p=mv

m = mass (kg)

v = velocity (m/s)

p = Momentum

- “the quantity of motion”
- kg m/s Not a combined unit

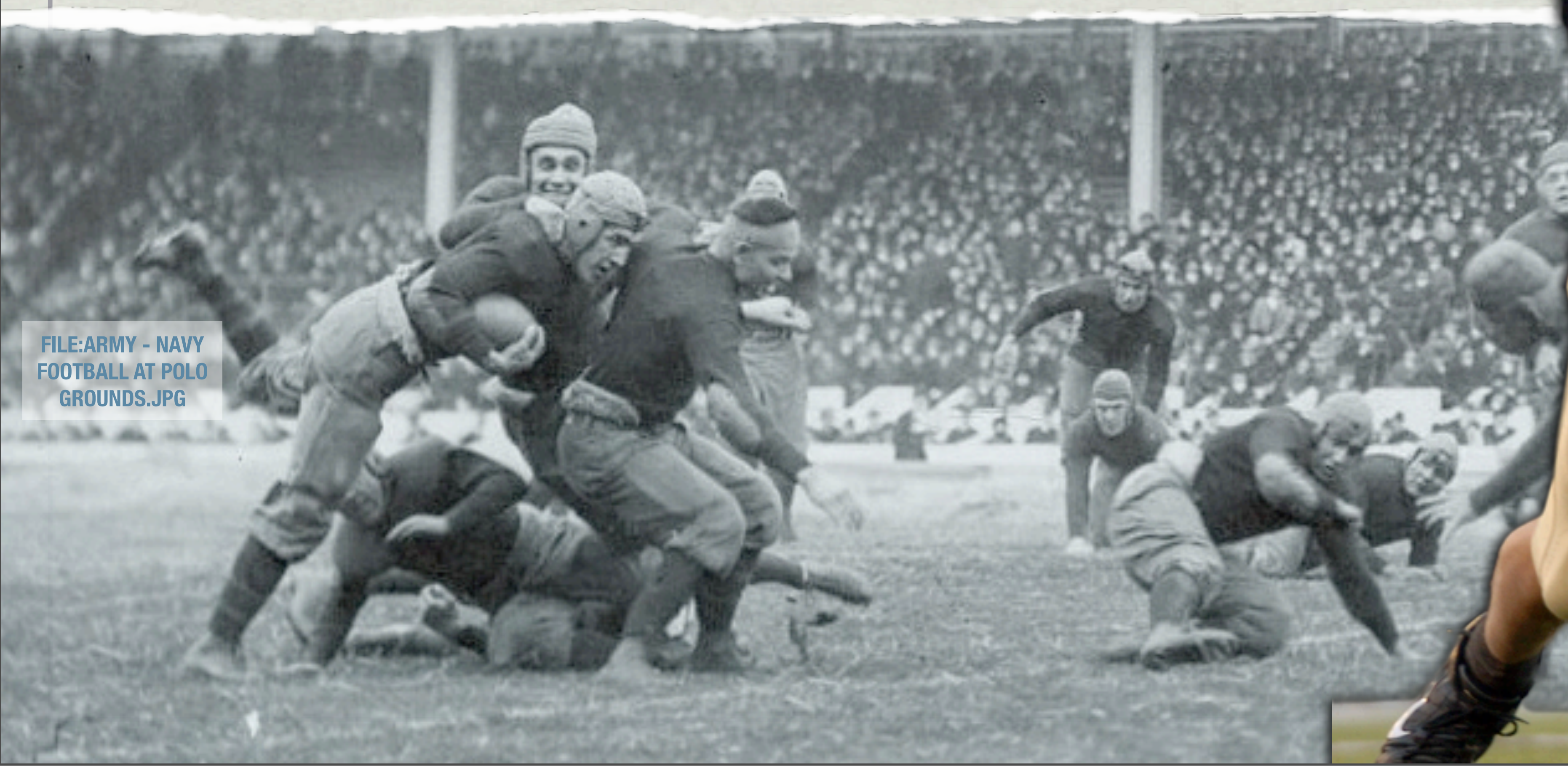
Conservation of Momentum

- In a closed system, the total momentum will remain a constant.

$$p_i = p_f$$

Perfectly Inelastic Collisions

- In the inelastic collision the two objects have the same final velocity

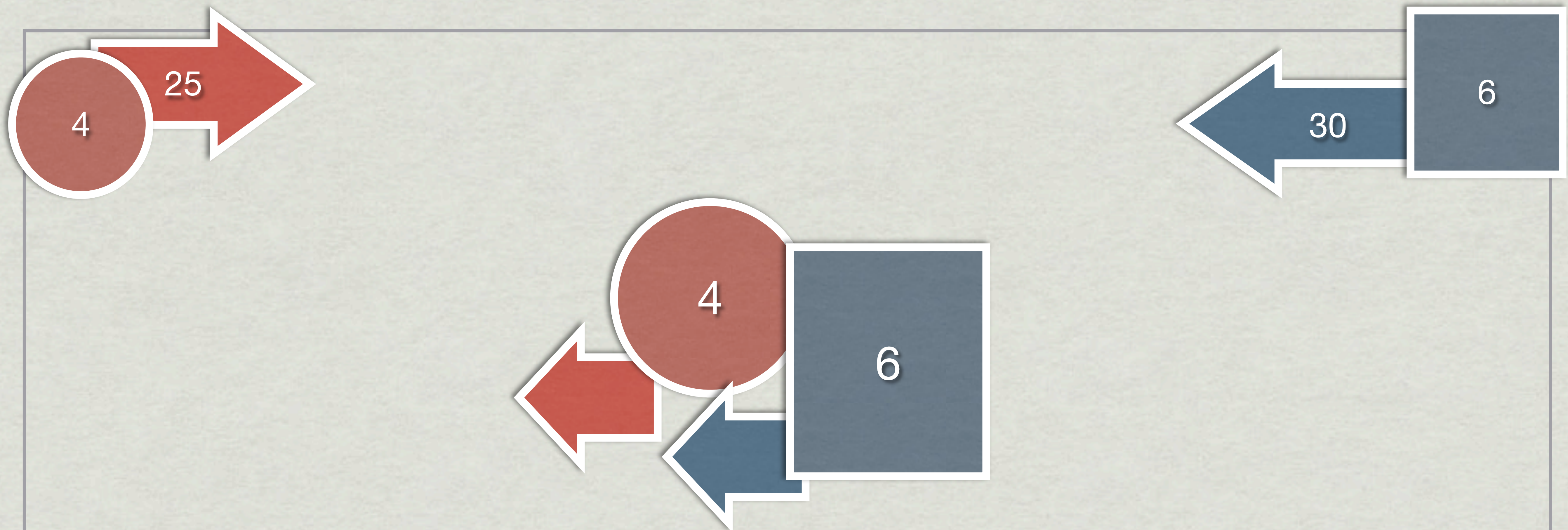


Example 1

Find the Final Velocity:
Perfectly Inelastic Collision

Mass A :
$V_i = 25$ m/s
m = 4 kg

Mass B :
$V_i = -30$ m/s
m = 6 kg



From Conservation of Momentum

$$4(25) + 6(-30) = 4\,(V_{af}) + 6\,(V_{bf})$$

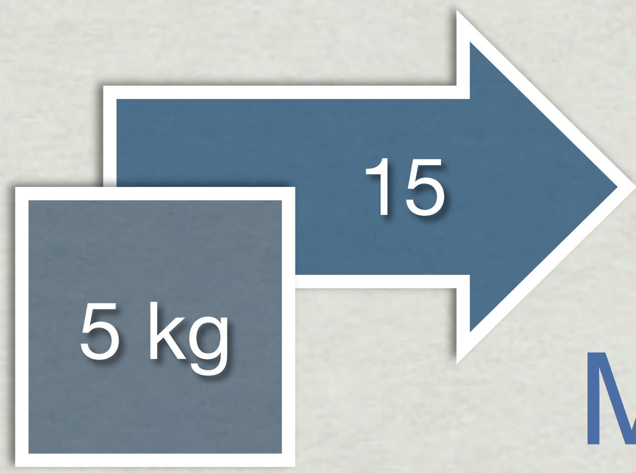
$$100 + (-180) = 10\,V_f$$

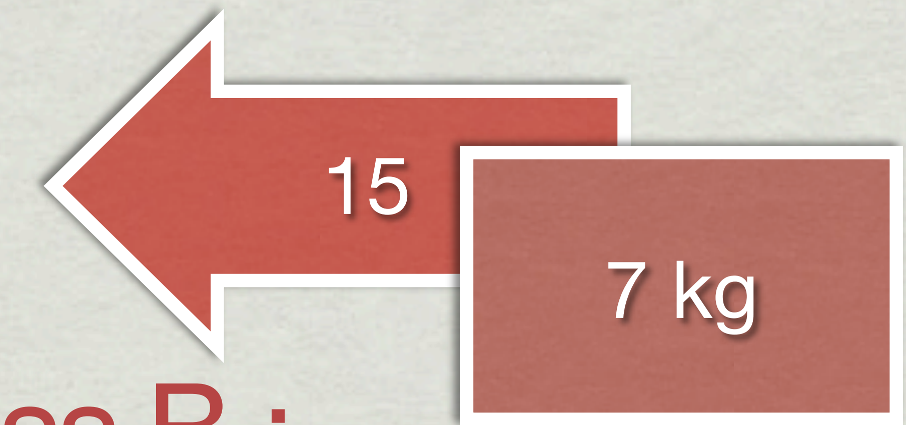
$$-80 = 10\,V_f$$

$$V_f = -8 \text{ m/s}$$

Example 2

- Find the Final Velocity of 1 car in an inelastic collision. They do not stick together, but one of the final velocities is known.





Mass A :

$V_i = 10$ m/s

$m = 5$ kg

Mass B :

$V_i = -15$ m/s

$m = 7$ kg

$V_f = 4$ m/s

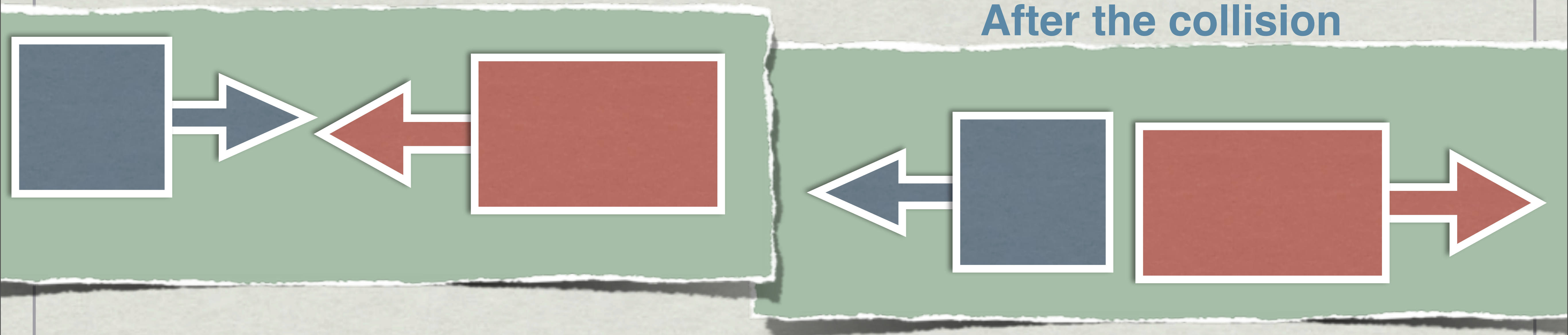
Mass A :
$V_i = 10$ m/s
$m = 5$ kg

Mass B :
$V_i = -15$ m/s
$m = 7$ kg

Mass B :
$m = 7$ kg
$V_f = 4$ m/s

Before the collision

After the collision



From Conservation of Momentum

$$5\,(10) + 7\,(-15) = 5\,V_{af} + 7\,(4)$$

$$50 + (-105) = 5\,V_{af} + 28$$

$$-83 = 5\,V_{af}$$

$$V_{af} = -16.6 \text{ m/s}$$

Coefficient of Restitution

Definition/Summary

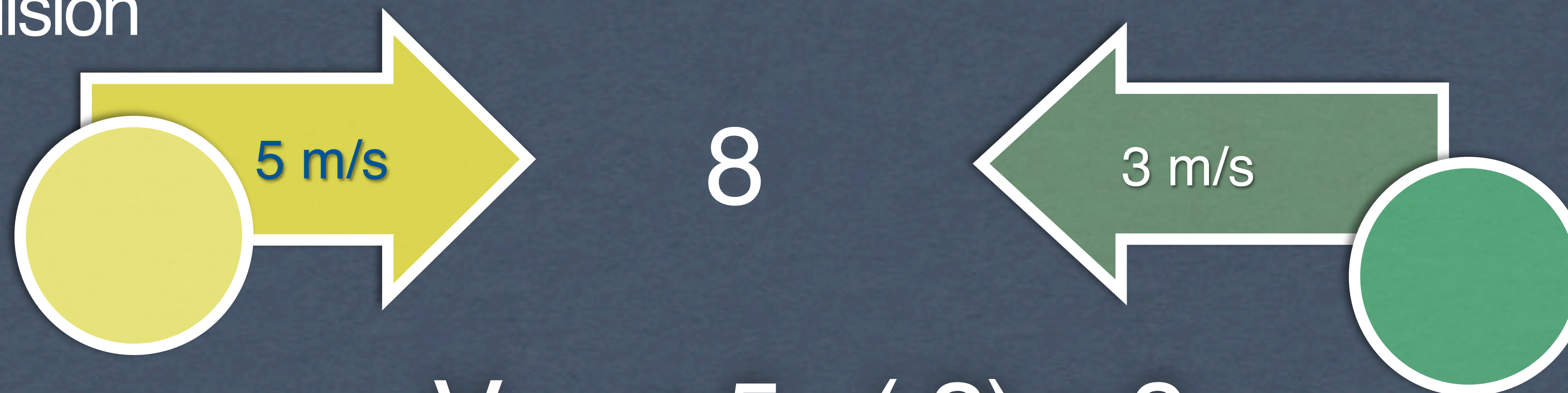
For a collision between two objects, the coefficient of restitution is the ratio of the relative speed after to the relative speed before the collision.

The coefficient of restitution is a number between 0 (perfectly inelastic collision) and 1 (elastic collision) inclusive.

HTTP://WWW.PHYSICSFORUMS.COM/LIBRARY.PHP?DO=VIEW_ITEM&ITEMID=270

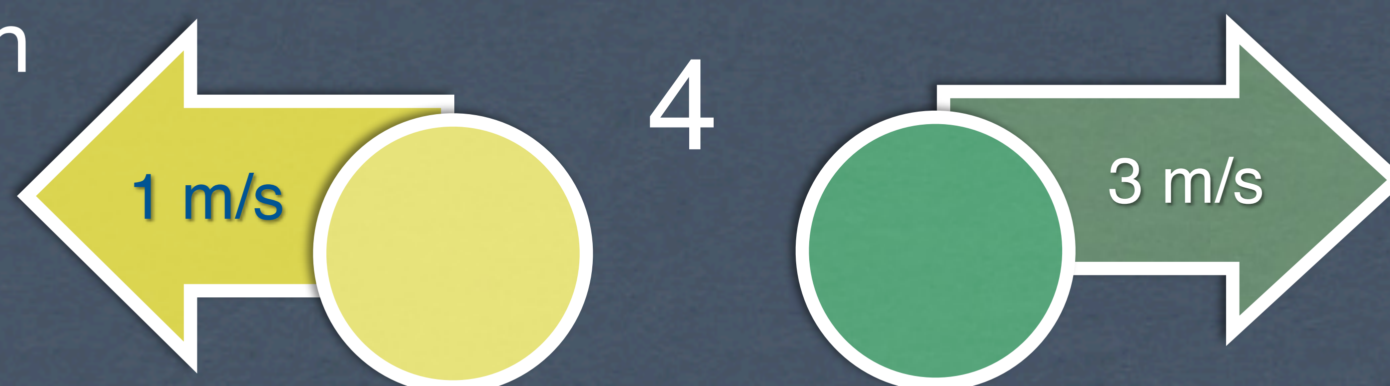
Approach then Separation

Approaching the collision



$$V_{app} = 5 - (-3) = 8$$

Separating after the collision



$$V_{sep} = 3 - (-1) = 4$$

Coefficient of Restitution

- The coefficient is the ratio of the relative approach velocity to the relative separation velocity in a collision
- $e = V_{sep} / V_{app}$
- $\mathbf{e = (V_{bf} - V_{af}) / (V_{ai} - V_{bi})}$
- For the example: $e = 0.5$

Coefficient Of Restitution

e = 0 Perfectly Inelastic

- they stick together

e = 1 Perfectly Elastic

- Kinetic Energy is conserved
- they bounce Perfectly

0 < e < 1 Inelastic

e > 1 Impossible movie theme

Inelastic Example 3

- A bumper car hits a wall with a velocity of 25 m/s.
- e = 0.4
- Find the final velocity of the car.

Inelastic Example 3

A bumper car hits a wall with a velocity of 25 m/s.
e = 0.4
Find the final velocity of the car.

From Elasticity

$$e = (V_{bf} - V_{af}) / (V_{ai} - V_{bi})$$

$$0.4 = (0 - (V_f)) / (25 - 0)$$

$$0.4\,(25) = -(V_f)$$

$$10 = -V_f$$

$$-10 \text{ m/s} = V_f$$

Example 4

- Find the Final Velocity of each car in a perfectly elastic collision. e = 1.0

From Conservation of Momentum

$$5(10) + 7(-15) = 5\,V_{af} + 7\,V_{bf}$$

From Elasticity

$$10 - (-15) = 1.0\,(V_{bf} - V_{af})$$

$$25 + V_{af} = V_{bf}$$

To solve, use both equations and substitution

$$50-105 = 5\,V_{af} + 7(25 + V_{af})$$

$$-55 = 5\,V_{af} + 175 + 7\,V_{af}$$

$$-230 = 12\,V_{af}$$

$V_{af} = -\,19.2$ m/s $\qquad V_{bf} = 5.8$ m/s

2D Collision

Car:

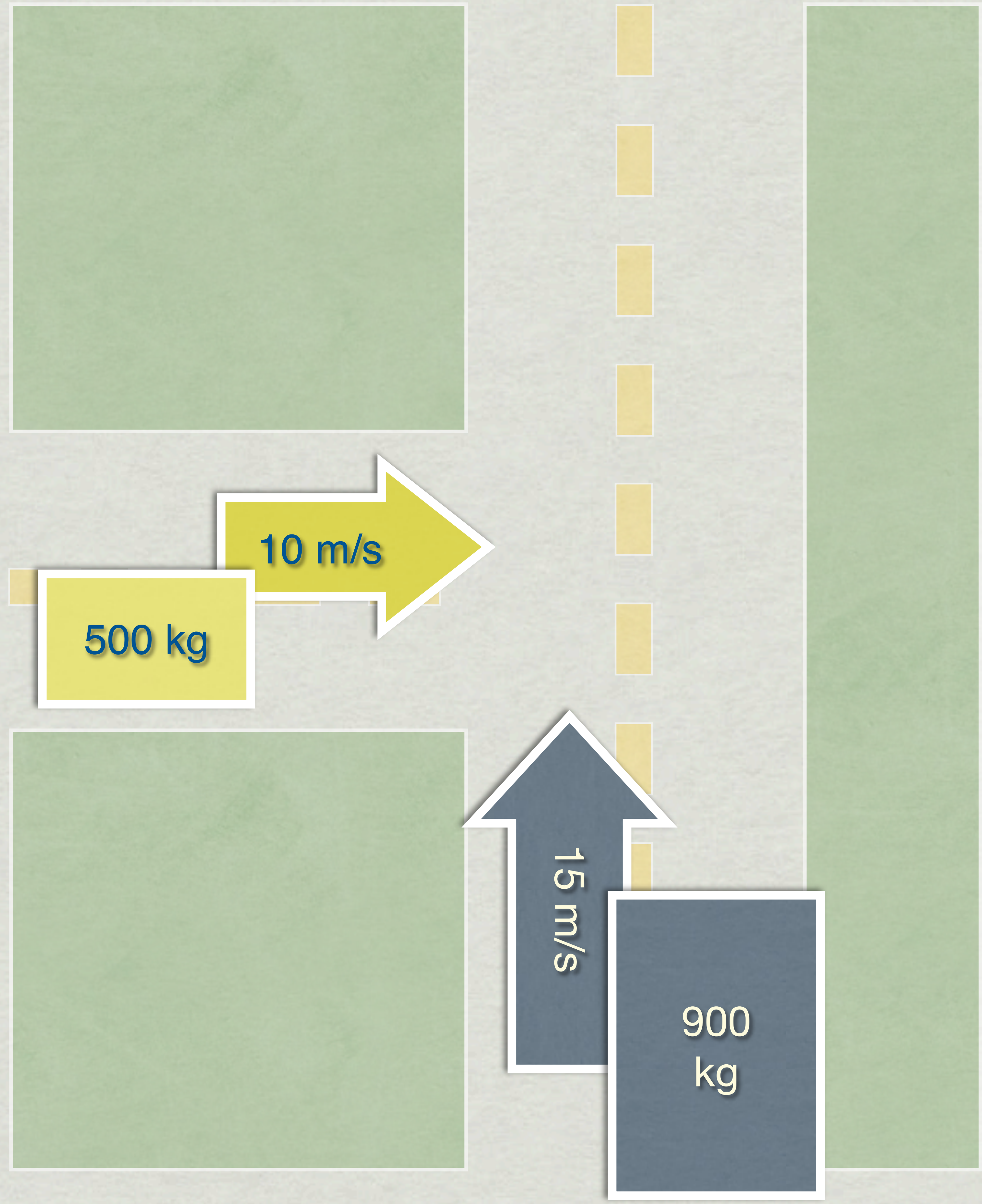
V_i = 10 m/s, E

m = 500 kg

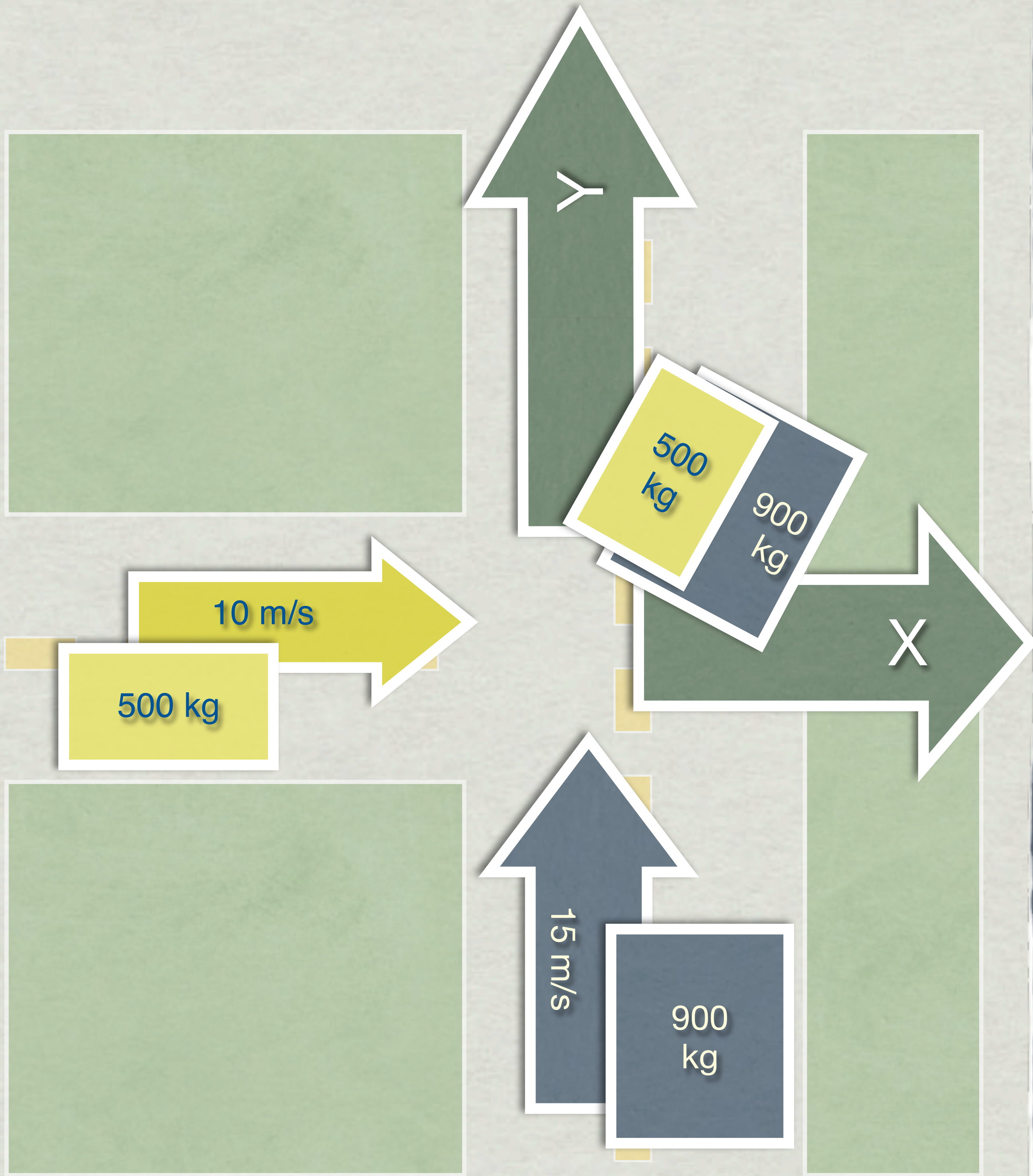
Truck :

V_i = 15 m/s, N

m = 900 kg



2D Collision



After the Collision
the masses combine together to 1400 kg

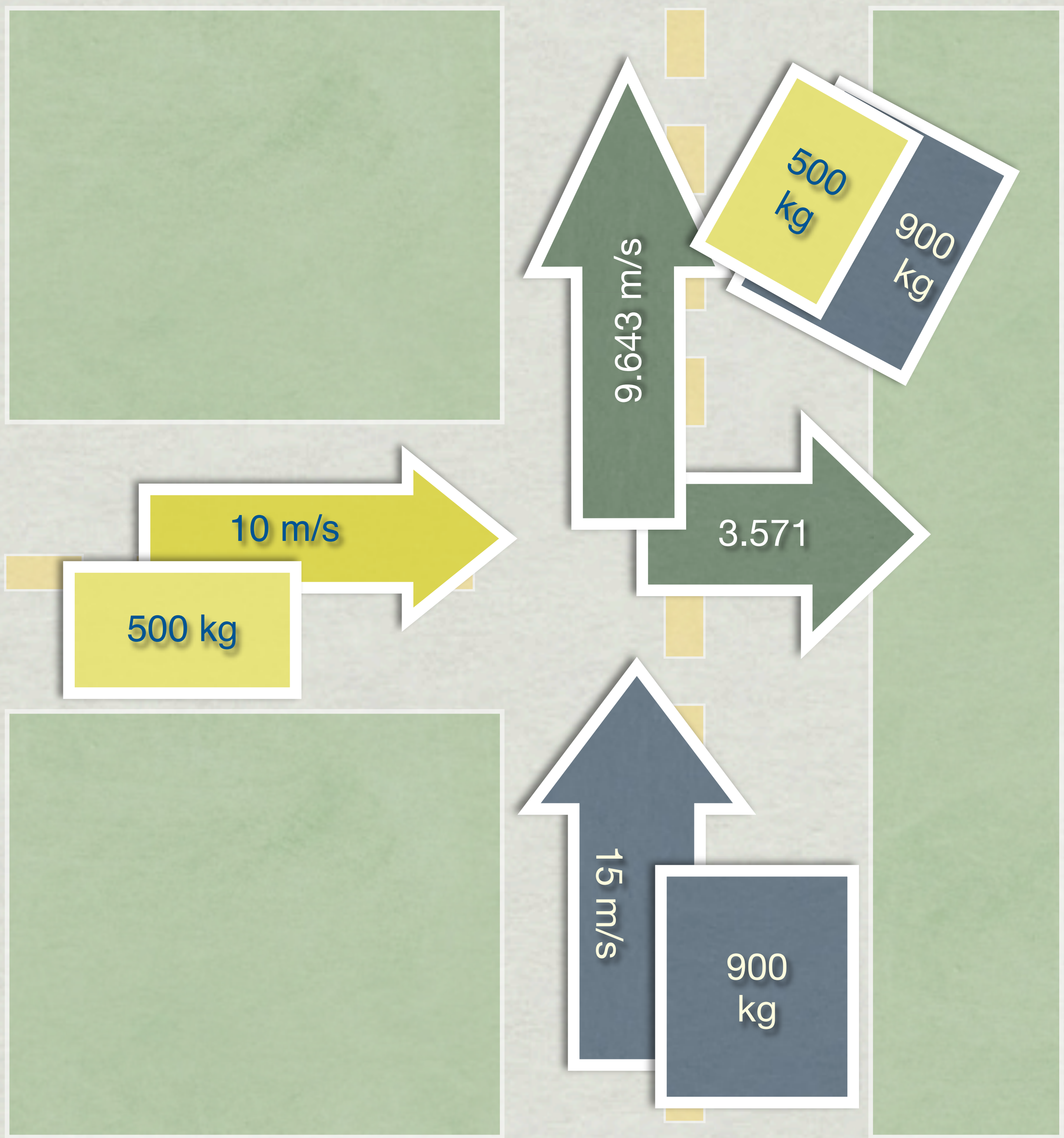
In the X Direction
the truck has no initial momentum

$$(500)(10) = (1400)(V_x)$$

In the Y Direction
the car has no initial momentum

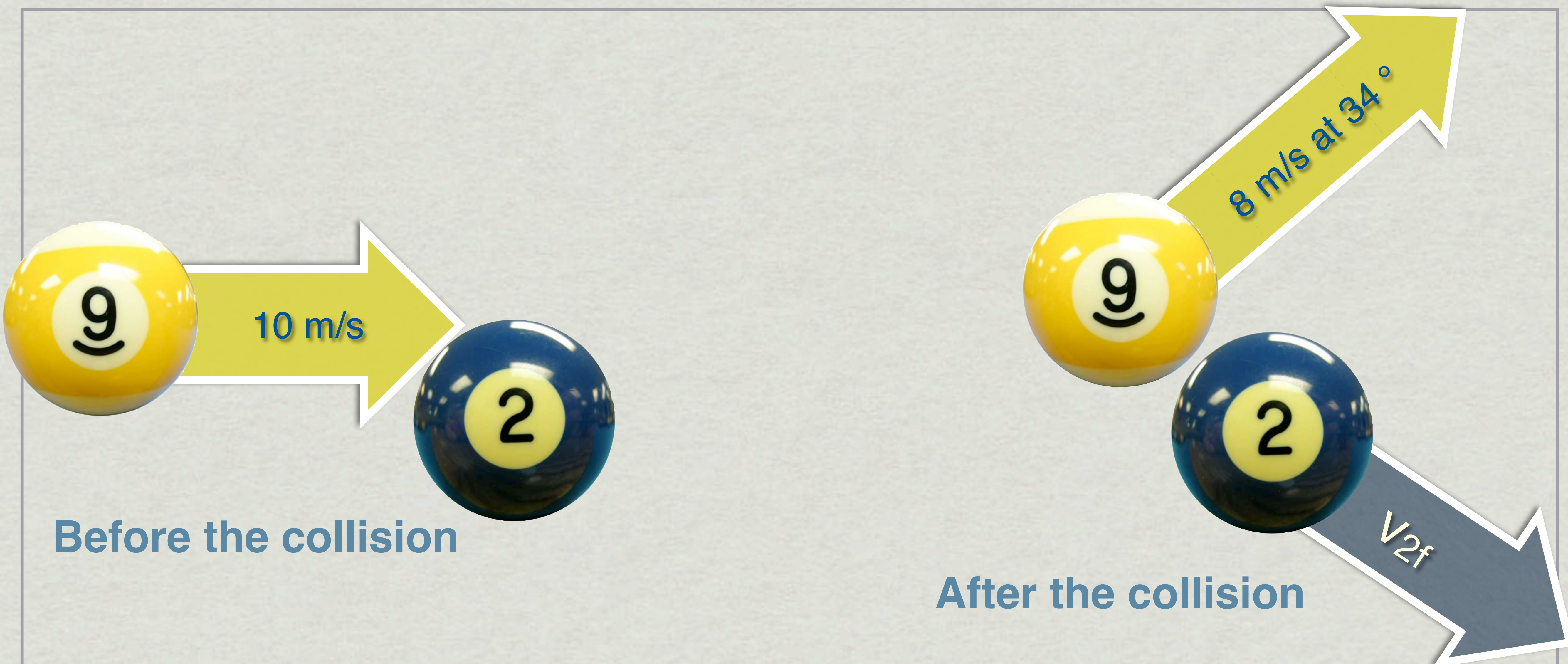
$$(900)(15) = (1400)(V_y)$$

2D Collision



Find the resultant velocity using the pythagorean theory and $\tan^{-1}$.

If the question asks for momentum, multiply by the total mass



In the X

$$(0.35)(10) + 0 = (0.35)(8\cos 34) + (0.3)(V_x)$$

In the Y

$$0 + 0 = (0.35)(8\sin 34) + (0.3)(V_y)$$

In the X

$(0.35)(10) + 0 = (0.35)(8 \cos 34) + (0.3)(V_x)$

$V_x = 3.92$ m/s

In the Y

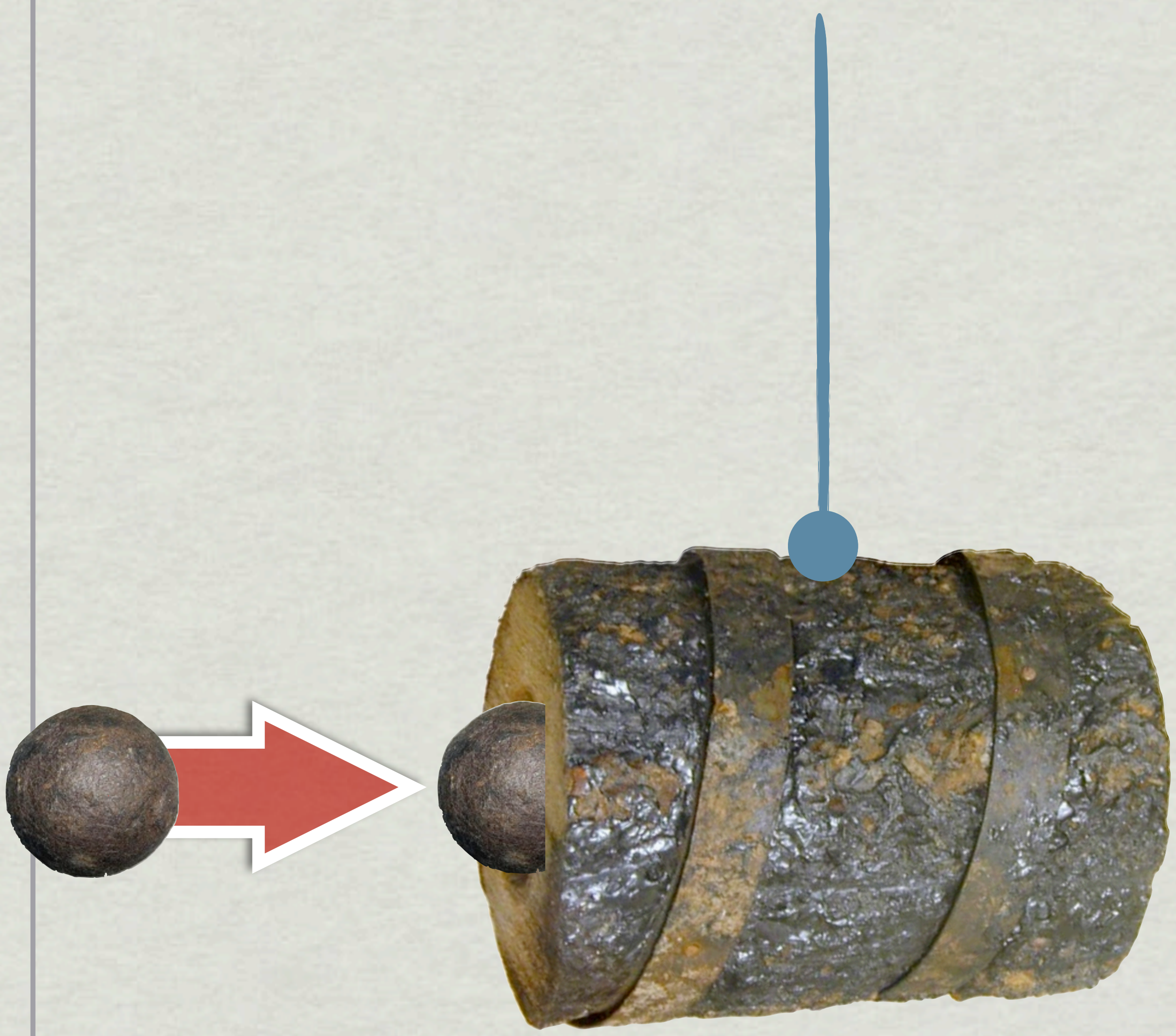
$0 + 0 = (0.35)(8 \sin 34) + (0.3)(V_y)$

$V_y = -5.22$ m/s

V = 6.5 m/s, at 307°

Ballistic Pendulum

A projectile is fired into a log that is suspended by a wire.
Perfectly Inelastic Collision
Find the final height of the log.



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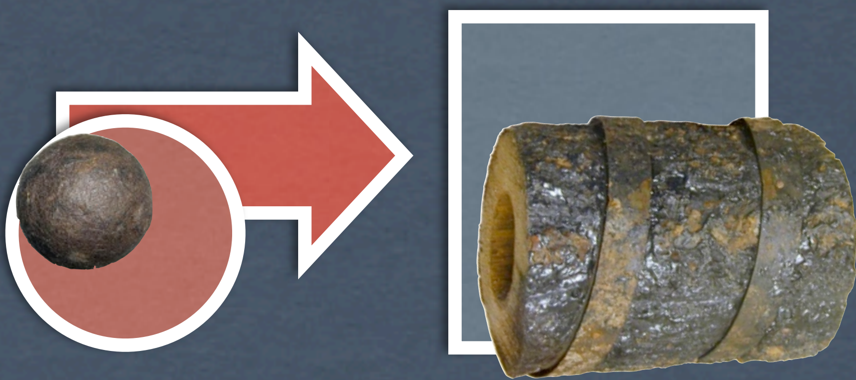
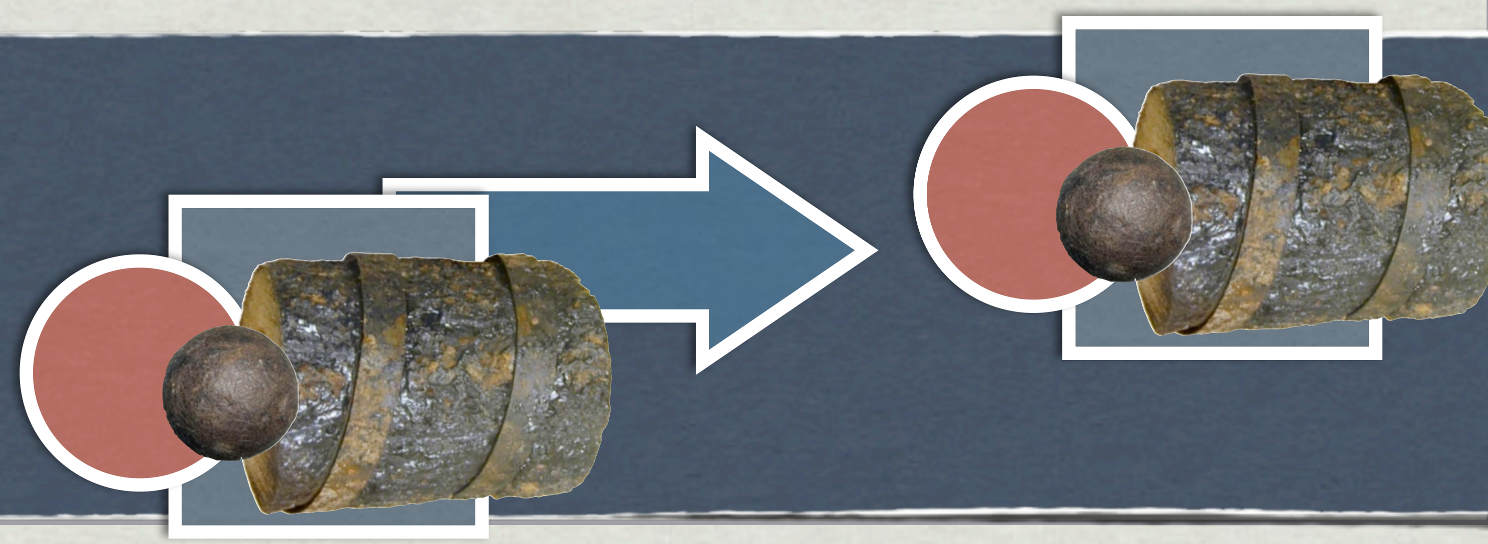


Conservation of Momentum
any time you observe a collision

Velocity after the collision
Perfectly Inelastic makes it easy to solve

Conservation of Energy
Kinetic to Potential



Conservation of Momentum

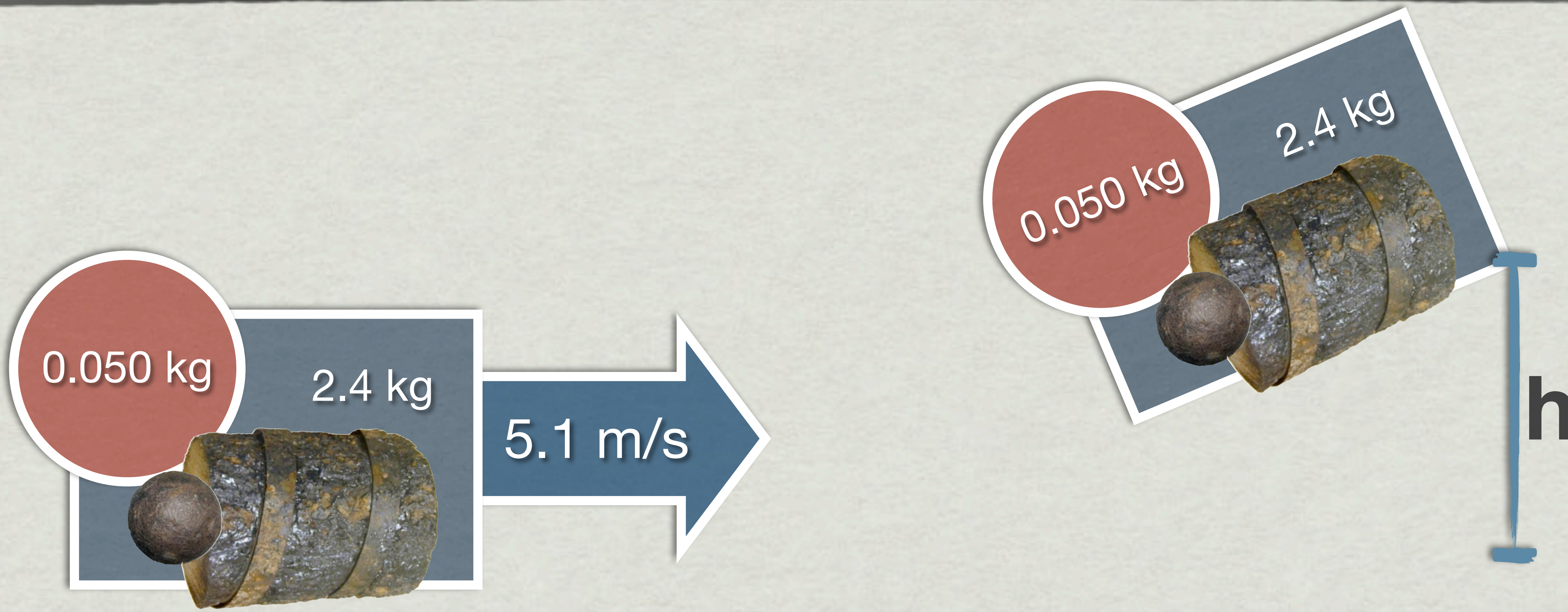
any time you observe a collision

From Conservation of Momentum

$$0.05(250) + 0 = 2.45(V_f)$$

$$V_f = 5.1 \text{ m/s}$$

Conservation of Energy
Kinetic to Potential



From Conservation of Energy

$$\frac{1}{2}mv^2 = mgh$$

$$\frac{1}{2} \times 2.45 \times (5.1)^2 = 2.45 \times 9.8 \times h$$

$$h = 1.33 \text{ m}$$

Impulse

equal to the change in momentum

$$J = F\,t$$

F	Force	(N)
t	time	(s)
J	Impulse	(Ns)

Impulse = Δp

Change in Momentum

A 300 g tennis ball hits a wall at a speed of 36 m/s. It bounces back at a new speed of 20 m/s after being in contact with the wall for 0.4 seconds.

Find; p_i , p_f , Δp , J , F , e

Change in Momentum

A 300 g tennis ball hits a wall at a speed of 36 m/s. It bounces back at a new speed of 20 m/s after being in contact with the wall for 0.4 seconds.

Find; p_i , p_f , Δp , J , F , e

$$p_i = mv_i$$

$$p_i = 0.300\ (36)$$

$$p_i = 10.8 \text{ kg m/s}$$

$$p_f = mv_f$$

$$p_f = 0.300\ (-20)$$

$$p_f = -6.0 \text{ kg m/s}$$

Change in Momentum

A 300 g tennis ball hits a wall at a speed of 36 m/s. It bounces back at a new speed of 20 m/s after being in contact with the wall for 0.4 seconds.

Find; p_i , p_f , Δp , J , F , e

$$\Delta p = p_f - p_i$$

$$\Delta p = (-6) - (10.8)$$

$$\Delta p = -16.8 \text{ kg m/s}$$

$$J = \Delta p$$

$$J = -16.8 \text{ kg m/s}$$

$$J = -16.8 \text{ Ns}$$

Change in Momentum

A 300 g tennis ball hits a wall at a speed of 36 m/s. It bounces back at a new speed of 20 m/s after being in contact with the wall for 0.4 seconds.

Find; p_i , p_f , Δp , J , F , e

$$J = F\,t$$

$$-16.8\ \text{Ns} = F\,(0.4)$$

$$F = -42\ \text{N}$$

$$e = v_s / v_a$$

$$e = 20\text{m/s} / 36\text{m/s}$$

$$e = 0.55$$