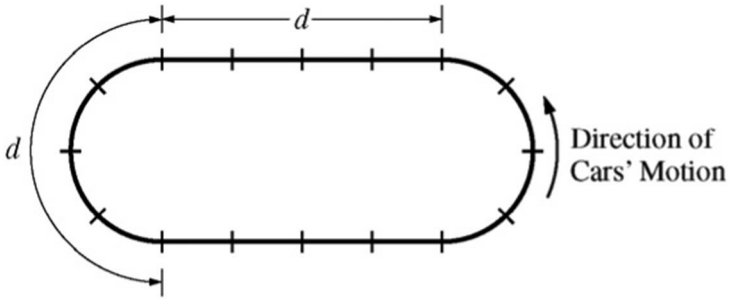


3. The figure above represents a racetrack with semicircular sections connected by straight sections. Each section has length d, and markers along the track are spaced $d/4$ apart. Two people drive cars counterclockwise around the track, as shown. Car X goes around the curves at constant speed v_c, increases speed at constant acceleration for half of each straight section to reach a maximum speed of $2v_c$, then brakes at constant acceleration for the other half of each straight section to return to speed v_c. Car Y also goes around the curves at constant speed v_c, increases speed at constant acceleration for one-fourth of each straight section to reach the same maximum speed $2v_c$, stays at that speed for half of each straight section, then brakes at constant acceleration for the remaining fourth of each straight section to return to speed v_c.

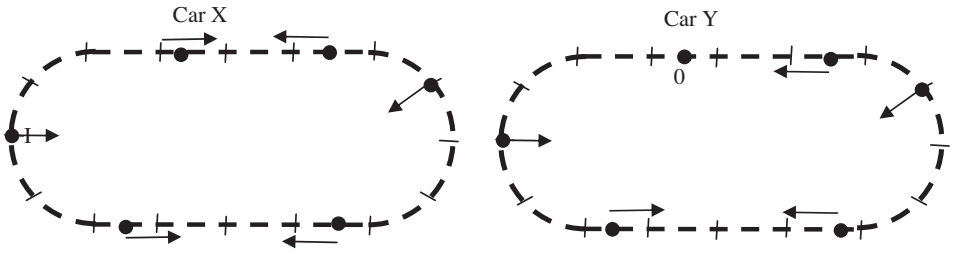
   (a) On the figures below, draw an arrow showing the direction of the net force on each of the cars at the positions noted by the dots. If the net force is zero at any position, label the dot with 0.

   (b) i. Indicate which car, if either, completes one trip around the track in less time, and justify your answer qualitatively without using equations.
   ii. Justify your answer about which car, if either, completes one trip around the track in less time quantitatively with appropriate equations.

   (c) Explain how your equations in part (b) ii reexpress your reasoning in part (b) i. Do not simply refer to any final results of your calculations, but instead indicate how terms in your equations correspond to concepts in your qualitative explanation.

Scoring Guidelines for Free-Response Question 3 (12 points)

(a) (3 points)



For correct directions of the net forces at all the locations on the semicircular sections (i.e., all directed generally toward the center of the circle)	1 point
For correct directions of the net forces at all the locations on the bottom straightaways (i.e., directed toward the center of the segment)	1 point
For correct directions of the net forces at all locations on the top straightaway (i.e., both rightmost arrows directed toward the left, the left one for car *X* directed toward the right, and the left one for car *Y* equal to zero)	1 point

(b) (7 points total)

i) (2 points)

For realizing that the difference in time is only on the straightaways — 1 point

For correct reasoning leading to Car *Y* taking a shorter time on the straightaways — 1 point

Example:

Car *X* takes longer to accelerate and does not spend any time traveling at top speed. Car *Y* accelerates over a shorter time and spends time going at top speed. So car *Y* must cover the straightaways in a shorter time. Curves take the same time, so car *Y* must overall take a shorter time.

ii) (5 points)

The time to travel each curve is d/v_c. Answers can be expressed in terms of d/v_c or $t_c = d/v_c$ or some other defined unit of time. The calculations below will use $t_c = d/v_c$.

For stating that the time to travel each curve is d/v_c — 1 point

For correct kinematics expressions that allow determination of the time it takes for one segment of acceleration on the straightaways — 1 point

Example: $D = v_c t_1 + \frac{1}{2} a t_1^2, a = (2v_c - v_c)/t_1 = v_c/t_1$,

For work that shows an understanding of how to determine the time that car *X* and car *Y* each spend accelerating — 1 point

For work that shows an understanding of how to determine the time that car *Y* spends at constant speed — 1 point

For correctly determining the total straightaway times for each car — 1 point

Calculating the time for car *X* to travel one straightaway:

$$\frac{d}{2} = v_c t_1 + \frac{1}{2} a t_1^2, a = (2v_c - v_c)/t_1 = v_c/t_1$$

$$t_1 = \frac{d}{3v_c} = \frac{t_c}{3}, \text{total time is} \frac{2t_c}{3}$$

Calculating the time for car Y to travel one straightaway:

Doing the calculation shown above using the distance of acceleration $d/4$ gives the result that one section of acceleration takes a time $t_c/6$.

The time for car Y to travel one constant speed section on the straightaway is $(d/2)/2v_c = (t_c/4)$.

Adding three segments to get the total time for one straightaway gives $7t_c/12$.

The calculations show that car *Y* takes less time on a straightaway, and both cars take the same time on the curves, so car *Y* overall takes less time.

(c) (2 points)

For linking math to one aspect of qualitative reasoning that explains the difference in times — 1 point

For linking math to all other qualitative reasoning that explains the difference in times — 1 point

Examples:

The only difference in the calculations for the time of one segment of linear acceleration is the difference in distances. That shows that car *X* takes longer to accelerate. The equation $(d/2)/2v_c = (t_c/4)$ corresponds to car *Y* traveling for a time at top speed.

Substituting $a = v_c/t_1$ into the displacement equation in part (b) ii gives $D = (3/2)v_c t_1$. This shows that a car takes less time to reach its maximum speed when it accelerates over a shorter distance. This means car *Y* reaches its maximum speed more quickly and therefore spends more time at its maximum speed than car *X* does, as argued in part (b) i.