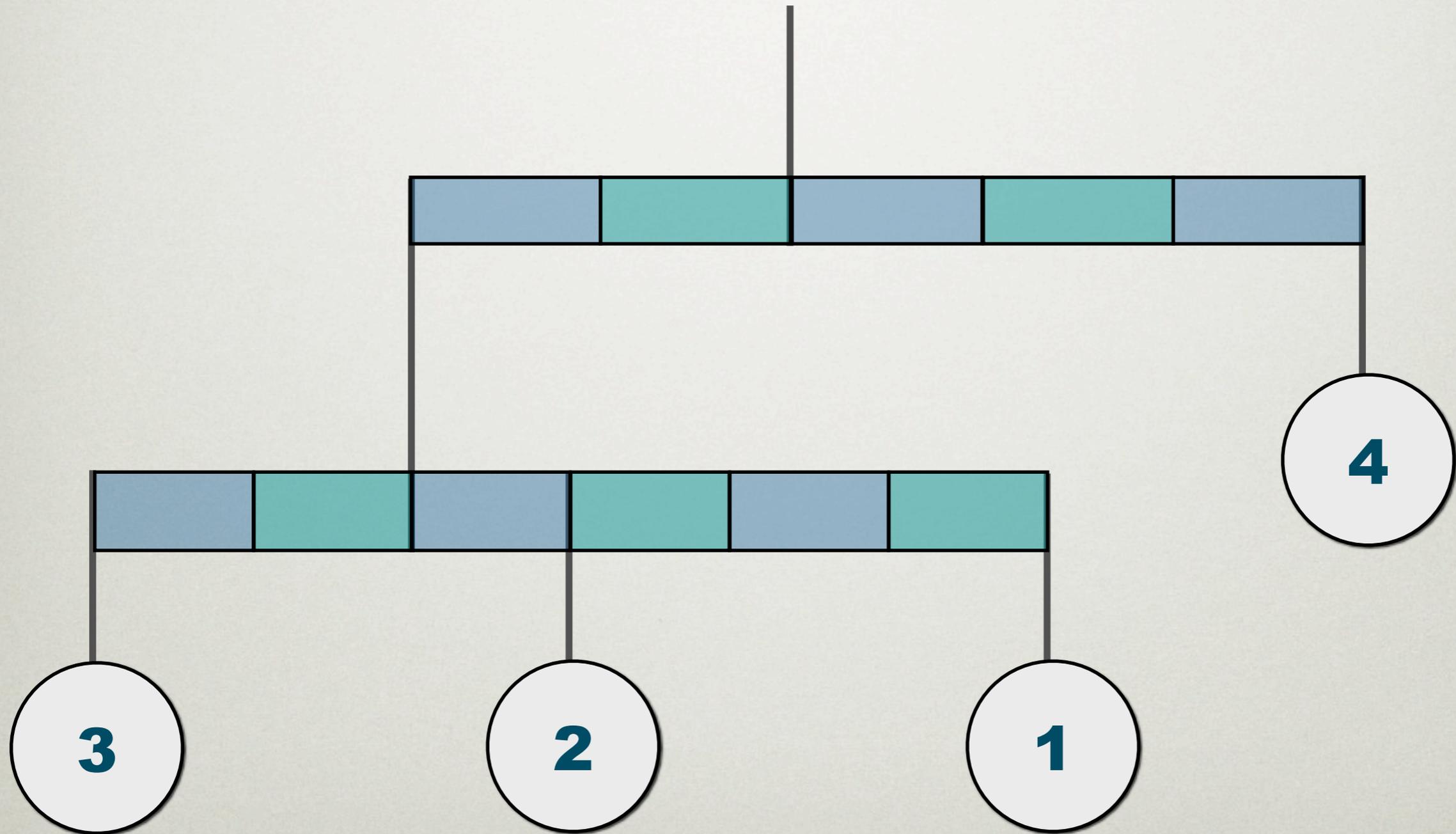
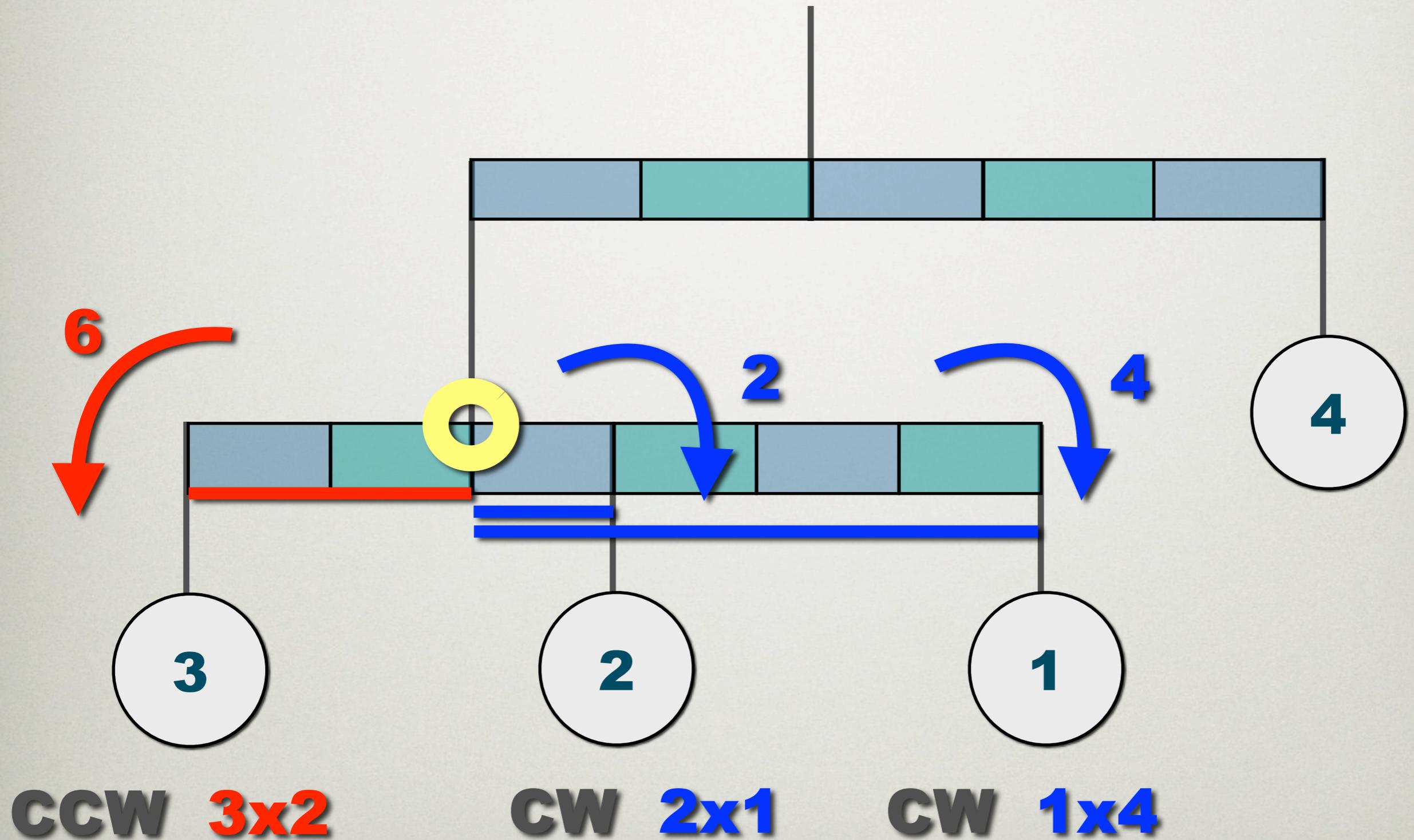


# EQUILIBRIUM PROBLEMS

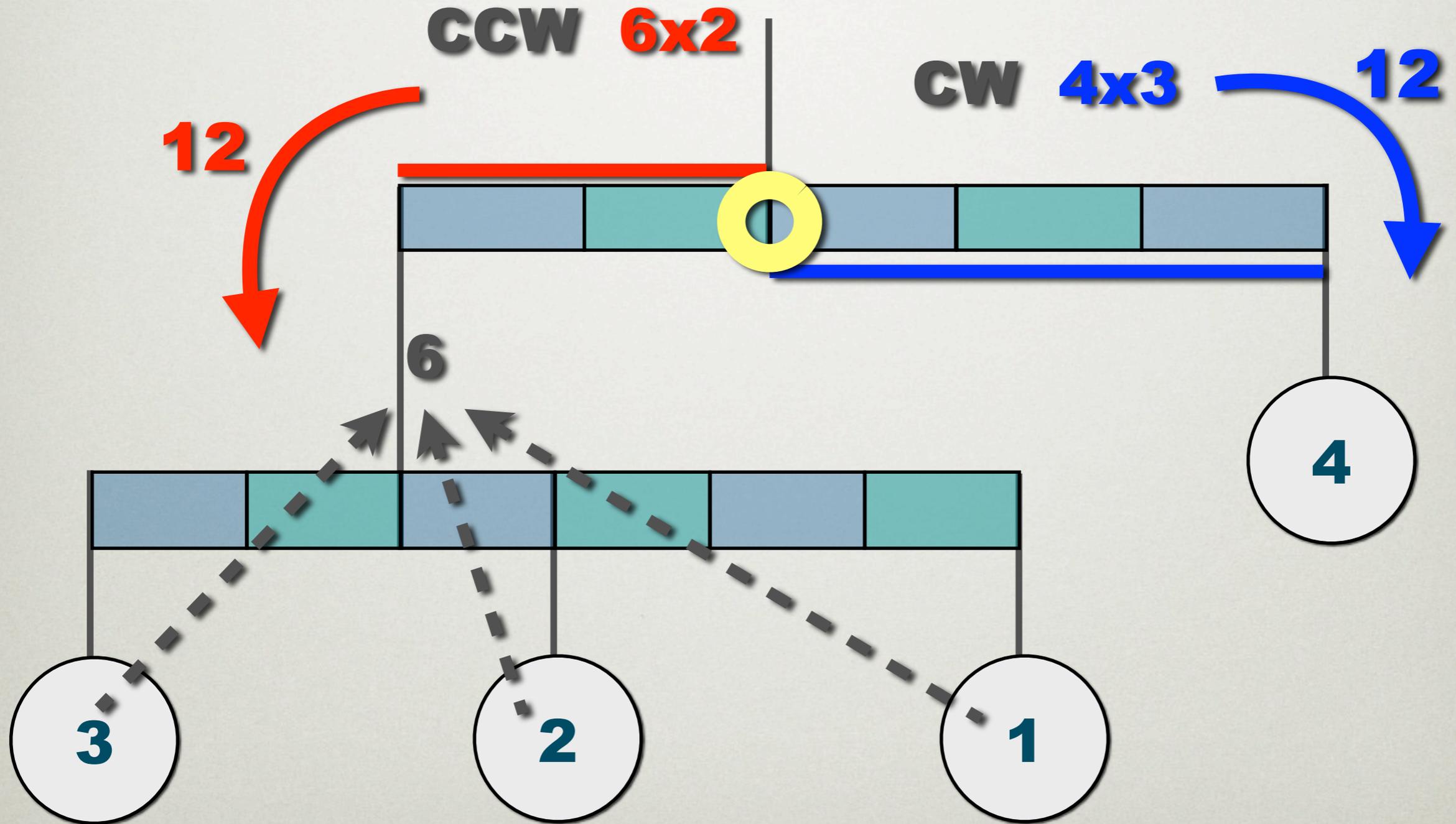
# WHY would this work?



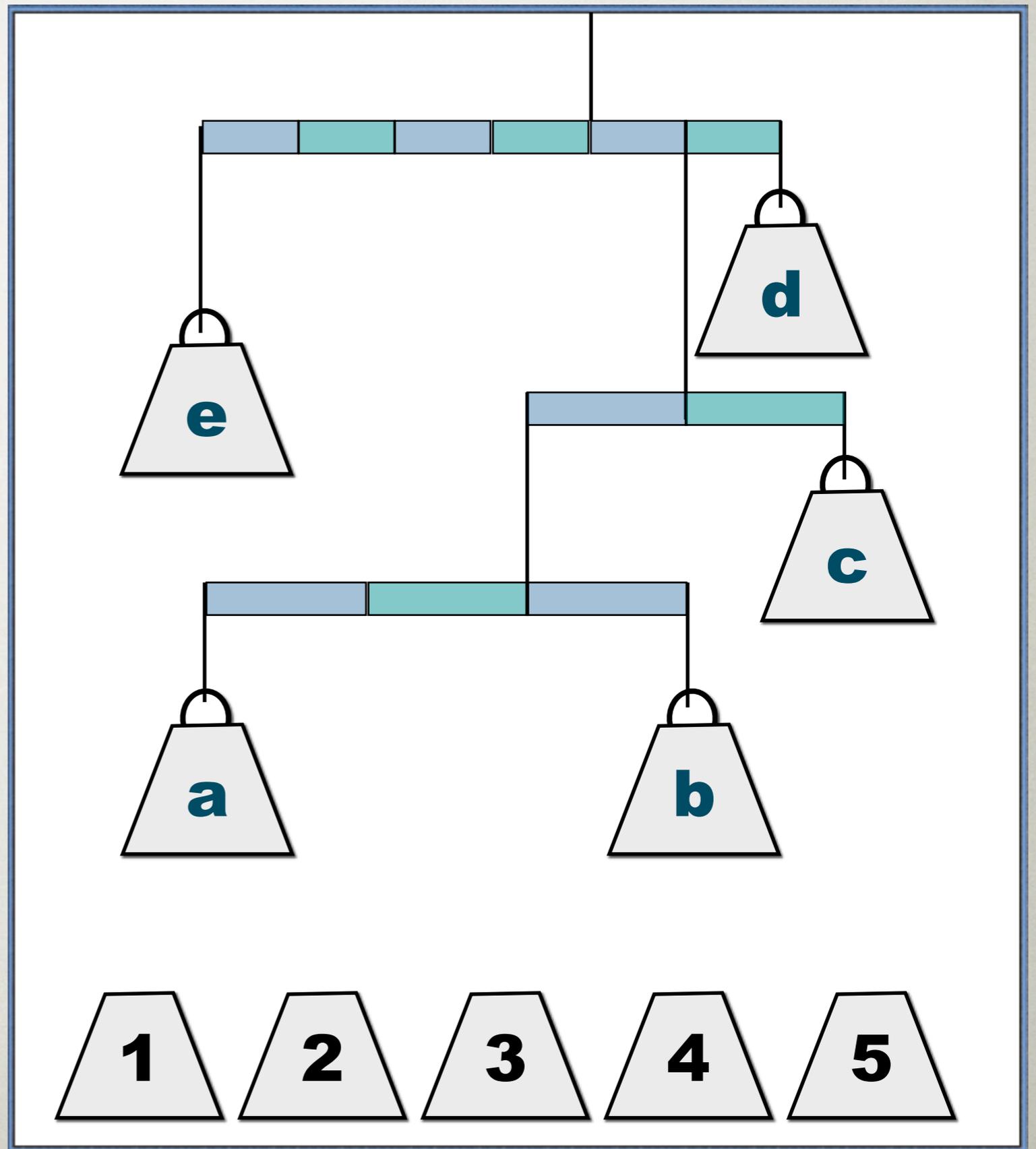
# The Bottom Bar First



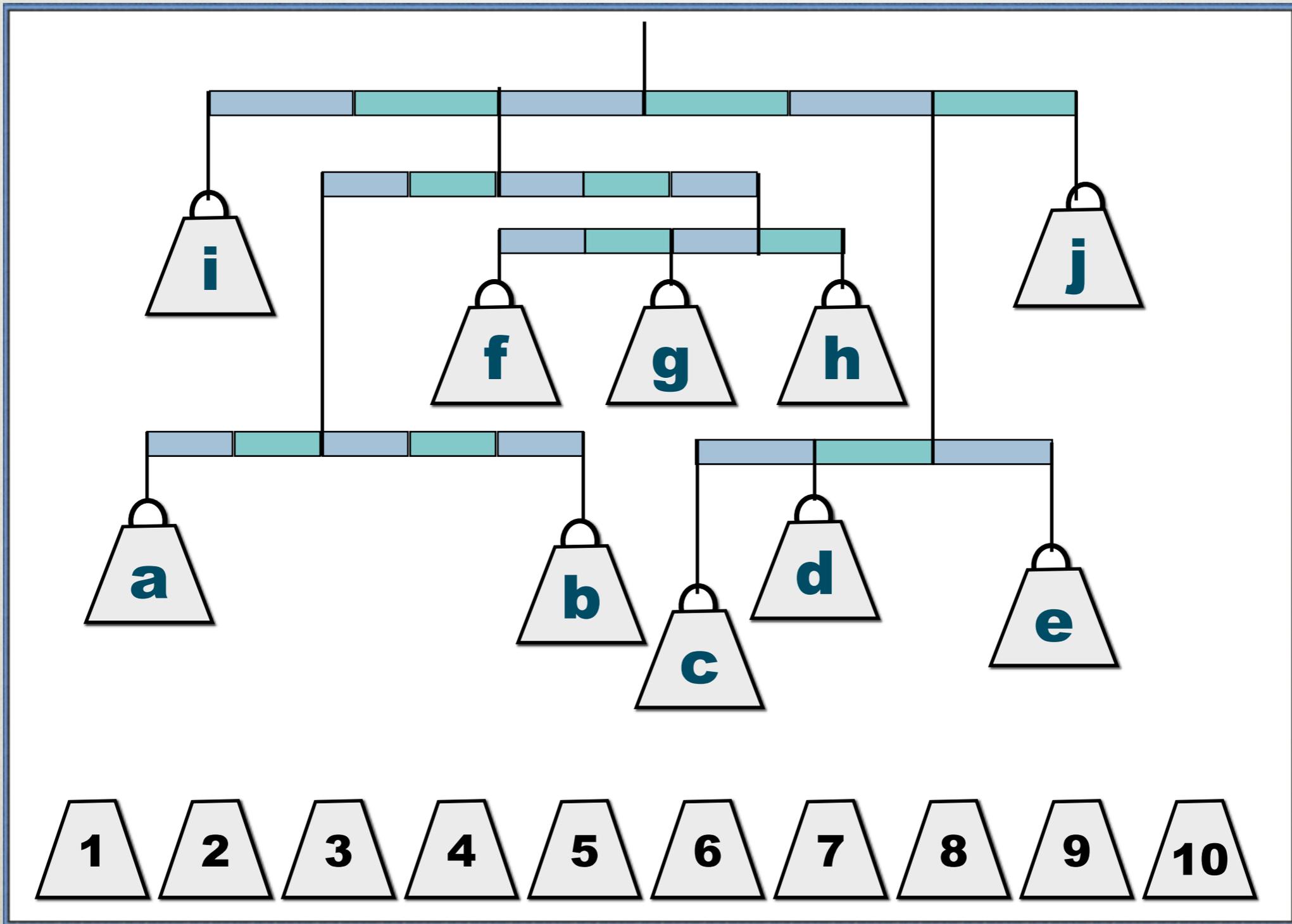
# The Top Bar



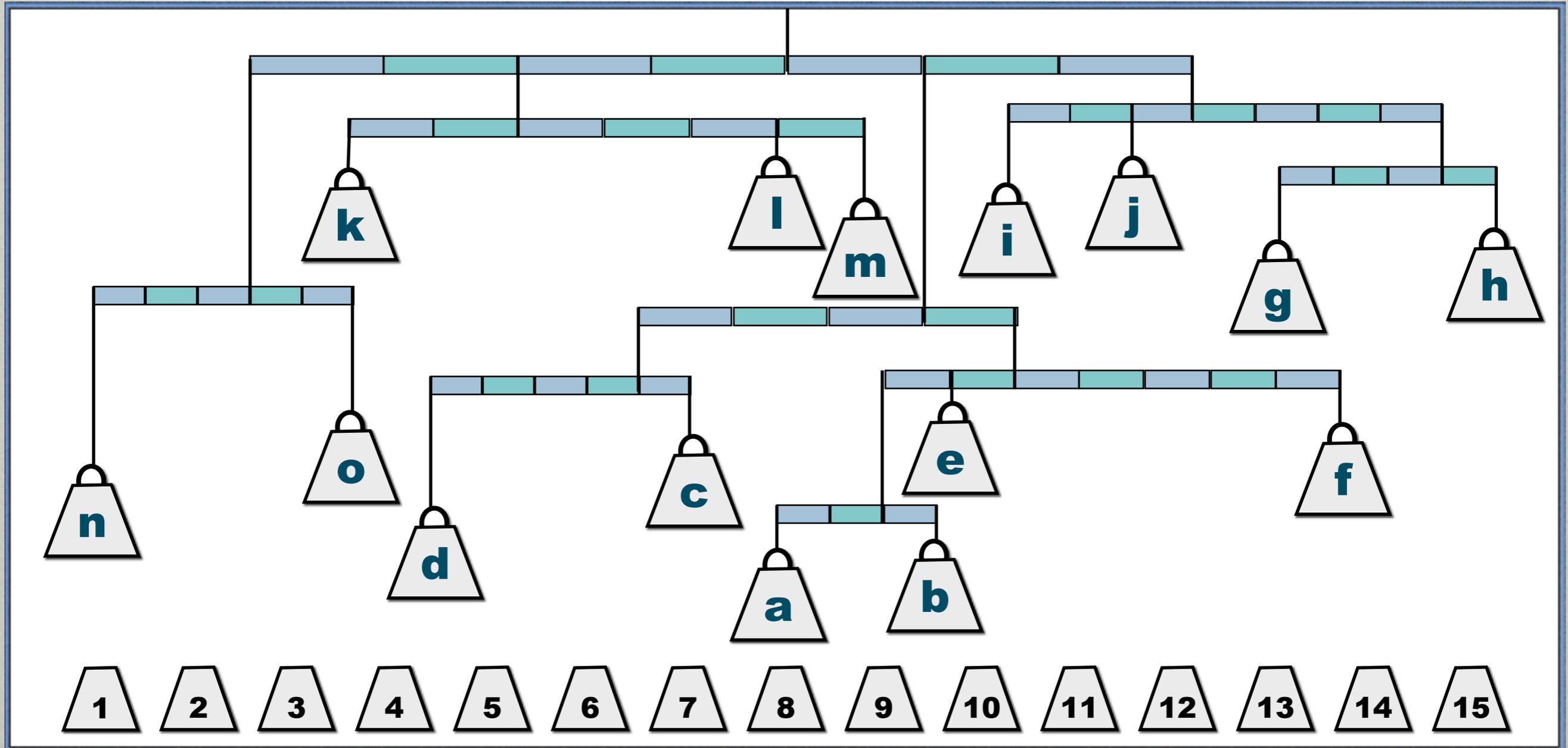
# Easy Sample



# Middle Example



# The "IMPOSSIBLE" one



# Torque

*What is the BEST way to open a door?*

- The most force
- The right direction
- Far from the hinges



$$T = F d \sin\theta$$

**T**orque (Nm)

**F**orce (N)

**d**istance (m)

*or r for radius or l for lever arm*

$\theta$  the angle between F and d

*so, push on a door at a right angle with a force of 25N,  
80cm from the hinges...*

$$\text{Torque (Nm)} = \mathbf{25N \ 0.8m \ sin90^\circ}$$

# Three States of Equilibrium

*Vertical Forces -*

“Ups” equal “Downs”

*Horizontal Forces-*

“Lefts” equal “Rights”

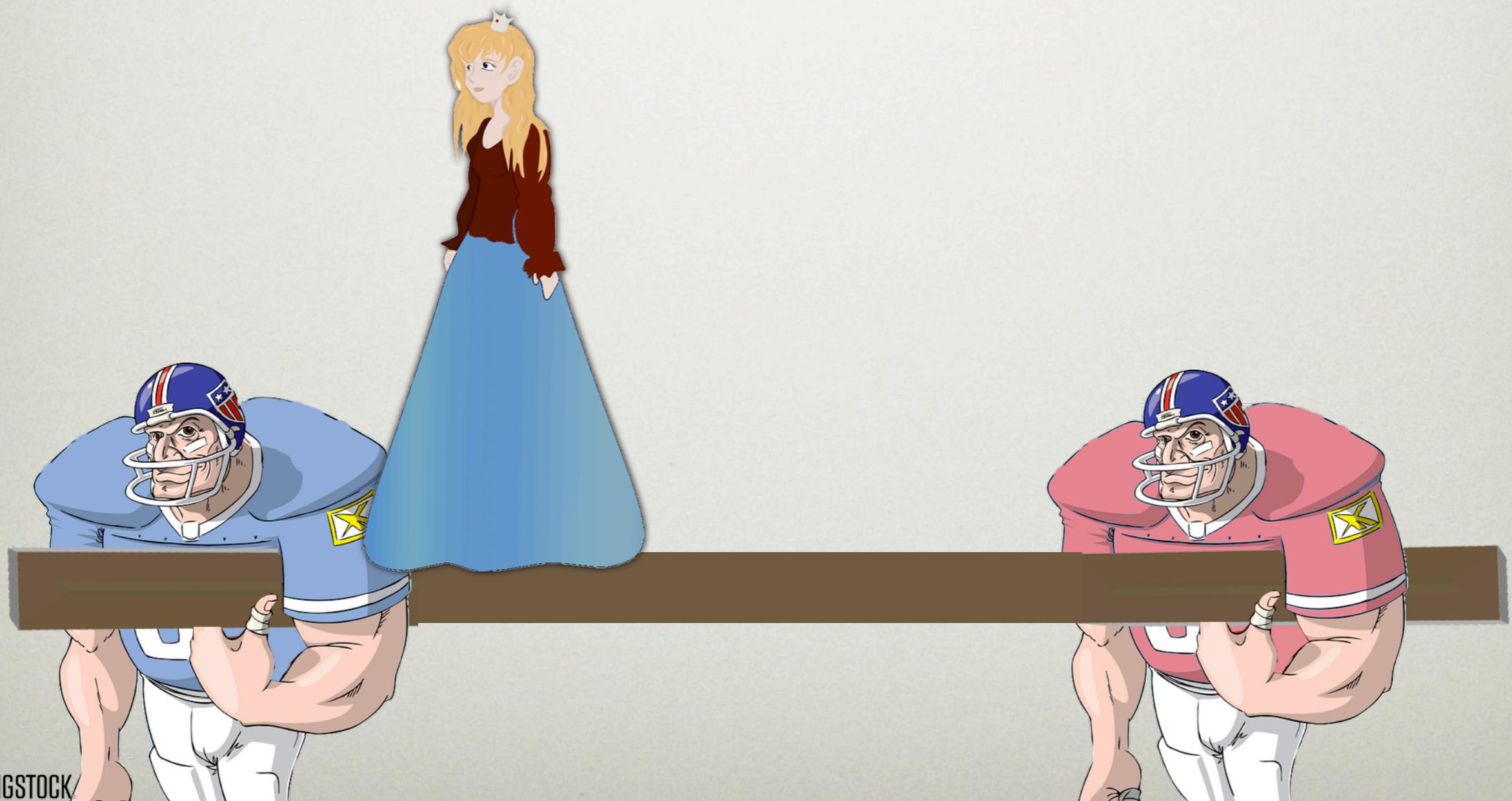
*Torques -*

“Clockwise” equals “Counter-clockwise”

# EQUILIBRIUM WITH RIGHT ANGLES

# A Simple Problem

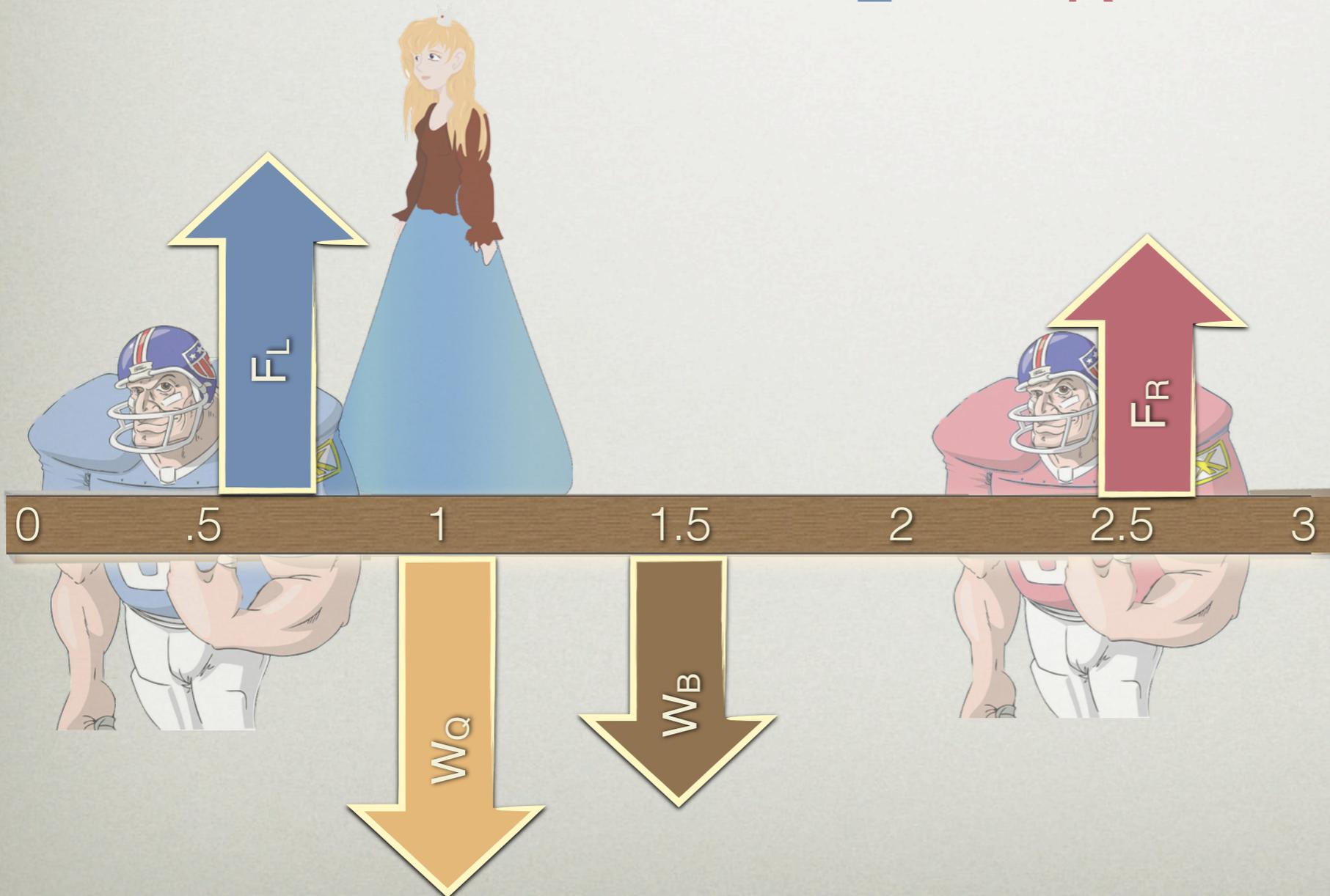
- The 500 N homecoming queen stands 1 m from the end of a 3 m, 350 N board as shown.
- Two players stand 0.5 m from each end. Find all Forces!



# Vertical (y direction) Forces

$$\underline{\text{Up} = \text{Down}}$$

$$F_L + F_R = 500 + 350$$

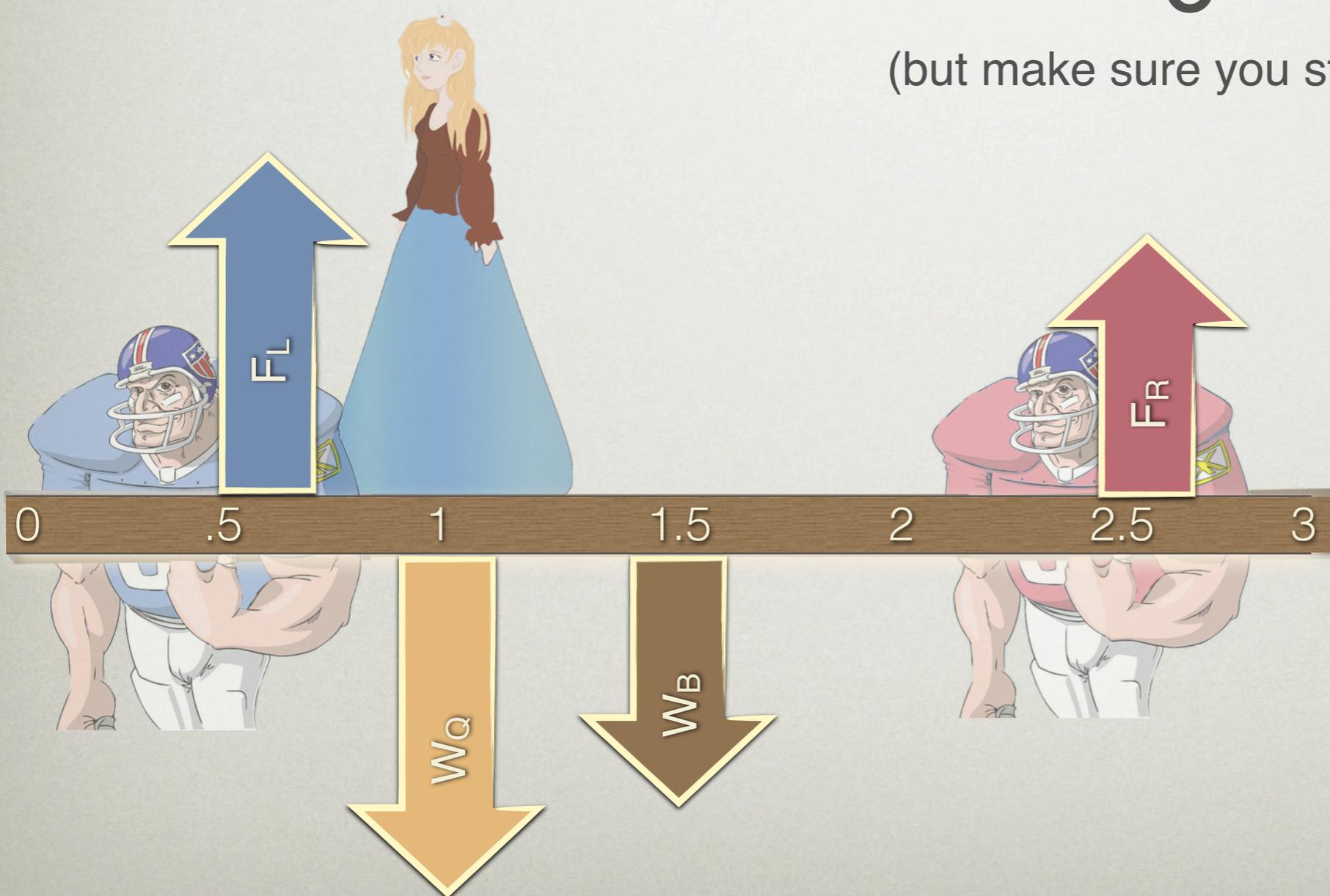


# Horizontal (x direction) Forces

Left = Right

$$0 = 0$$

(but make sure you state this anyway)

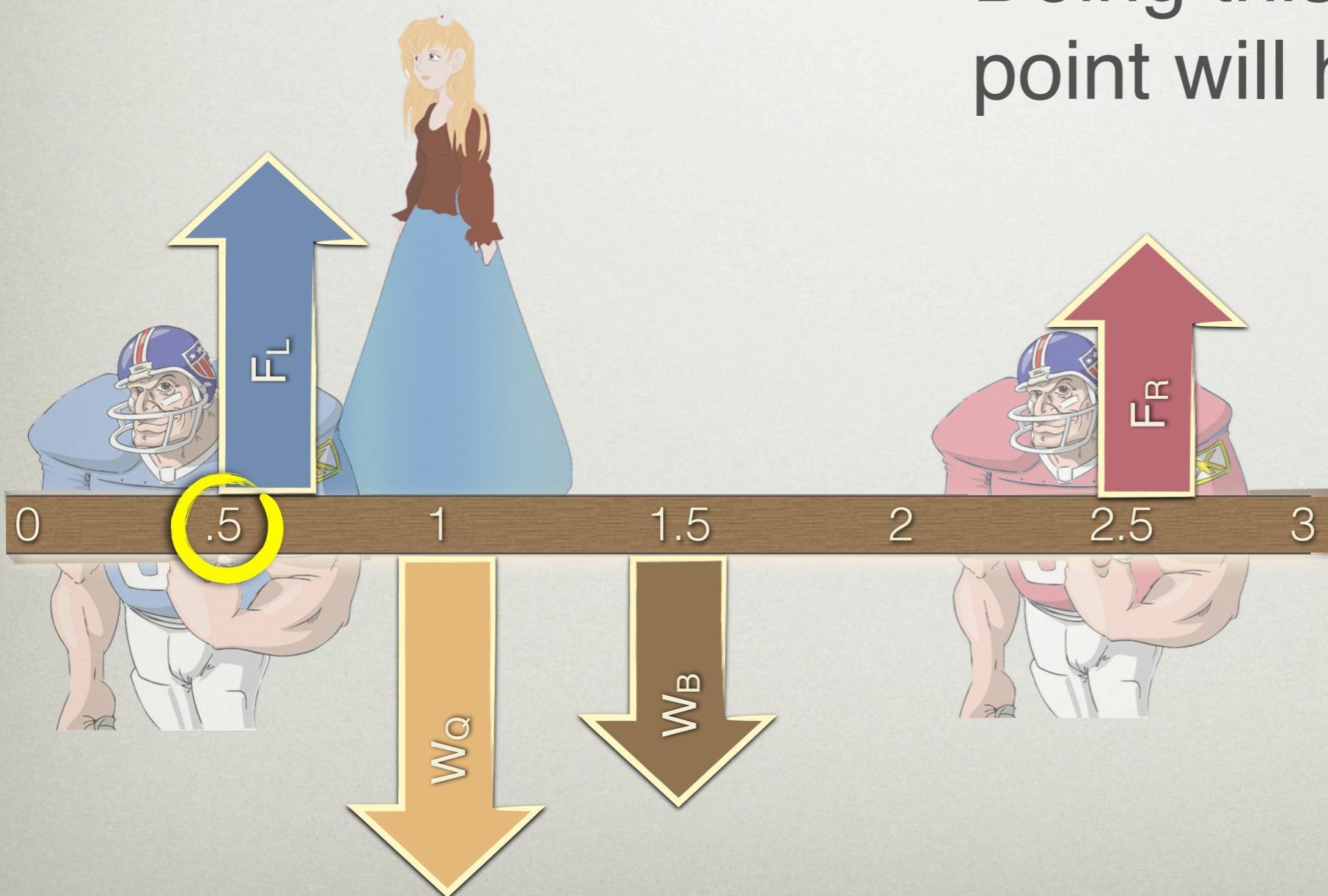


# Torque- *Turning Forces*

pick a point where it is not turning

Pick any Axis

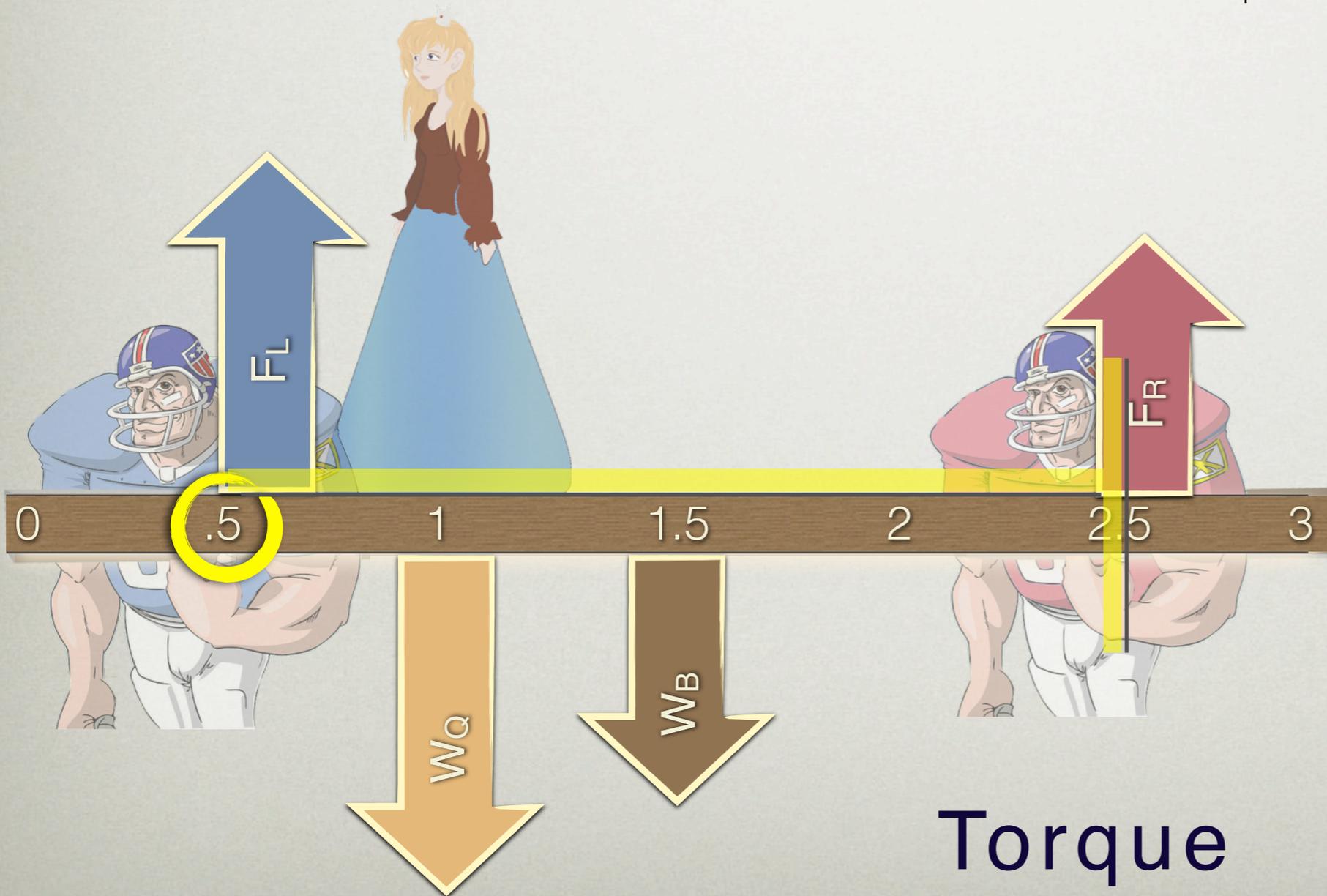
Doing this at a variable point will help



Start at one end

Player R

$$\frac{CCW}{F_R(2)\sin 90} = \frac{CW}{}$$



Torque  $T = F d \sin \theta$

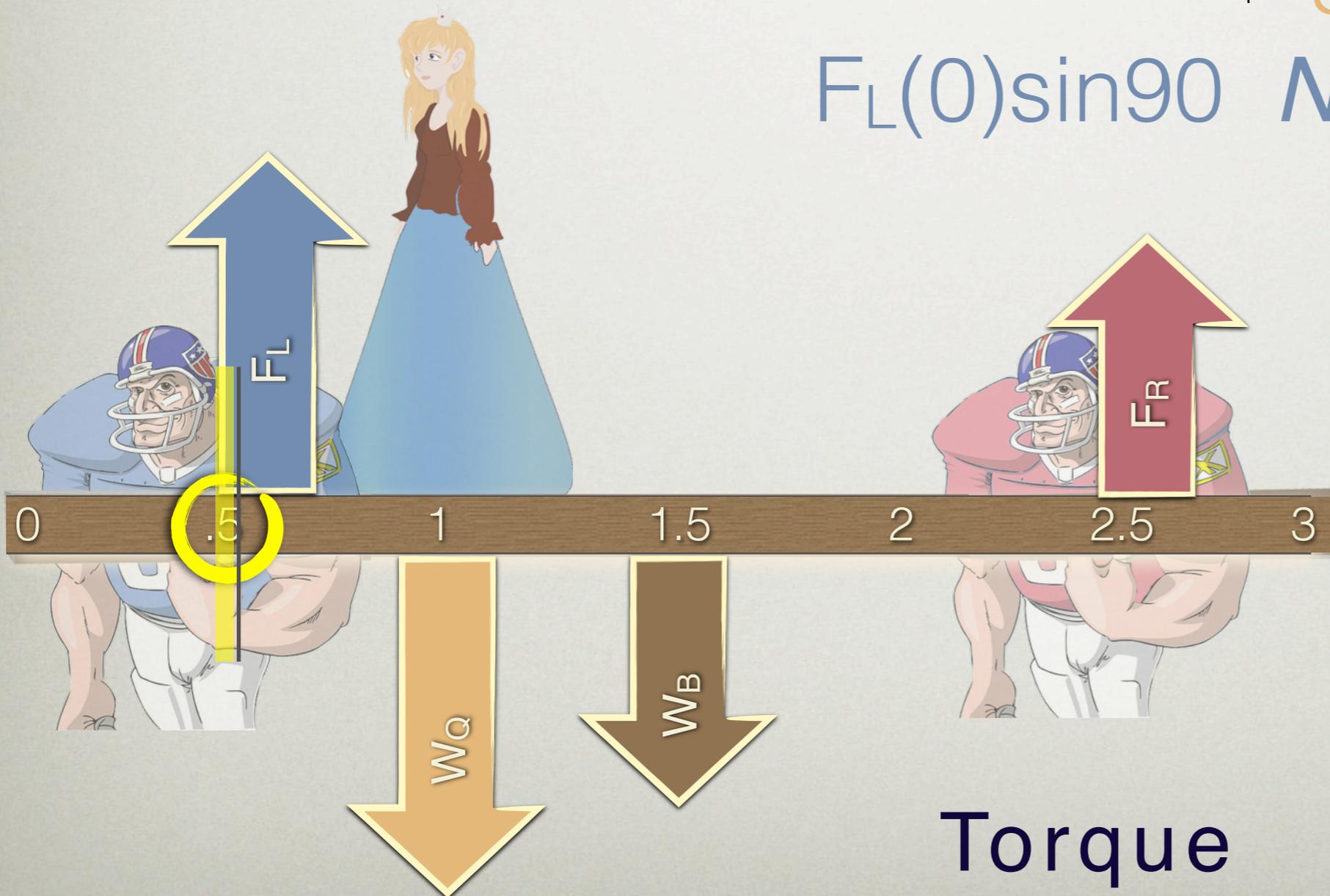
Player L

$$\underline{CCW} = \underline{CW}$$

$$F_R(2)\sin 90$$

$$350(1)\sin 90$$
$$500(0.5)\sin 90$$

$$F_L(0)\sin 90 \quad \text{No Torque}$$



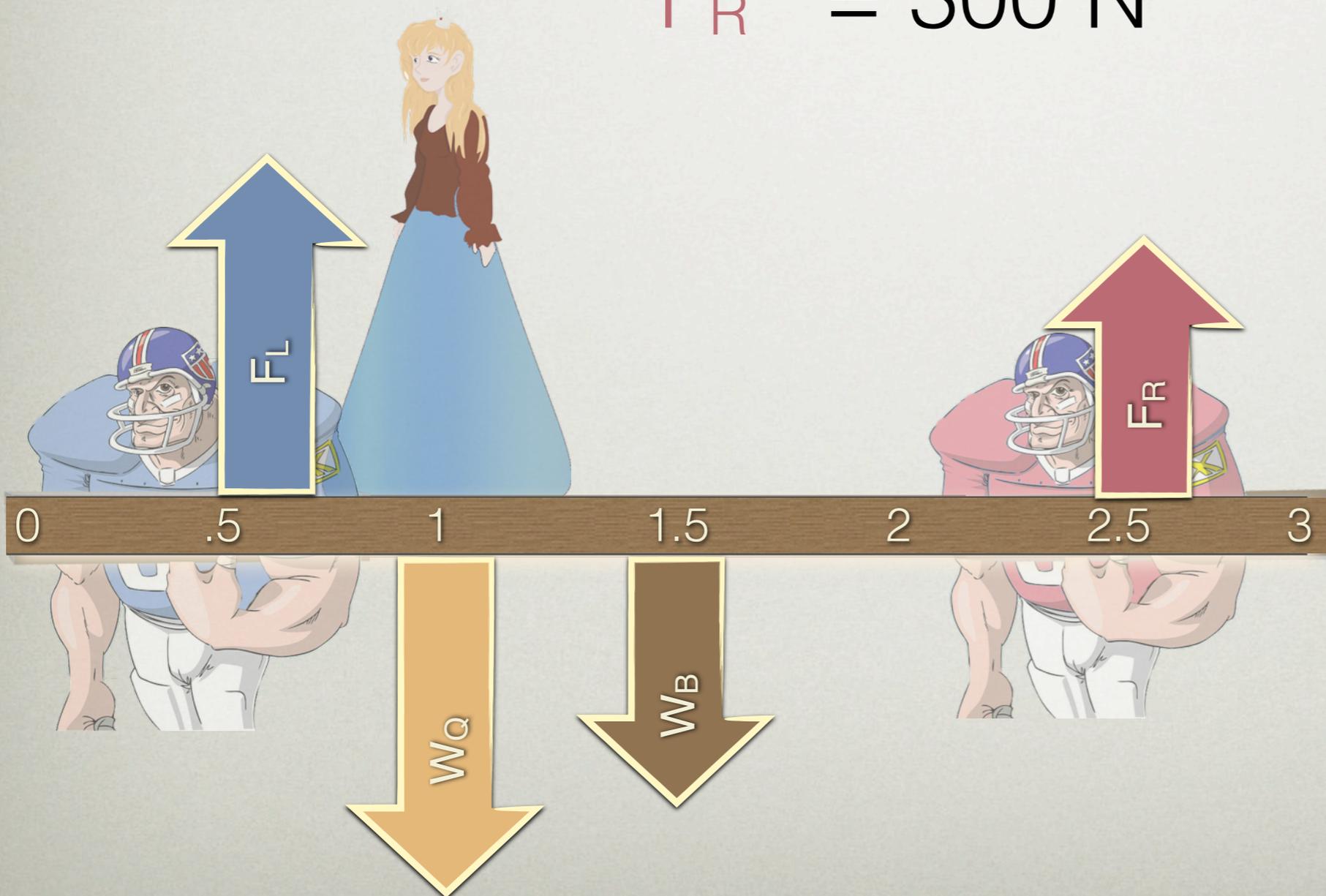
Torque  $T = F d \sin \theta$

$$\underline{CCW} = \underline{CW}$$

$$F_R(2)\sin 90 = 350(1)\sin 90 + 500(0.5)\sin 90$$

$$2 F_R = 350 + 250$$

$$F_R = 300 \text{ N}$$



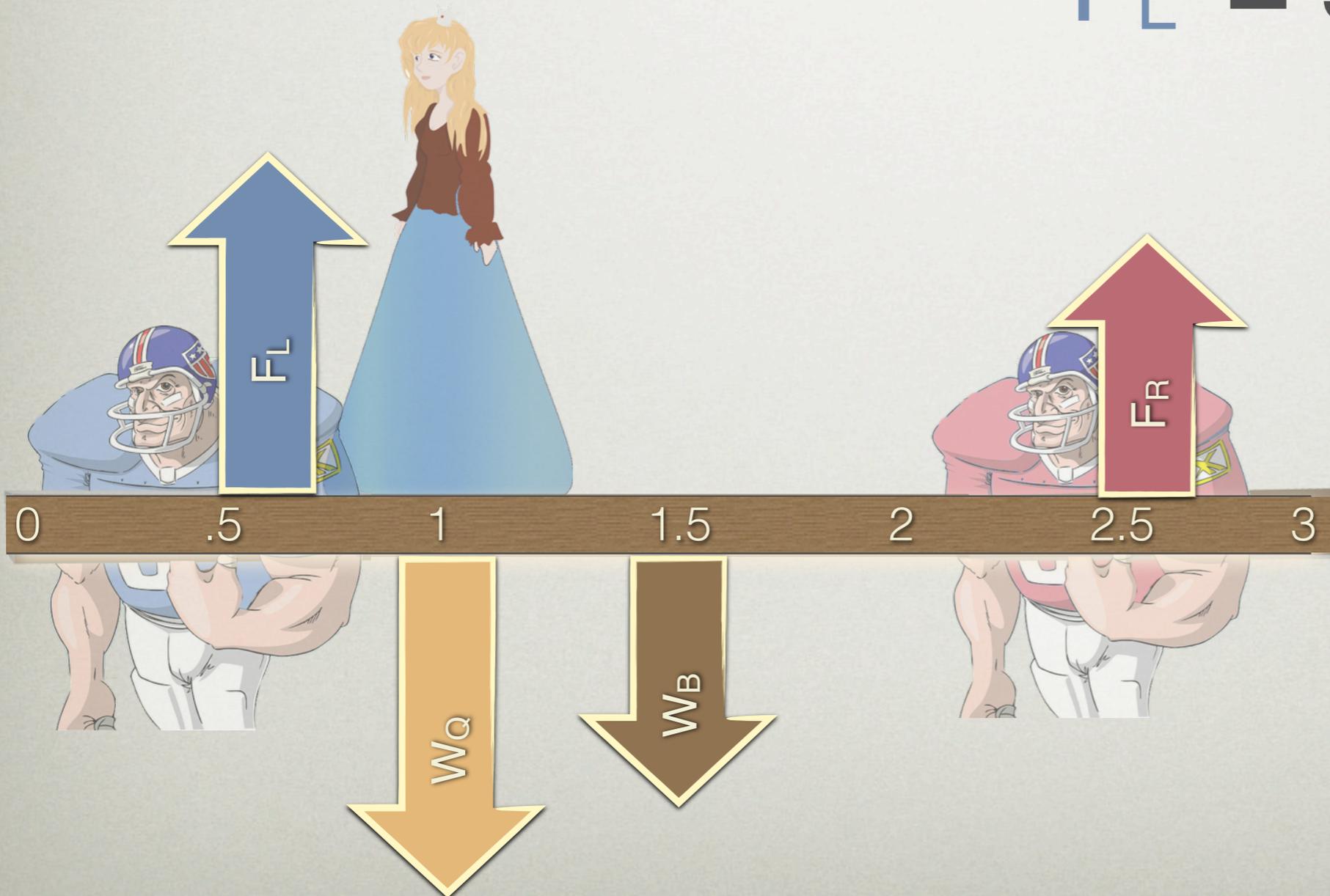
Up = Down

Solve for all variables using the up=down and left=right equations again if needed.

$$F_L + F_R = 500 + 350$$

$$F_L + 300 = 850$$

$$F_L = 550 \text{ N}$$



# Check

If you have picked a good place for your axis of rotation, you should have only simple algebra to handle.

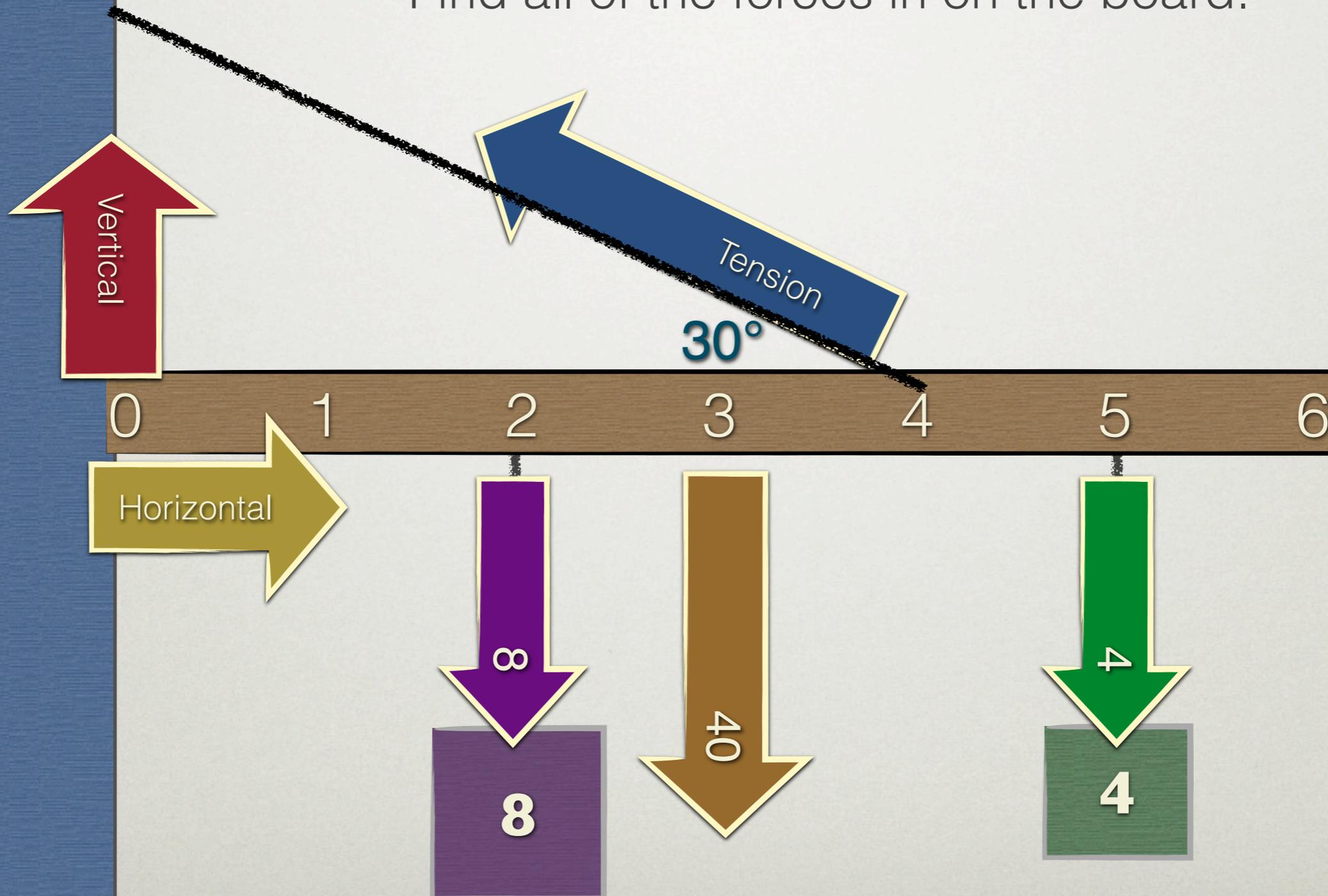
If you have time, pick another axis of rotation and be sure that you get the same answer.

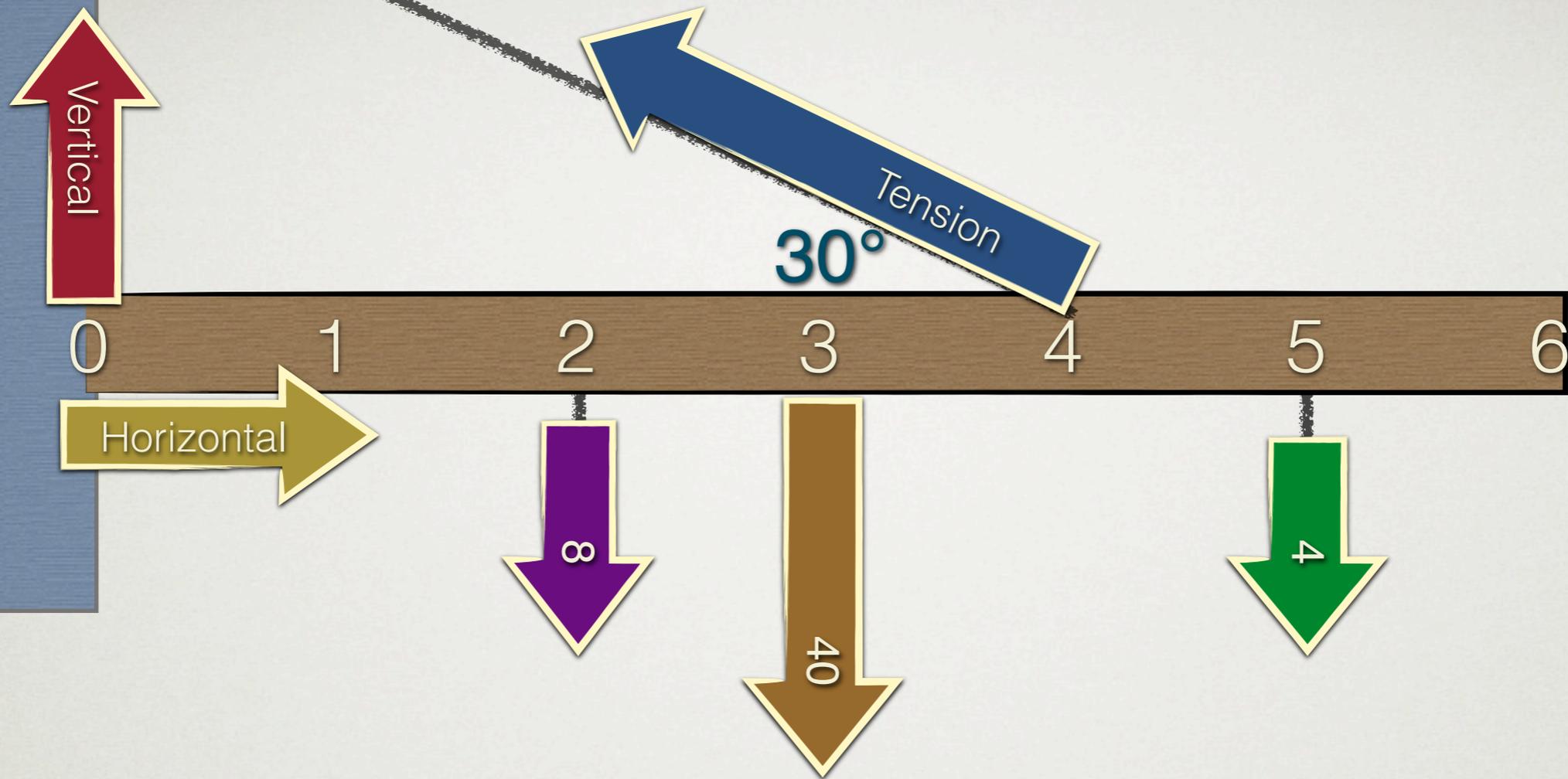
# PART 2 - CRANE BOOMS



A 6 foot, 40 pound board is used to support two blocks as shown. A rope is connected 4 feet from the wall and forms a  $30^\circ$  angle with the board

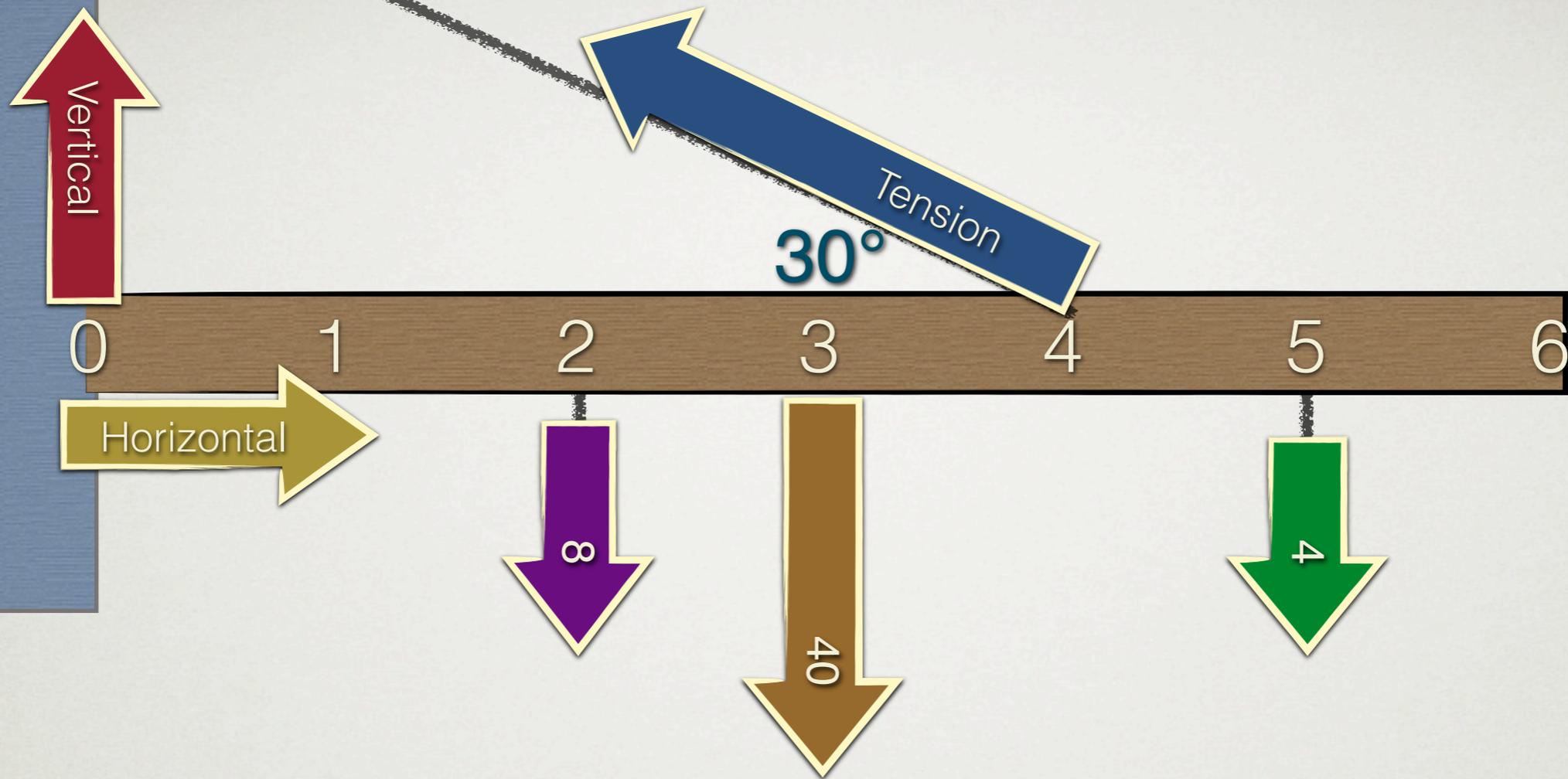
Find all of the forces in on the board.





$$\underline{Up} = \underline{Down}$$

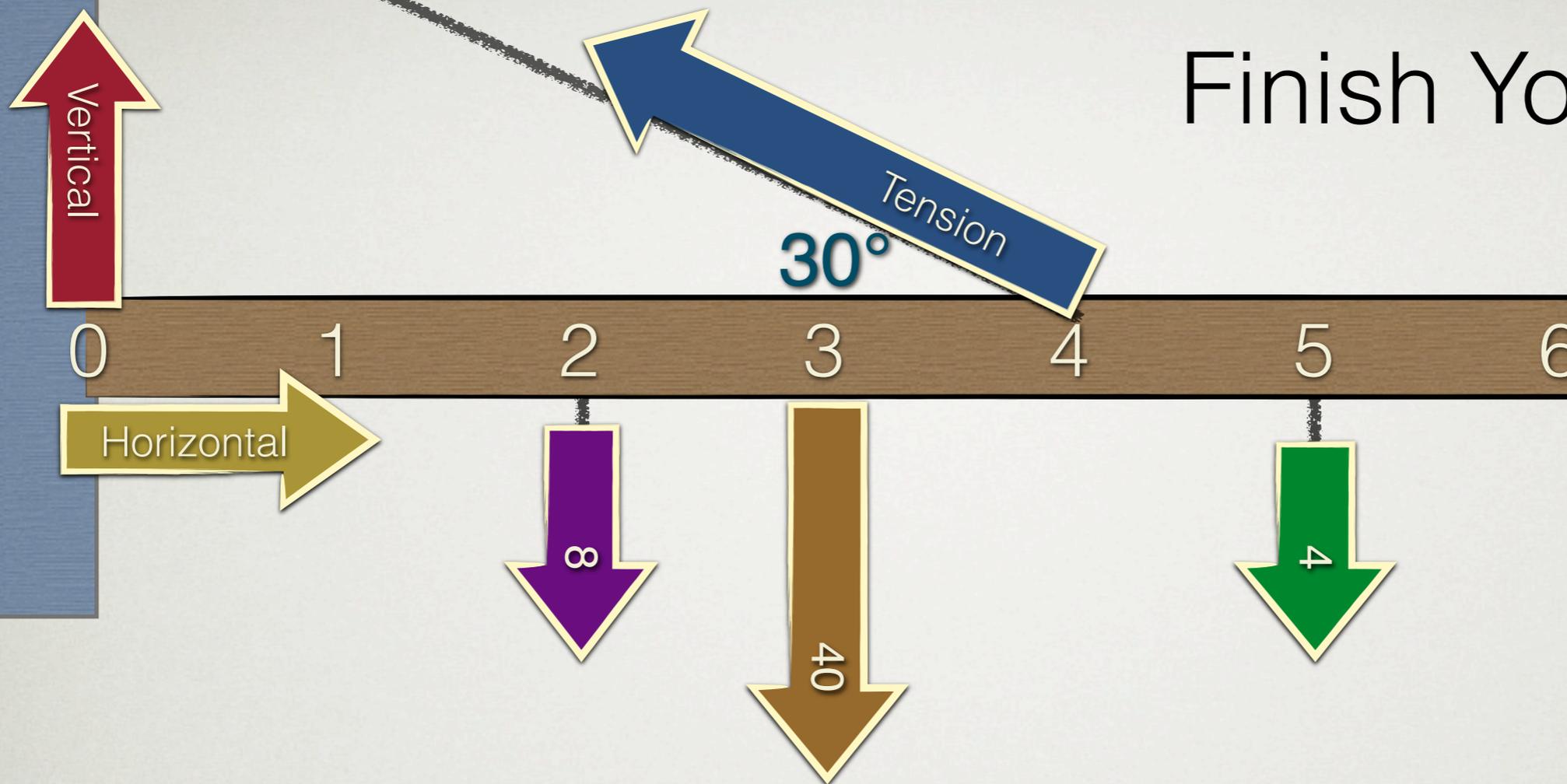
$$V + T \sin 30 = 8 + 40 + 4$$



Left = Right

$$T \cos 30^\circ = H$$

Finish Your Math



$$\underline{CCW = CW}$$

$$T(4)\sin 30 = 4(5)\sin 90 + 40(3)\sin 90 + 8(2)\sin 90$$

$$T(2) = 4(5) + 40(3) + 8(2)$$

$$T = 78 \text{ lb}$$

$$\underline{L = R}$$

$$T \cos 30^\circ = H$$

$$H = 67.5 \text{ lb}$$

$$\underline{U = D}$$

$$V + T \sin 30 = 8 + 40 + 4$$

$$V = 13 \text{ lb}$$

THE OBJECT CREATES THE  
ANGLE

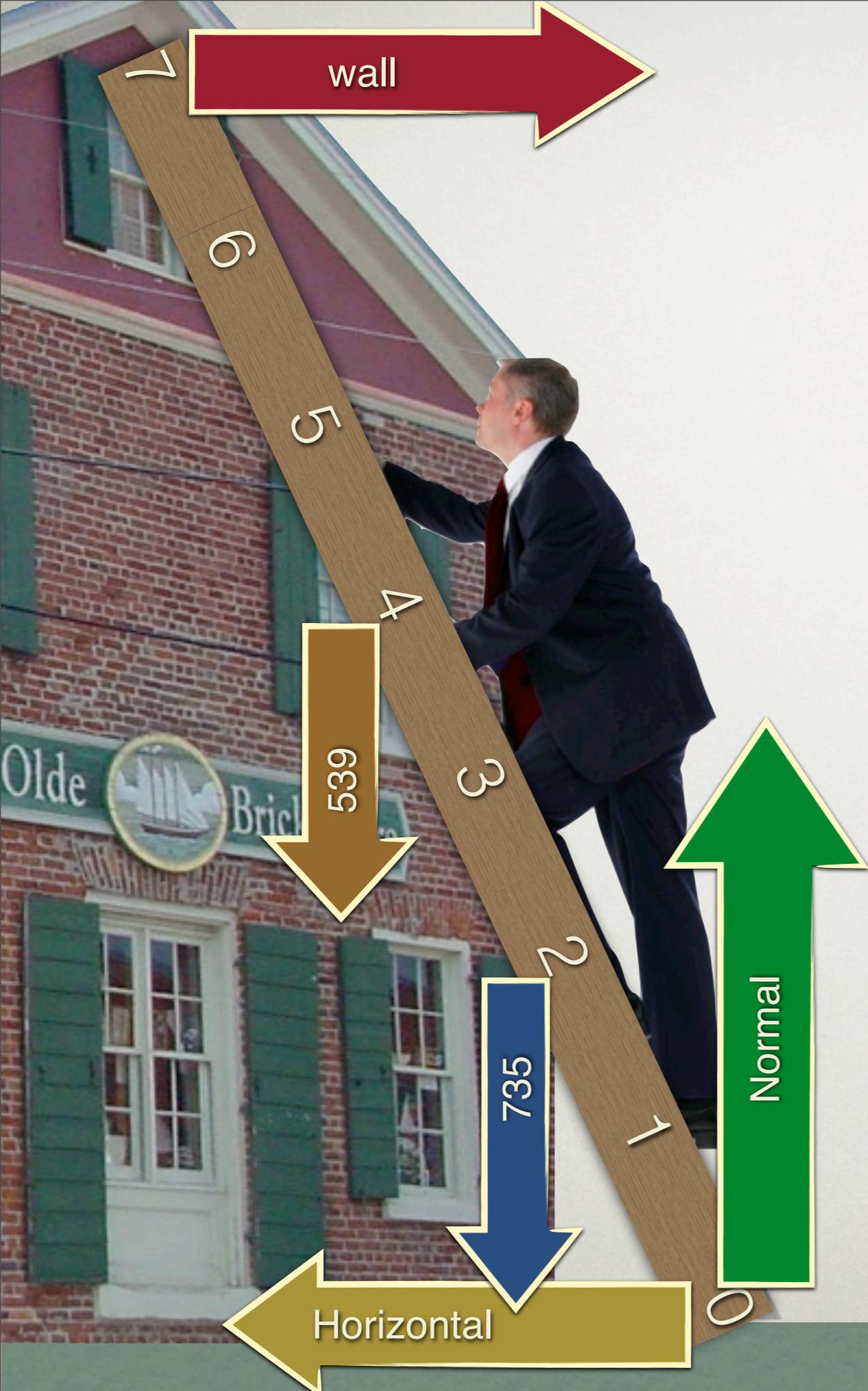
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A LADDER PROBLEM



# Climbing a Ladder Find all Forces

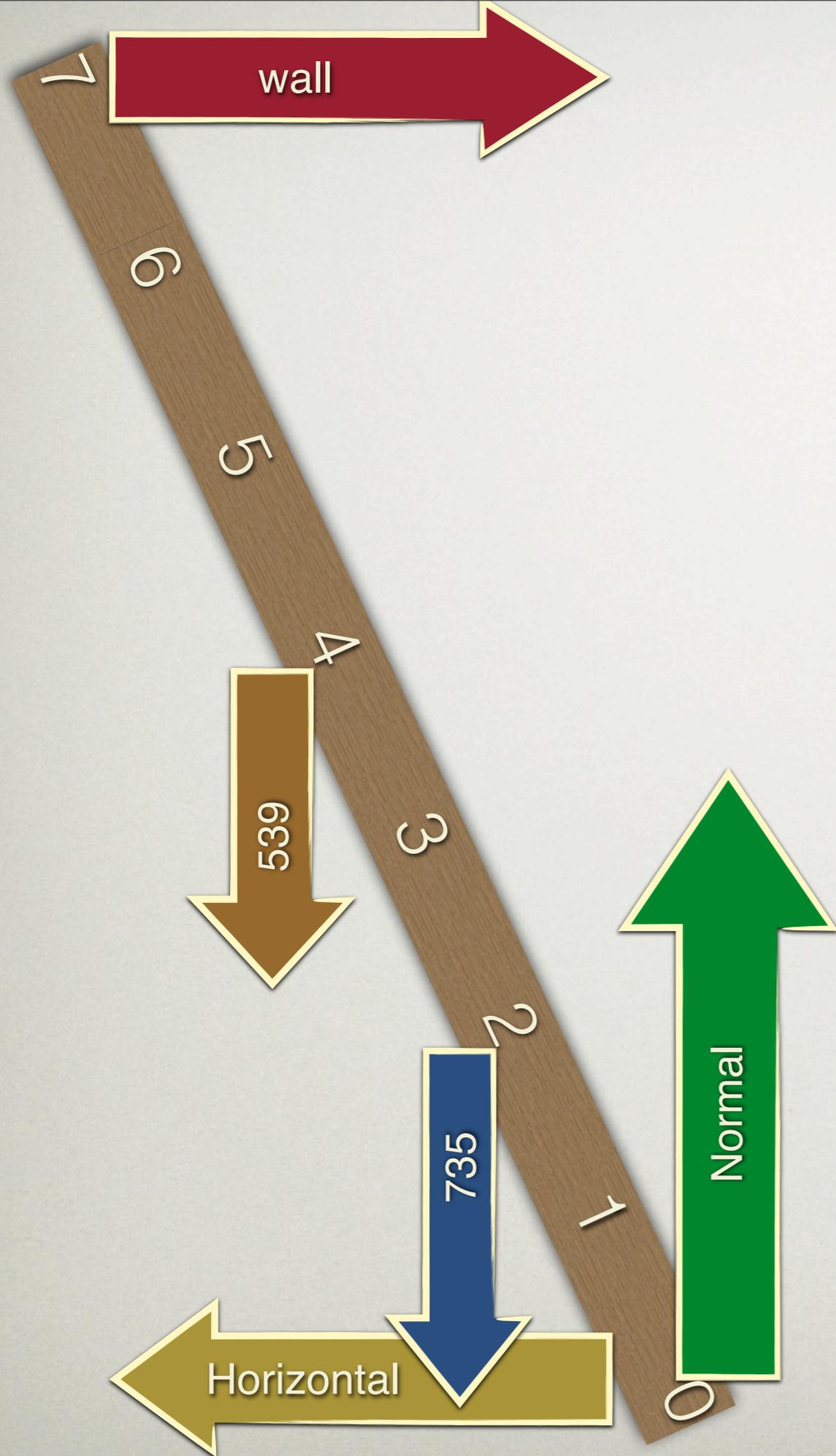
A 75 kg man has climbed 2 meters up a 7 meter ladder. The ladder has a mass of 55kg, and it forms a  $60^\circ$  angle with the ground. Find all forces.



# Climbing a Ladder

## Find all Forces

- Weight of Ladder
  - $55 \times 9.8$
- Weight of Climber
  - $75 \times 9.8$
- Normal
  - “Wall”
- Horizontal



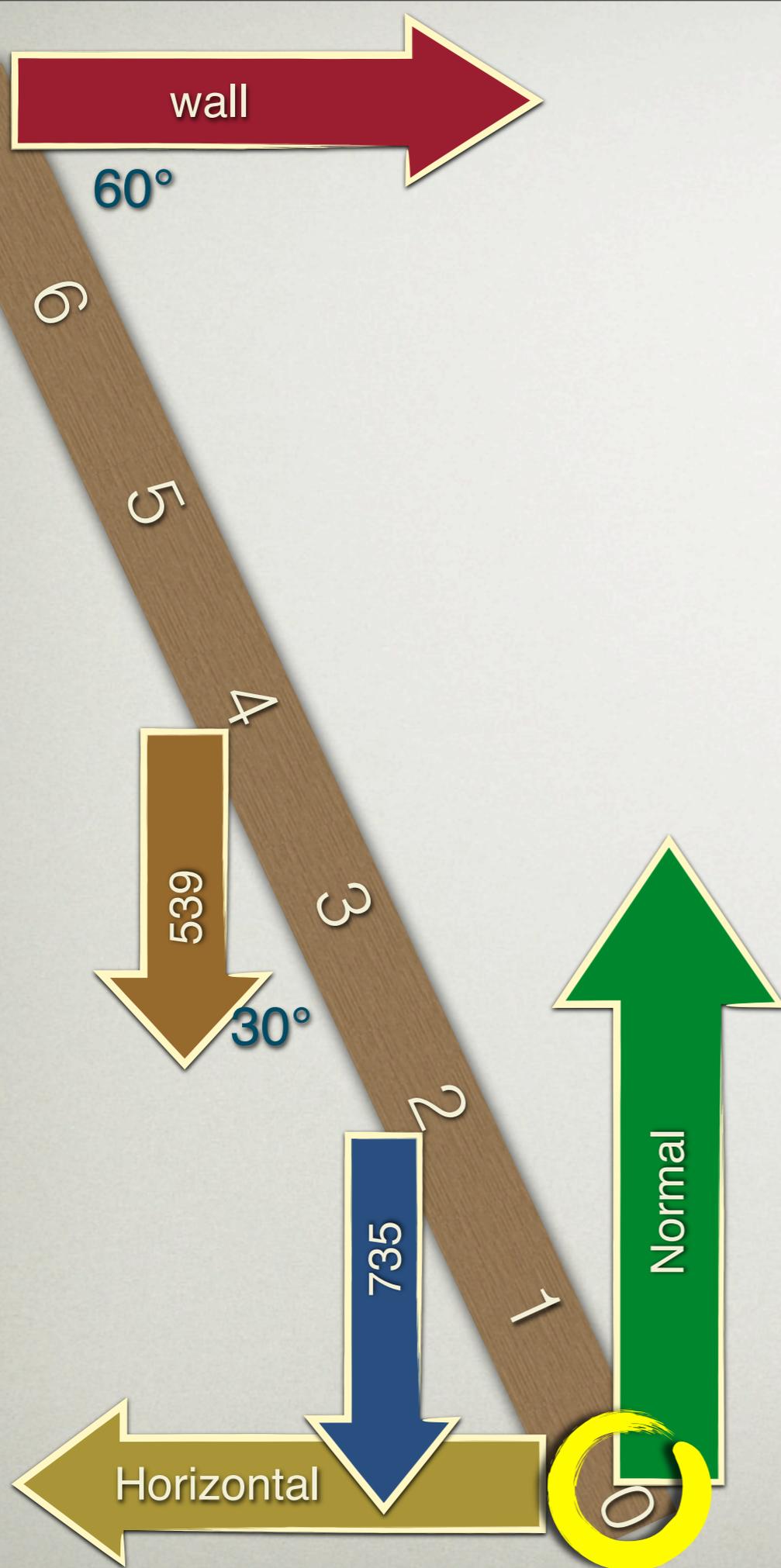
$$\underline{U = D}$$

$$N = 735 + 539$$

$$N = 1274 \text{ N}$$

$$\underline{L = R}$$

$$H = W$$



$$\underline{CW} = \underline{CCW}$$

$$W(7)(\sin 60)$$

$$539(3.5)(\sin 30)$$

$$735(2)(\sin 30)$$

$$W = 276.8 \text{ N}$$

Now finish your math

$$H = W$$

$$H = 276.8 \text{ N}$$

And Maybe...

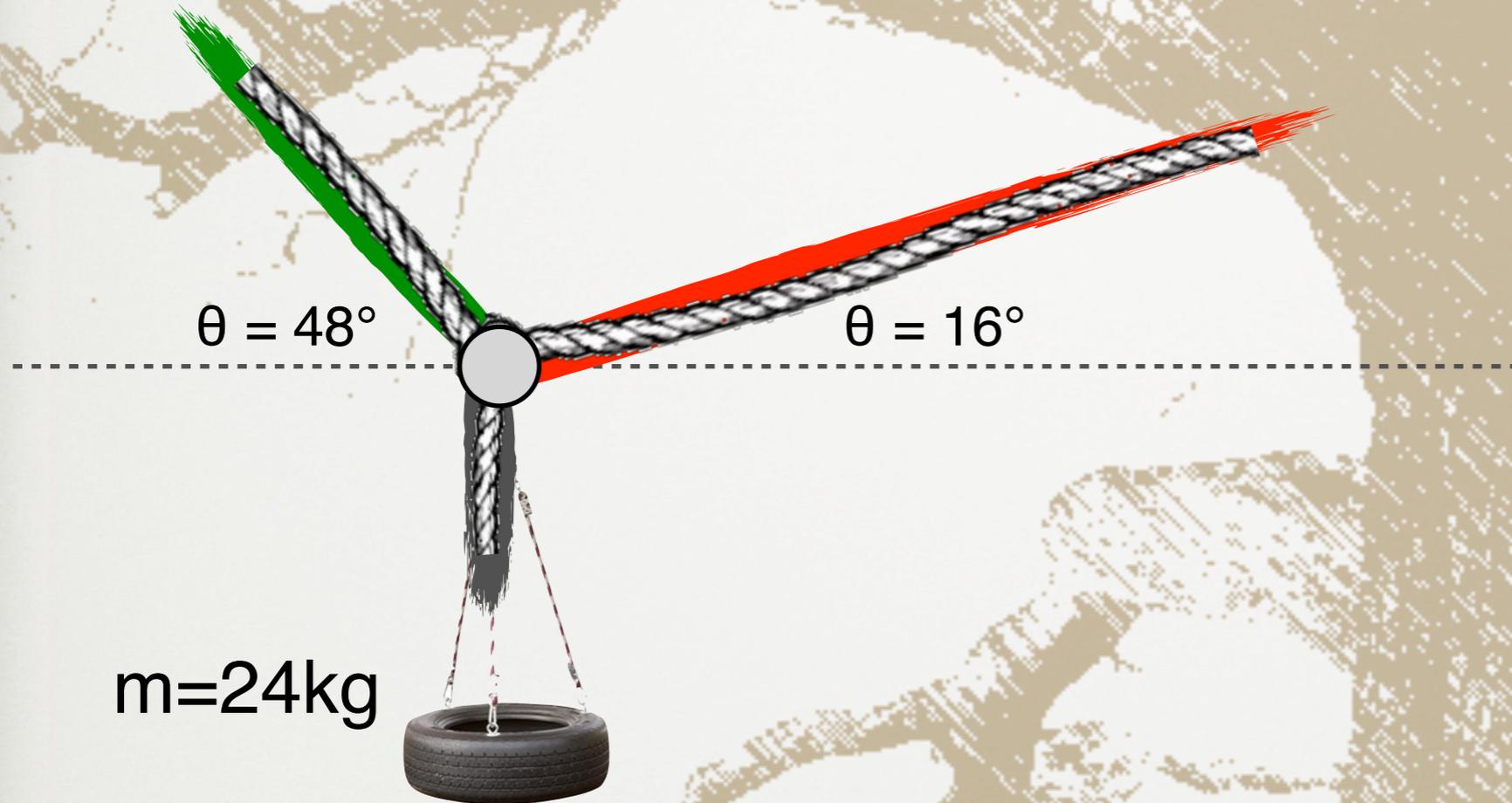
$$F = \mu N$$

$$276.84 = \mu 1274$$

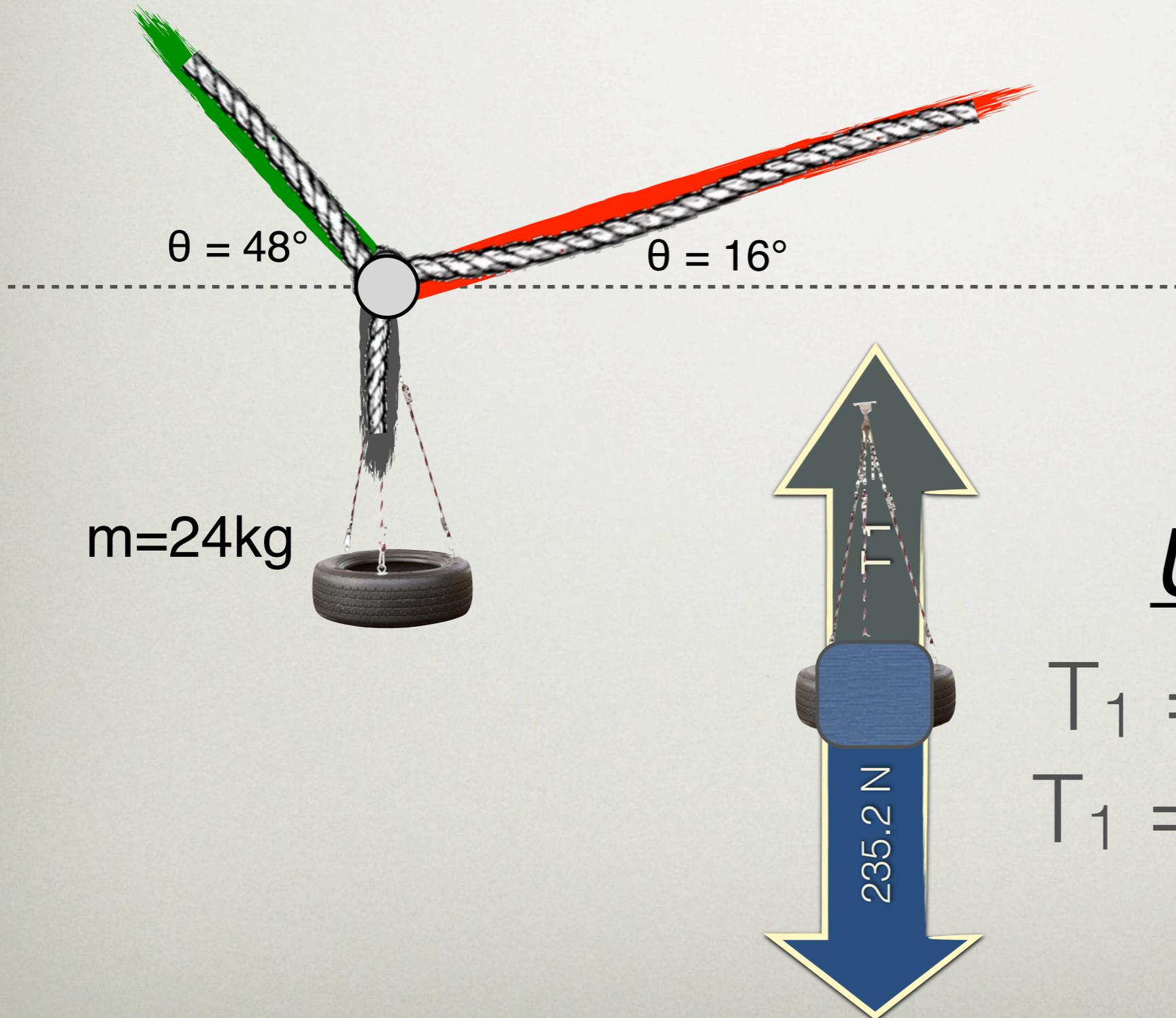
# EQUILIBRIUM WITH ANGLES

## THE ROPES

# Find all Forces



First, isolate the tire

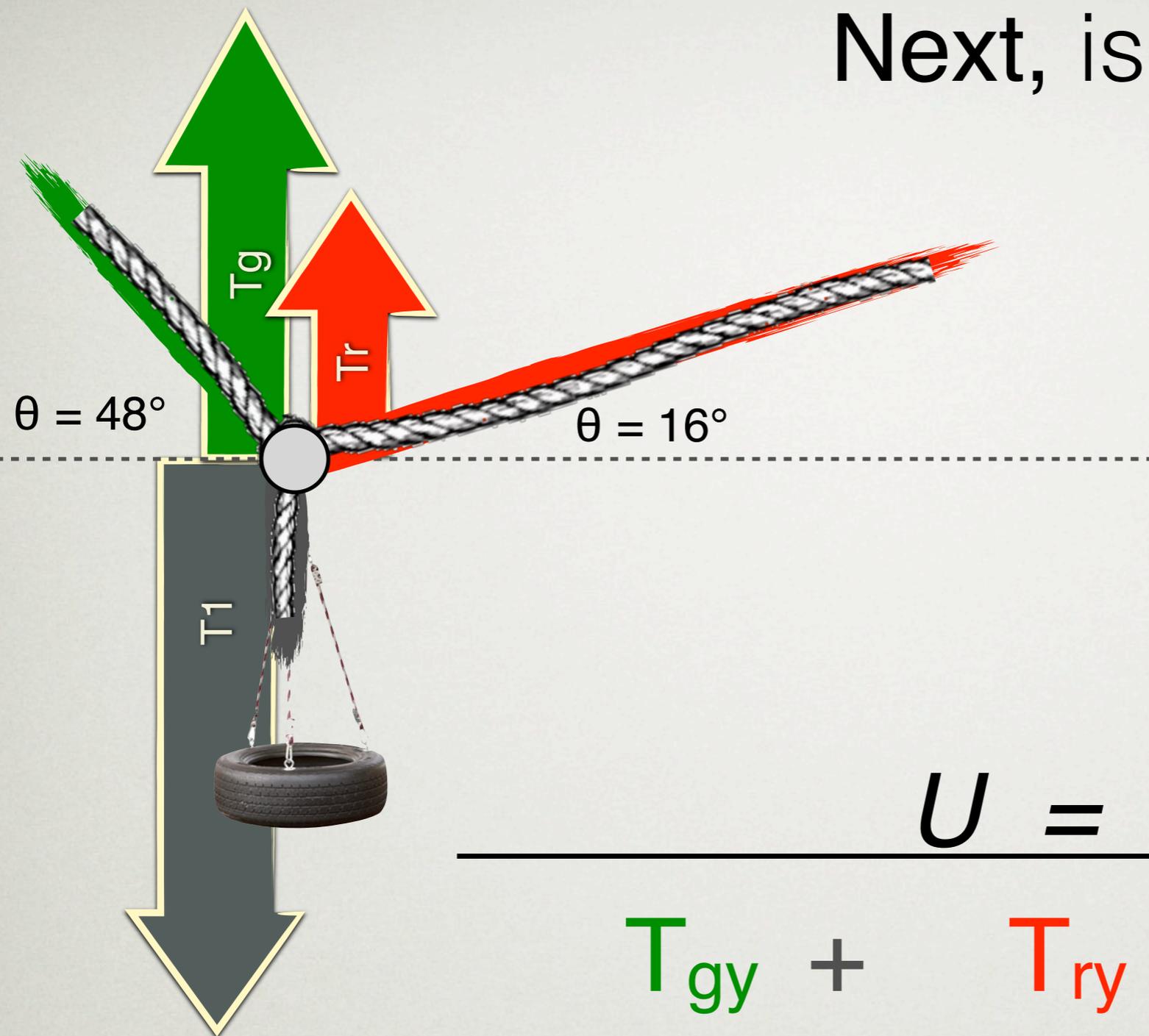


$$\underline{U = D}$$

$$T_1 = 24(9.8)$$

$$T_1 = 235.2\text{ N}$$

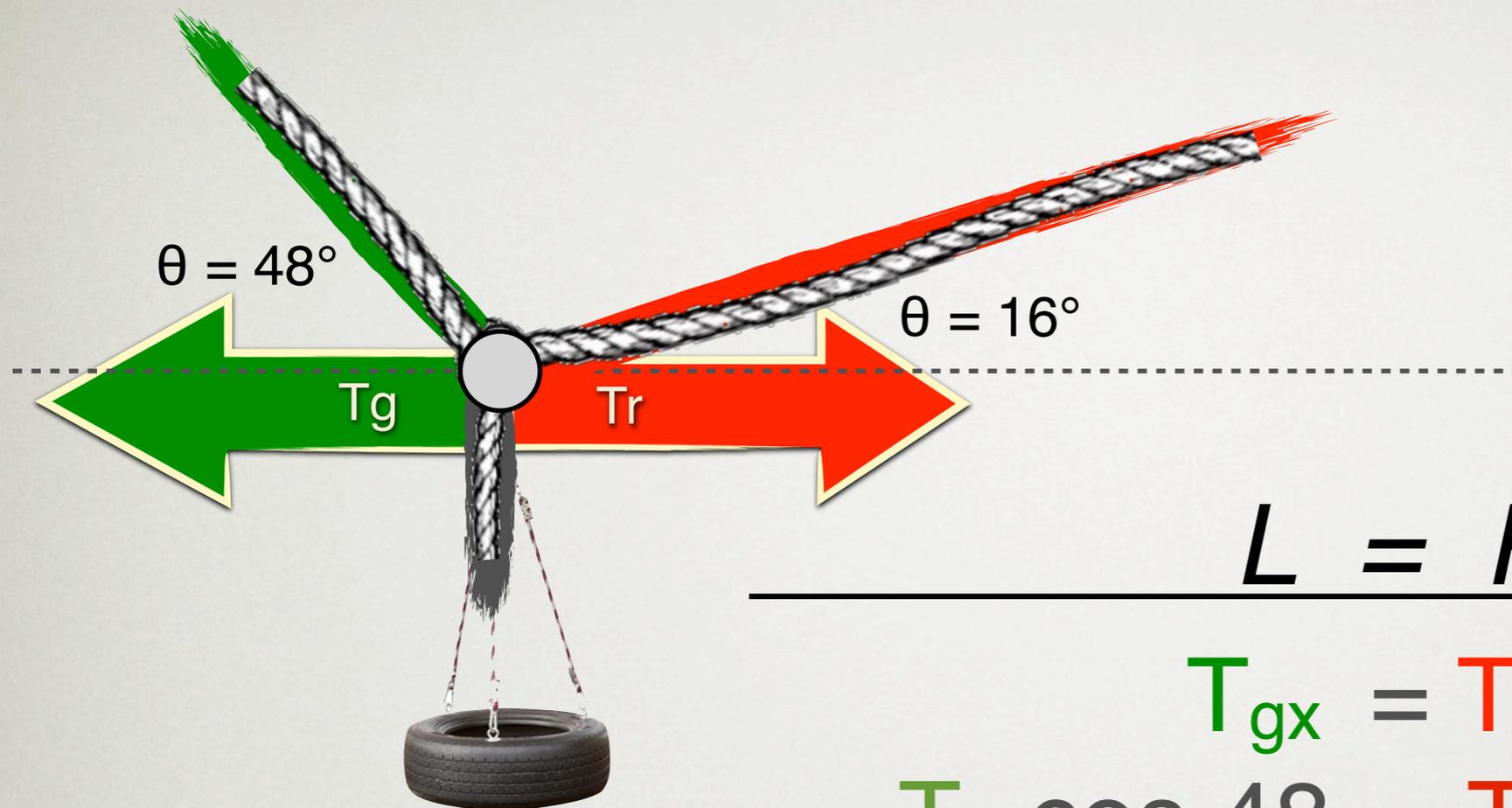
Next, isolate the “knot”



$$U = D$$

$$T_{gy} + T_{ry} = T_1$$
$$T_g \sin 48 + T_r \sin 16 = 235.2 \text{ N}$$

$$T_g = 316.49 - 0.371(T_r)$$



---


$$L = R$$

$$T_{gx} = T_{rx}$$

$$T_g \cos 48 = T_r \cos 16$$

from  $u=d$ :

$$T_g = 316.49 - 0.371(T_r)$$

$$211.77 - 0.248 T_r = 0.961 T_r$$

$$211.77 = 1.209 T_r$$

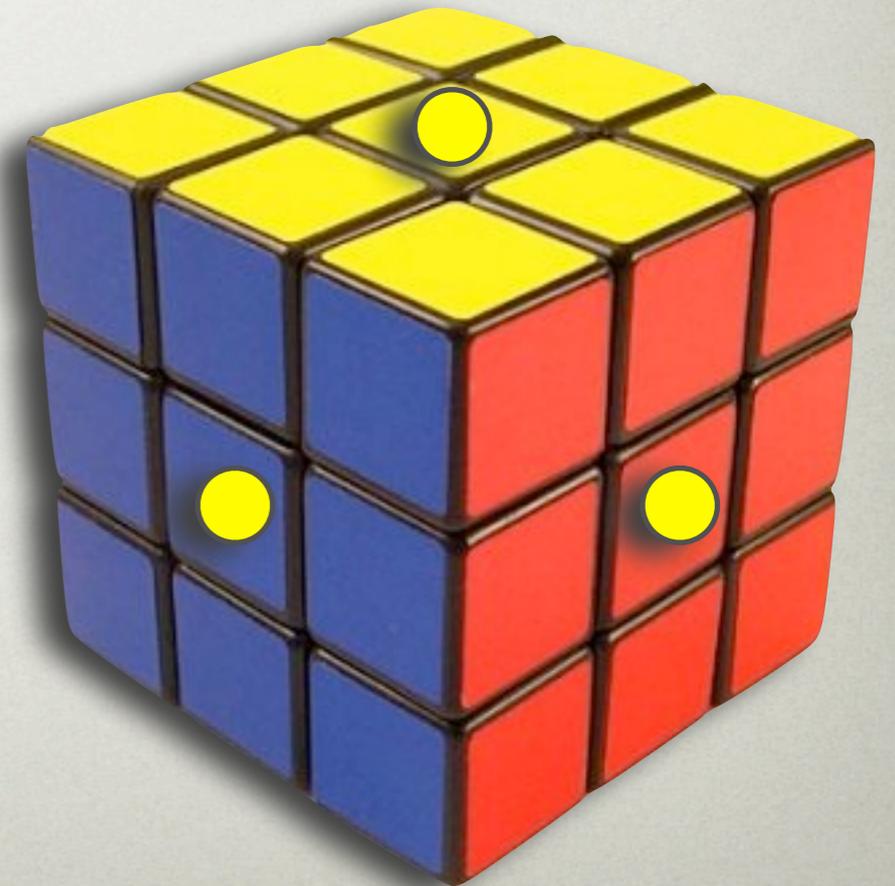
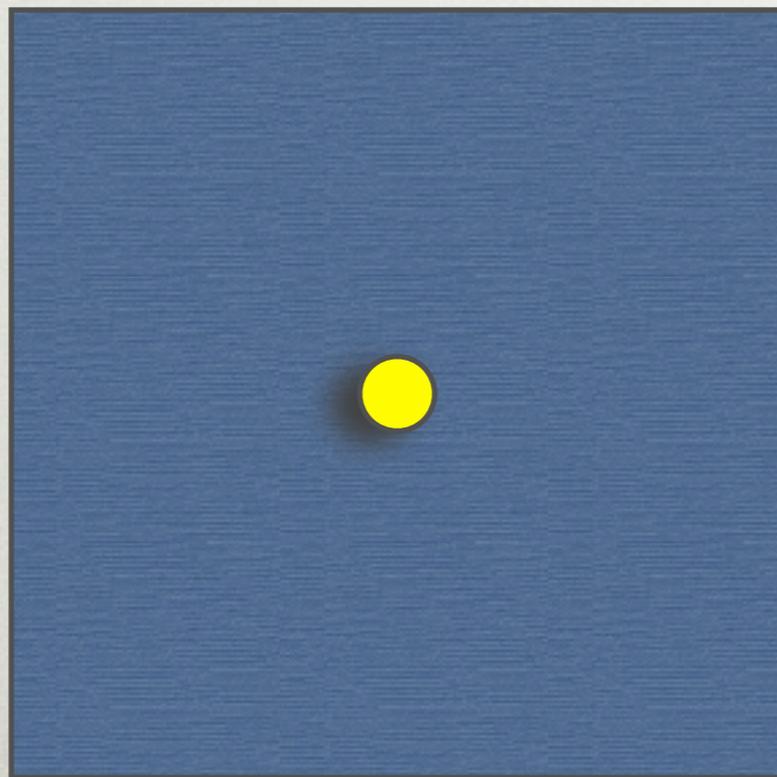
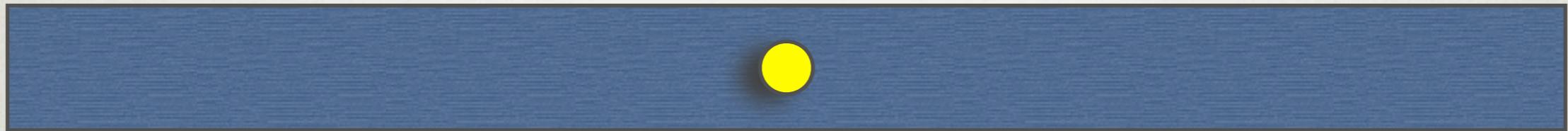
$$T_r = 175.1 \text{ N}$$

$$\therefore T_g = 251.5 \text{ N}$$

# CENTER OF MASS

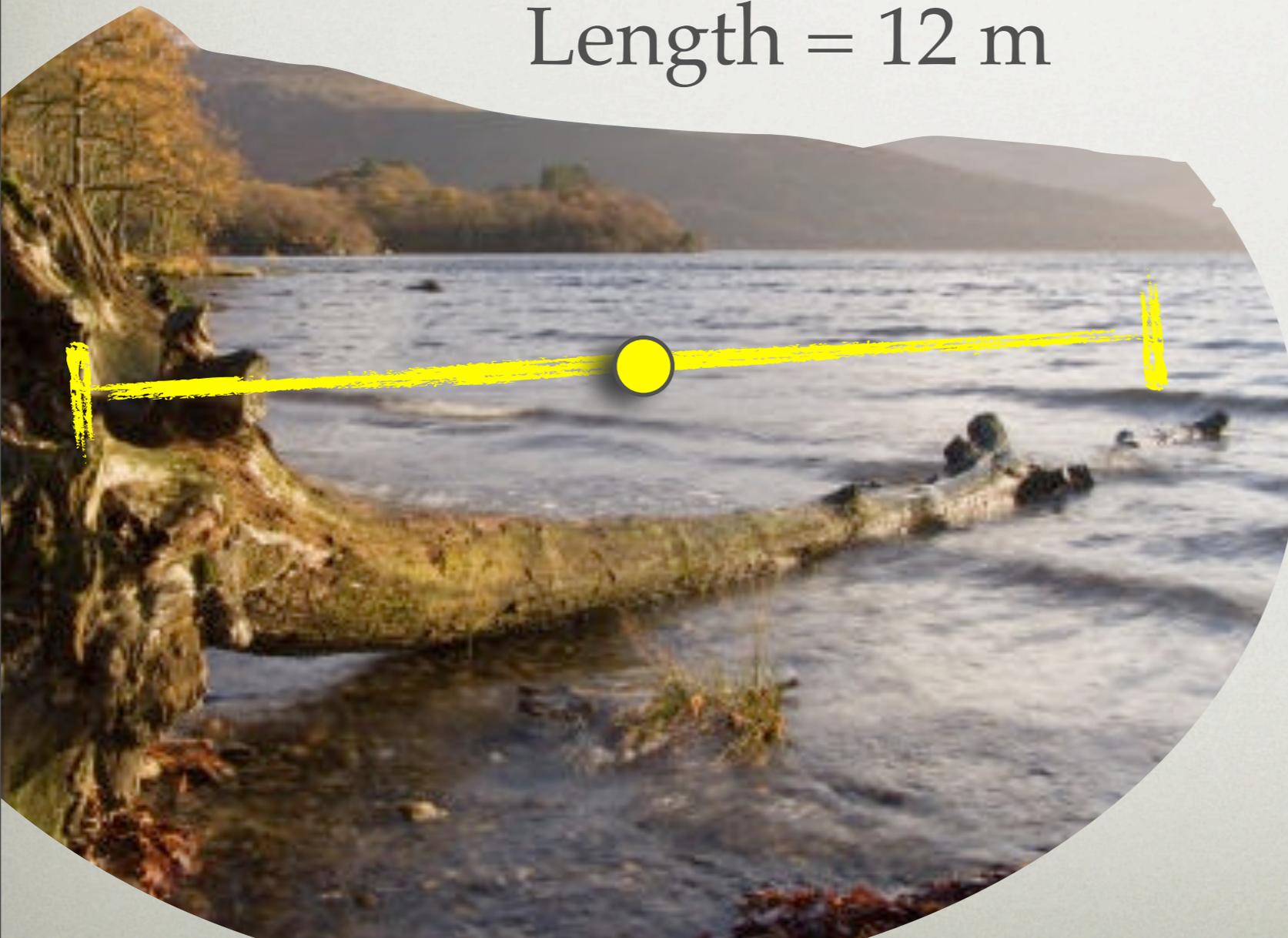
# Find the Center

*Easy sometimes*



# Find the Center

Length = 12 m



- Where is the center of mass for this tree?
- half the length?

# Find the Center



$$x_{cm} = \frac{x_1 m_1 + x_2 m_2 + x_3 m_3}{m_{total}}$$

# Find the Center



$$x_{cm} = \frac{15 \times 1 + 25 \times 6 + 50 \times 14}{15 + 25 + 50}$$

Find the  
Center

20 g

30 g

40 g

20 g

30 g

$$x_{cm} = 6.4 \text{ cm}$$

$x_{cm}$

=

$$20 \times 0$$

+

$$30 \times 10$$

+

$$40 \times 7$$

$$20 + 30 + 40$$

40 g

20 g

30 g

$$x_{cm} = 6.4 \text{ cm}$$

$$y_{cm} = 6.56 \text{ cm}$$

$$20 \times 10$$

+

$$30 \times 13$$

+

$$40 \times 0$$

$y_{cm}$

=

$$20 + 30 + 40$$

40 g