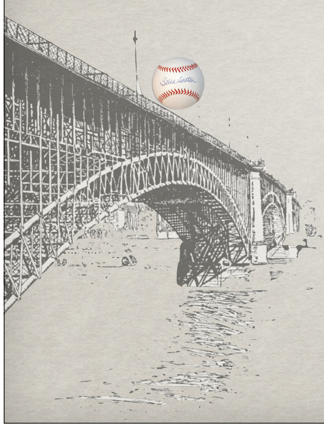
Projectile Motion

Definitions:

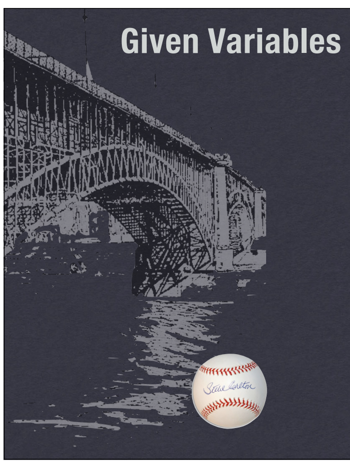
* Simple Projectile Motion:
  * The motion of a body thrown or fired with an initial velocity v_o in a gravitational field.
* Projectile:
  * A kinematic object whose motion is influenced by only the force of gravity.
* Trajectory:
  * The path through space followed by a projectile.

Linear Motion



* A baseball is dropped from the top of a bridge 200m above the water.
* When does it hit the base? (*water*)
* What is its final velocity?

Given Variables



* A baseball is dropped from the top of a bridge 200m above the water.
* When does it hit the ground?
* What is its final velocity?

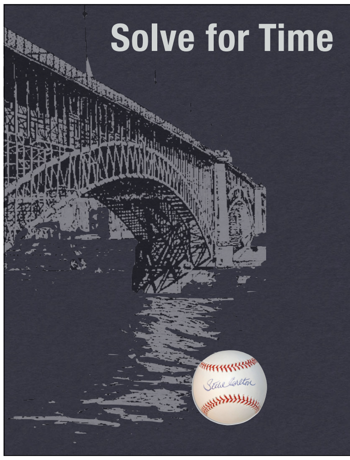
$Y_i = 200$ m

$Y_f = 0$ m

$a = -9.8$ m/s^2

$v_i = 0$ m/s

Solve for Time



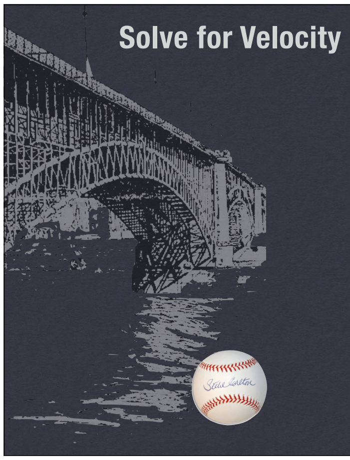
- A baseball is dropped from the top of a bridge 200m above the water.
- When does it hit the ground?
- What is its final velocity?

$$y_f = y_i + v_y t + \tfrac{1}{2} a t^2$$

$$0 = 200 + 0 + \tfrac{1}{2}(-9.8) t^2$$

$$t = \pm 6.39\text{s}$$

Solve for Velocity



$$v_f^2 = v_i^2 + 2 a d$$

$$v_f^2 = 0 + 2(-9.8)(-200)$$

$$v_f = \pm 62.61 \text{ m/s}$$

$\pm$ choose the one that makes sense

Reaction Time

- A meter stick is dropped, and is caught by a second student.
- The meter stick fell 16.5cm

- How much time did it take to be caught?
- What is its final velocity?

Reaction Time

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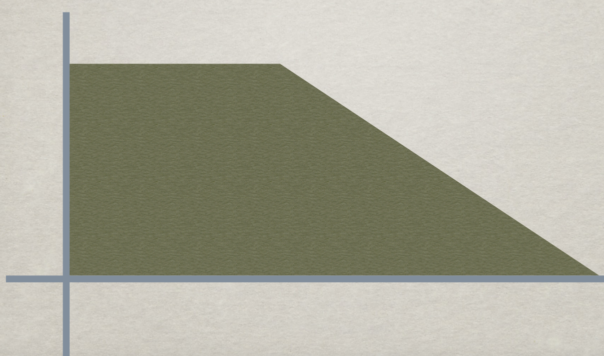
- How much time did it take to be caught?
- What is its final velocity?

Sample Problem

- A driver traveling at 30.0 km/hr sees the light turn red at the intersection. If his reaction time is 0.600 s, and the car can decelerate at 4.50 m/s^2, find the stopping distance of the car.

Picture the velocity graph

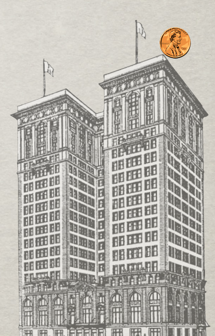


Grab the "givens"

- A driver traveling at 30.0 km/hr sees the light turn red at the intersection. If his reaction time is 0.600 s, and the car can decelerate at 4.50 m/s^2, find the stopping distance of the car.

A little more complicated

- A penny is thrown, straight up in the air, with an upward velocity of 15 m/s from the top of a 90m building.
- How high does it go?
- When does it land?
- What is its final velocity?



Solve: Height at the Top

- **$v_f^2 = v_i^2 + 2\,a\,d$**
- $0 = 15^2 + 2(-9.8)d$
- $d = 11.5$m
- $Y_{max} = h = 101.5$m

Solve: Time at the Top

- **$vf = vi + at$**
- $0 = 15 + (-9.8)t$
- $t = 1.53$ s

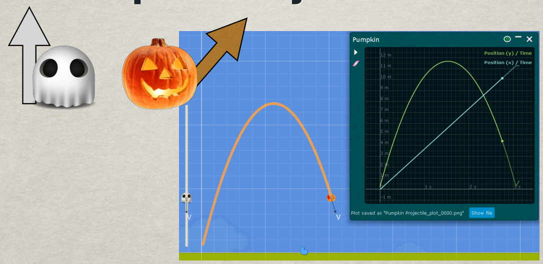
Solve: Velocity at the Bottom

- **$v_f^2 = v_i^2 + 2\,a\,d$**
- $v_f^2 = 0 + 2(-9.8)(-101.5)$
- $v_f = \pm 44.6$ m/s
  - $\pm$ means you have to decide “up” or “down”

Solve: Time at the Bottom

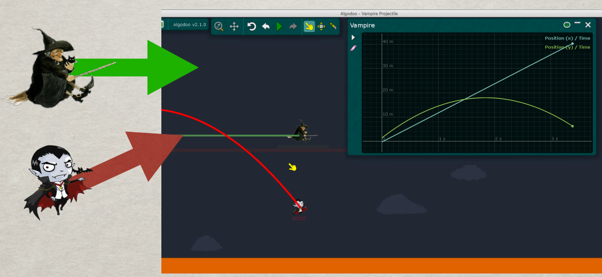
- **$y_f = y_i + v_y\,t + ½\,a\,t^2$**
- $0 = 101.5 + ½\,(-9.8)\,t^2$
- $t = 4.55$s
- Total Time = 6.08s

Pumpkin Projectile



Each new trial had the same Y velocity, and increased the X velocity.

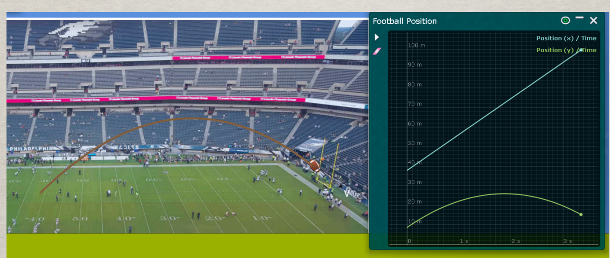
Vampire Projectile



Each new trial had the same X velocity, and increased the Y velocity.

Football Projectile

Each new trial had the same speed but at different angles.



See what one company is doing.

WAYMO

https://youtu.be/B8R148hFxPw

Step 1 - Find out what people think.

Partners, 30 points

* Using Poll Everywhere, you and a partner will create and analyze a 15 question survey about the public perception of AV.
* 3+ q - What does the responder currently feel about autonomous vehicles?
* 3+ q - What is happening currently in AV Safety and Development (Waymo safety report or..)
* 3+ q - What will be some possible advantageous use cases (lets talk)
* 3+ q - Other considerations (Focus on your book chapter)
* Present and consider your results.

Step 2 - Deeper Investigation.

Individual, 30 points

* Explain your book reading as if you were being interviewed. Think podcast discussion.
* 2 page submitted
  * Why you selected the topic.
    * A good long paragraph.. 150+ words?
  * Summary of findings.
    * 2-3 paragraphs 250-300 words?
  * Conclusion Thoughts
    * A good long paragraph.. 100-150 words?

Step 3 - Solve the Physics.

small groups, 40 points

* First, use Logger Pro to analyze the velocity of a car driving through Tower Hill's campus.
* Also, find the acceleration of a car that is approaching a walkway or stop sign.
* Submit screen captures of the motion analysis of your videos, and the graphs of position/time and velocity/time for each.

Problem 1

- Your car drives down the road towards a stop sign. Based on your values for velocity and acceleration, how far from the stop sign would the driver need to be in order to stop?

Problem 2

- How long should it take you to drive "home" *(either partner)*
- You must go the speed limit *(exactly)* at all times.
- You will hit every other stop light, requiring a "stop" a "start" and a 45s "wait"

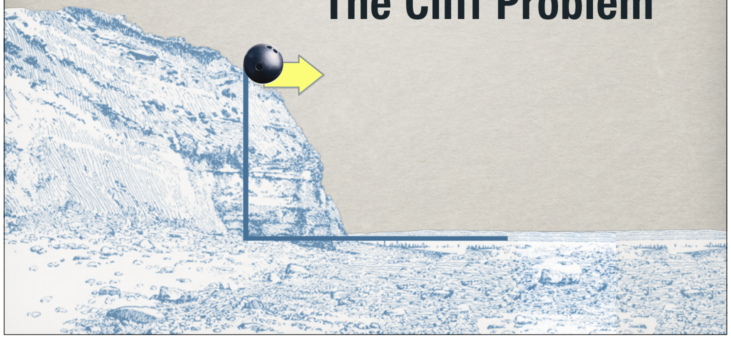
Problem 3

- Your car drives down the road in front of the school. The driver notices a student about to cross the road ahead. What is the minimum distance required to provide student safety? Include a 0.8 s reaction time.
- The student walks at 3 miles per hour.
- The student is 12 feet from the side of the road when you see them.
- The lane you are in is 16 feet wide.

Problem 4

- Your car drives down the road in front of the school, following another driver. The front car applies their brakes to allow a student to cross the street.
- You were only looking down for 1 second, and you react quickly (0.4s). Because you "slam" your brakes, your car can provide twice the acceleration of a normal stop.
- How far behind the other car would you need to drive, so that you can safely stop without hitting them?

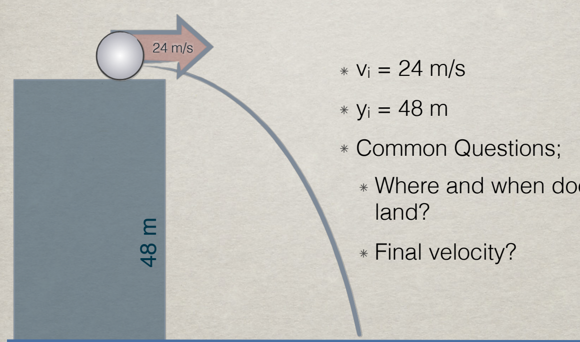
The Cliff Problem



The Cliff

- Problems of this style have an Initial Velocity that is Horizontal
- "x" Velocity is constant
- Common Questions;
  - Find Time
  - Find Range
  - Find Final "y" Velocity
  - Find Final Velocity

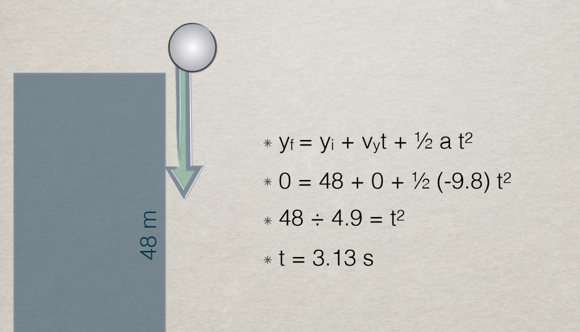
A Ball Rolls From a Cliff



- v_i = 24 m/s
- y_i = 48 m
- Common Questions;
  - Where and when does it land?
  - Final velocity?

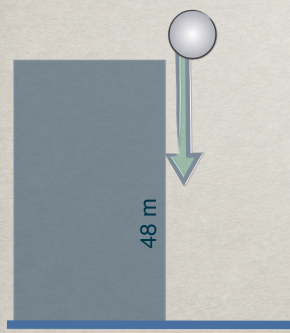
Ignore the "complicated" parabola

Find the time to land



- $y_f = y_i + v_y t + ½ a t^2$
- $0 = 48 + 0 + ½ (-9.8) t^2$
- $48 ÷ 4.9 = t^2$
- $t = 3.13$ s

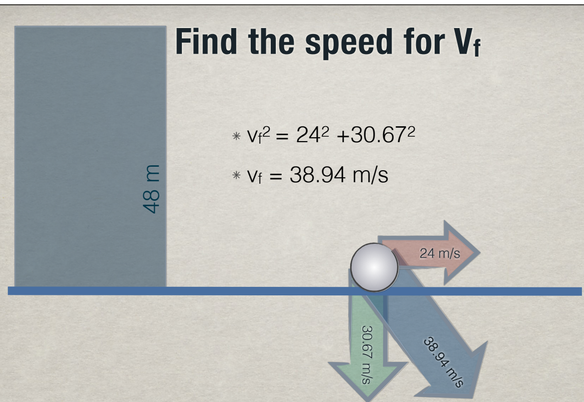
Find V_{fy}



- $v_f^2 = v_i^2 + 2\,a\,d$
- $v_{fy}^2 = 0 + 2\,(-9.8)\,(-48)$
- $v_{fy}^2 = 940.8$
- $v_{fy} = \pm\,30.67$ m/s

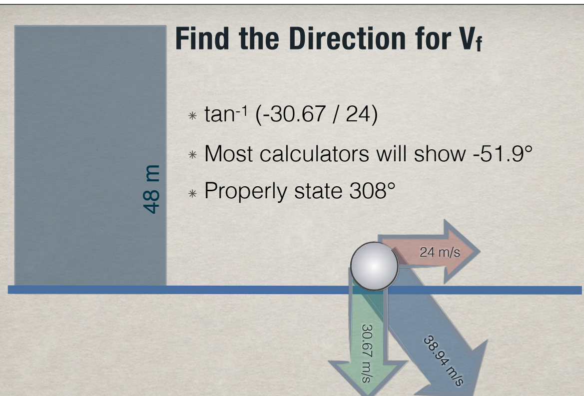
Find the speed for V_f

- $v_f^2 = 24^2 + 30.67^2$
- $v_f = 38.94$ m/s

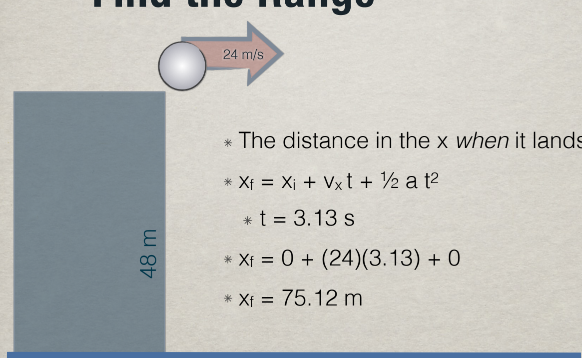


Find the Direction for V_f

- $\tan^{-1}(-30.67 / 24)$
- Most calculators will show -51.9°
- Properly state 308°



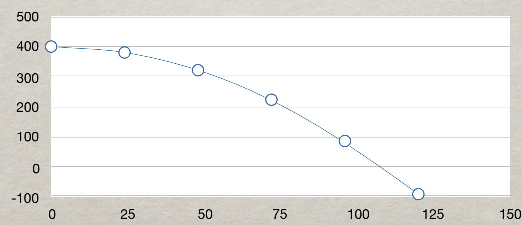
Find the Range



- The distance in the x *when* it lands
- $x_f = x_i + v_x\,t + \frac{1}{2}\,a\,t^2$
  - $t = 3.13$ s
- $x_f = 0 + (24)(3.13) + 0$
- $x_f = 75.12$ m

Location every 2 seconds

time	x velocity	y velocity	x position	y posititon
0	12	0	0	400
2	12	-19.6	24	380.4
4	12	-39.2	48	321.6
6	12	-58.8	72	223.6
8	12	-78.4	96	86.4
10	12	-98	120	.0000000000



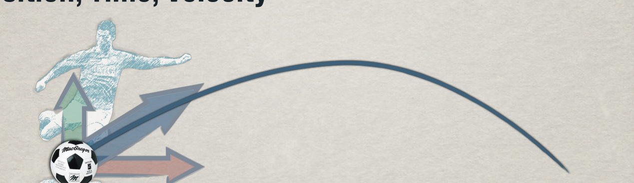
SOCCER BALL

* Initial angle θ for velocity
* Final and Initial Heights are equal

VECTORS

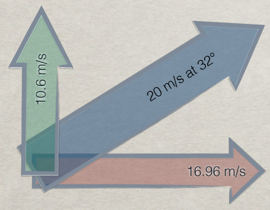
* As a vector at the start of a problem
* X velocity will not change
* Y velocity is changed by gravity
* As a vector at the end of a problem

The Soccer Ball
Position, Time, Velocity



* Common Questions;
  * Maximum Height
  * Where and when does it land?
  * Final velocity?

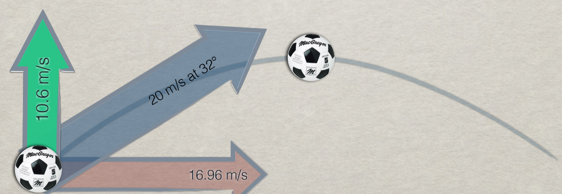
Initial Velocities



* v_i = 20 m/s at 32°
* v_{ix} = 20 cos 32°
* v_{iy} = 20 sin 32°
* a_y = -9.8 m/s^2
* a_x = 0 m/s^2

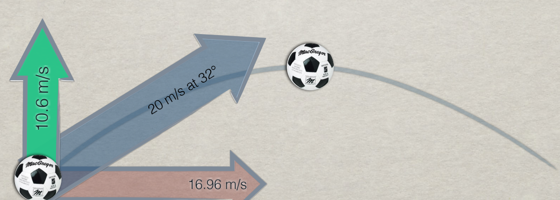
Maximum Height

* $v_f^2 = v_i^2 + 2\,a\,d$
* $0^2 = 10.6^2 + 2(-9.8)(y)$
* y = 5.73m



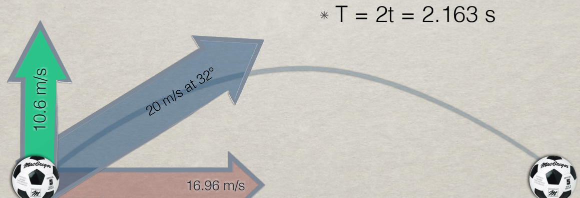
Time at the Top

* $v_f = v_i + at$
* 0 = 10.6 + (-9.8)t
* t= 1.08 s



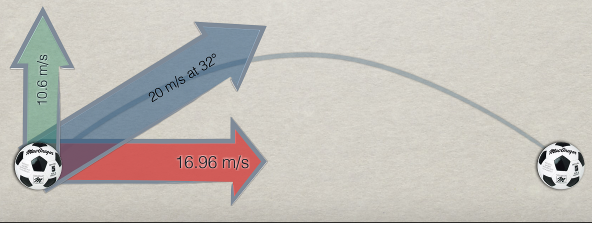
Time to the ground

* t_{top}= 1.08 s
* same distance, and acceleration?
* T = 2t = 2.163 s

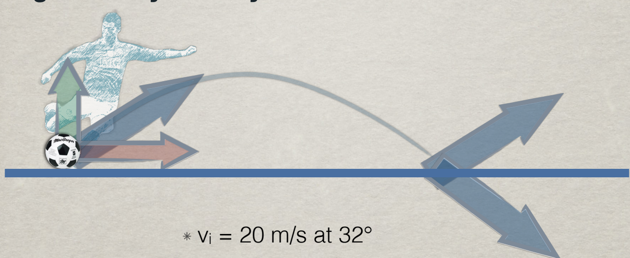


Range - the final x position

- $T = 2.163$ s
- $x = x \;\; + vT \;\; + ½\, aT^2$
- $x = 0 + (16.96)(2.16) + 0$
- $x_f = 36.7$ m



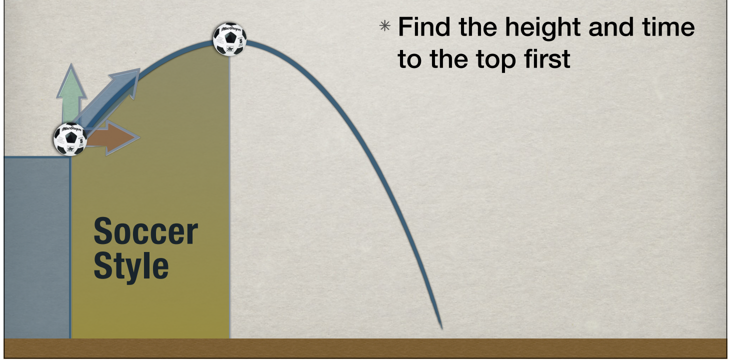
Final Velocity
Using some symmetry

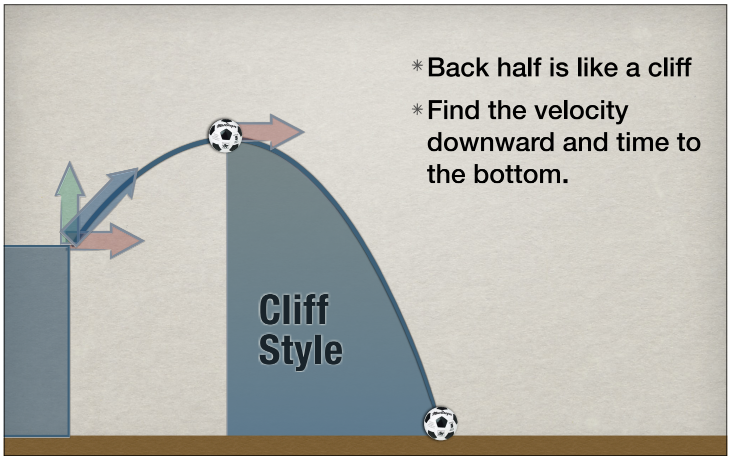


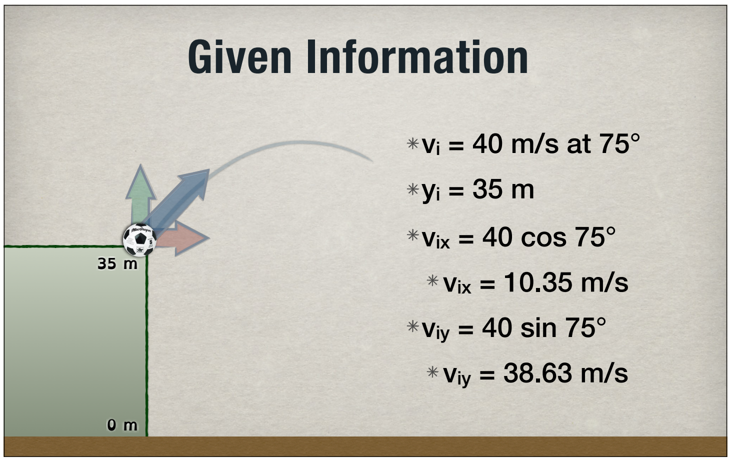
- v_i = 20 m/s at 32°
- v_f = 20 m/s at -32°
- v_f = 20 m/s at 328°

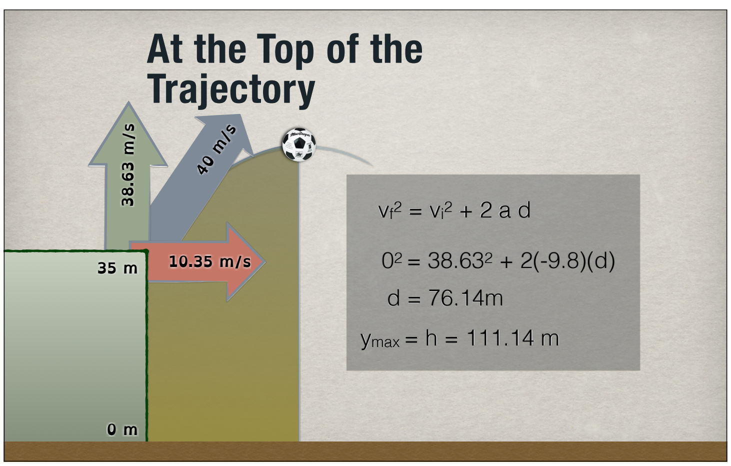
The Combination

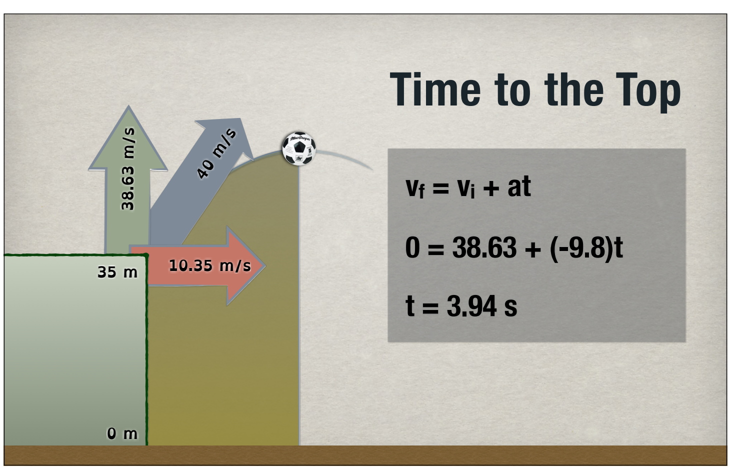
- Front Half is like a soccer problem to the top.
- Find the height and time to the top first

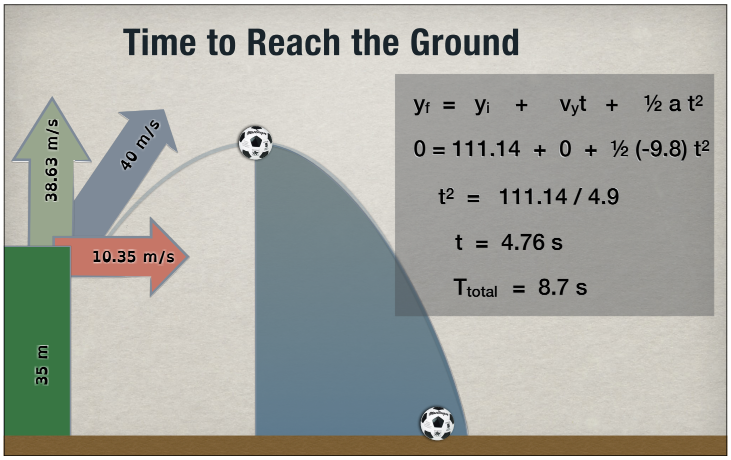


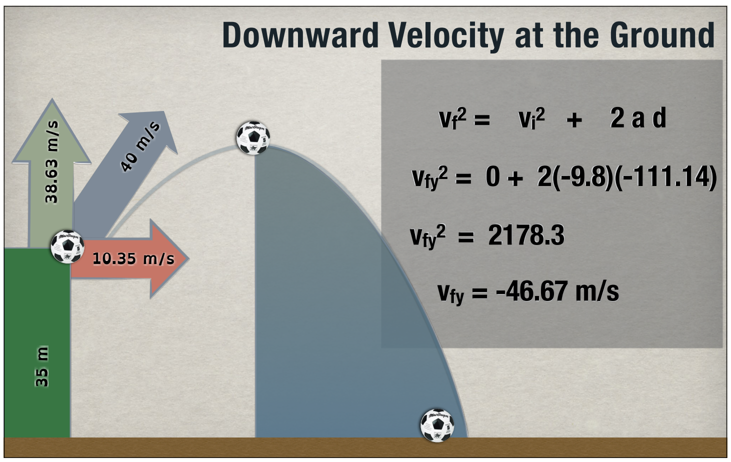

Back half is like a cliff
Find the velocity downward and time to the bottom.
Cliff Style

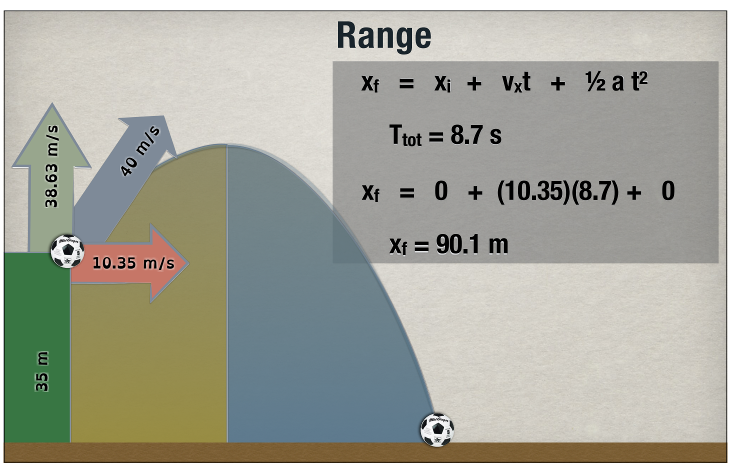

Given Information
v_i = 40 m/s at 75°
y_i = 35 m
v_{ix} = 40 cos 75°
v_{ix} = 10.35 m/s
v_{iy} = 40 sin 75°
v_{iy} = 38.63 m/s
35 m
0 m

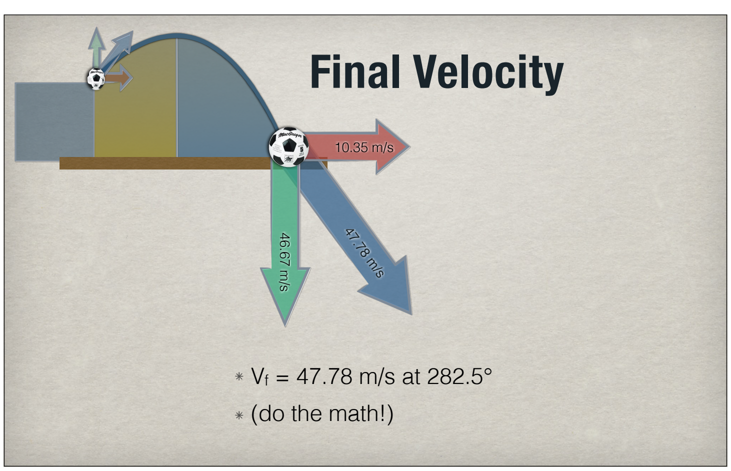

At the Top of the Trajectory
38.63 m/s
40 m/s
10.35 m/s
35 m
0 m
$v_f^2 = v_i^2 + 2ad$
$0^2 = 38.63^2 + 2(-9.8)(d)$
d = 76.14m
y_{max} = h = 111.14 m


Time to the Top
38.63 m/s
40 m/s
10.35 m/s
35 m
0 m
$v_f = v_i + at$
0 = 38.63 + (-9.8)t
t = 3.94 s


Time to Reach the Ground
$y_f = y_i + v_y t + ½ a t^2$
$0 = 111.14 + 0 + ½ (-9.8) t^2$
$t^2 = 111.14 / 4.9$
$t = 4.76$ s
$T_{total} = 8.7$ s
38.63 m/s
40 m/s
10.35 m/s
35 m


Downward Velocity at the Ground
$v_f^2 = v_i^2 + 2 a d$
$v_{fy}^2 = 0 + 2(-9.8)(-111.14)$
$v_{fy}^2 = 2178.3$
$v_{fy} = -46.67$ m/s
38.63 m/s
40 m/s
10.35 m/s
35 m


Range
$x_f = x_i + v_x t + ½ a t^2$
$T_{tot} = 8.7$ s
$x_f = 0 + (10.35)(8.7) + 0$
$x_f = 90.1$ m
38.63 m/s
40 m/s
10.35 m/s
35 m

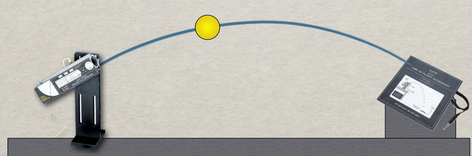

Final Velocity
10.35 m/s
46.67 m/s
47.78 m/s
* $V_f = 47.78$ m/s at 282.5°
* (do the math!)

Projectile Lab Experiment
Vertical Launch

Setting	Average Maximum Height
1 click	
2 clicks	
3 clicks	

Projectile Lab Experiment
Second Launch

Calculated Velocity	Assigned Angle	Predicted Range	Predicted Time



Projectile Lab Experiment
Third Launch

Calculated Velocity	Assigned Angle	Initial Height	Predicted Range	Predicted Time