Summary of Important Equations

Force on a charged particle in an external magnetic field
$F = qvB \sin \theta$ (22.1)

Definition of the magnetic induction
$B = \frac{F}{qv}$ (22.3)

Lorentz force where
$\mathbf{F} = \mathbf{F_e} + \mathbf{F_m}$ (22.4)

and

$F_e = qE$
$F_m = qvB \sin \theta$

Force on a current-carrying conductor in an external magnetic field
$F = I\ell B \sin \theta$ (22.12)

Biot-Savart law
$\Delta B = \frac{\mu_0 I \Delta \ell}{4\pi r^2} \sin \theta$ (22.13)

Total magnetic field
$\mathbf{B} = \Sigma \Delta \mathbf{B}$ (22.14)

Magnetic field at the center of a circular current loop
$B = \frac{\mu_0 I}{2r}$ (22.18)

Ampère's circuital law
$\Sigma B_{\parallel} \Delta \ell = \mu_0 I$ (22.20)

Magnetic field around a long straight wire
$B = \frac{\mu_0 I}{2\pi r}$ (22.21)

Magnetic field inside a solenoid
$B = \mu_0 n I$ (22.30)

Force on wire 1 caused by the magnetic field of wire 2
$F_1 = \frac{\mu_0 \ell I_1 I_2}{2\pi r}$ (22.32)

Torque on a current loop in an external magnetic field
$\tau = IAB \sin \theta$ (22.39)

Magnetic dipole moment of a current loop
$\mathbf{m} = IA\mathbf{n}$ (22.40)
$\mathbf{m} = NIA\mathbf{n}$ (22.41)

Torque on a current loop in an external magnetic field
$\tau = mB \sin \theta$ (22.42)

Questions for Chapter 22

†1. Should there be a law similar to Coulomb's law of electrostatics that shows the force between magnetic poles? What would be the advantages and disadvantages of such a law?

†2. In the very early days of nuclear physics, nuclear radiation was described in terms of alpha, beta, and gamma particles. How did Rutherford use a magnetic field to distinguish among these particles?

3. A charge in motion in a long straight wire with a drift velocity v_d generates a magnetic field around the wire. If you were to move parallel to the wire at the same velocity, would you still observe a magnetic field? If not, where did the field go?

4. Since a moving electric charge creates a magnetic field, does moving a magnet create an electric field?

5. Electric fields begin on positive electric charges and end on negative electric charges. If there are no isolated north and south poles, do magnetic fields begin and end anywhere or are they always continuous? Describe the magnetic field of a bar magnet from this point of view.

6. What is meant by an electromagnet?

7. How can a solenoid be used as a switch? Give an example of such a use in your car.

†8. How can you use a magnetic field to separate isotopes of a chemical element?

9. How can you make a bar magnet?

†10. What causes the earth's magnetic field? Is the field constant or does it change with time? Is it possible for the earth's poles to flip, that is, for the north pole to become the south and vice versa?

11. If you heat a bar magnet it loses its magnetism. Why?

†12. What does magnetism have to do with the Van Allen belts?

†13. What is magnetohydrodynamics?

Problems for Chapter 22

22.1 The Force on a Charge in a Magnetic Field—The Definition of the Magnetic Field B

1. A proton, moving at a speed of 1.62×10^3 m/s, enters a magnetic field of 0.250 T at an angle of 43.5°. Find the force acting on the proton.

2. An electron, moving at a speed of 3.00×10^6 m/s, enters a magnetic field of 0.200 T at an angle of 35.0°. Find the force acting on the electron.

3. An electron, moving at a speed of 3.00×10^5 m/s, enters a magnetic field of 0.250 T, at an angle of 30.0°. Find (a) the force on the electron and (b) the acceleration of the electron.

4. What is the force on a proton moving north to south at a speed of 3.00×10^5 m/s in the earth's magnetic field if the vertical component of the earth's magnetic field at that location is 25.0×10^{-6} T?

5. How fast must a proton move in a magnetic field of 2.50×10^{-3} T such that the magnetic force is equal to its weight?

6. Find the value of a magnetic field such that an electron moving at a speed of 2.50×10^4 m/s experiences a maximum force of 2.00×10^{-17} N.

7. An electron is accelerated through a potential difference of 1000 V. It then enters perpendicularly to a uniform magnetic field of 0.200 T. Find the radius of the circular orbit of the electron.

8. Find the value of the magnetic field necessary to cause a proton moving at a speed of 2.50×10^3 m/s to go into a circular orbit of 15.5-cm radius.

9. An electron has an energy of 100 eV as it enters a magnetic field of 3.50×10^{-2} T. Find the radius of the orbit.

10. A velocity selector has a magnetic field of 0.300 T. If a perpendicular electric field of 10,000 V/m is applied, what will be the speed of the particles that will pass through the selector?

11. Find the value of the magnetic field that is necessary for a particle, moving at a speed of 2.5×10^3 m/s, to move straight through an electric field of 500 N/C in a velocity selector.

12. Find the necessary value of the magnetic field in a velocity selector that has an electric field of 500 V/m such that an electron will have an orbital radius of 25.0 cm when it enters a secondary region where the magnetic field is 1.20×10^{-3} T.

22.2 Force on a Current-Carrying Conductor in an External Magnetic Field

13. A wire 35.0 cm long, carrying a current of 3.50 A, is placed at an angle of 40.0° in a uniform magnetic field of 0.002 T. Find the force on the wire.

14. What is the maximum force due to the horizontal component of the earth's magnetic field (20.0×10^{-6} T) acting on a 20.0-cm wire, carrying a current of 5.00 A?

15. Find the value of the magnetic field that will cause a maximum force of 7.00×10^{-3} N on a 20.0-cm straight wire carrying a current of 10.0 A.

22.5 The Magnetic Field at the Center of a Circular Current Loop

16. A circular loop of wire of radius 5.00 cm carries a current of 3.00 A. Find the magnetic induction at the center of the current loop.

17. A circular current loop of 10 turns carries a current of 5.00 A. If the radius of the loop is 5.00 cm, find the magnetic field at the center of the loop.

18. It is desired to neutralize the vertical component of the earth's magnetic field (20.0×10^{-6} T) at a particular point. A flat circular coil is mounted horizontally over this point. If the coil has 10 turns and has a radius of 10.0 cm, what current is necessary and in what direction should it flow through the coil?

19. How many loops of wire are necessary to give a magnetic field of 1.50×10^{-3} T at the center of a circular current loop carrying a current of 10.0 A, if the radius of the loop is 5.00 cm?

22.6 Ampère's Circuital Law

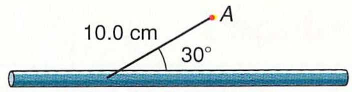
20. A long straight wire carries a current of 10.0 A. Find the magnetic field 5.00 cm from the wire.

21. A power line 10.0 m high carries a current of 200 A. Find the magnetic field of the wire at the ground.

22. A long straight wire carries a current of 10.0 A. How far from the wire will the magnetic field be (a) 1.00 T, (b) 0.100 T, (c) 1.00×10^{-2} T, and (d) 1.00×10^{-3} T?

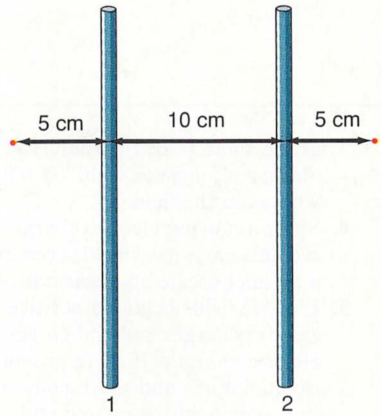
23. What current is necessary to generate a magnetic field of 0.100 T at a distance of 10.0 cm from a long straight wire?

24. Find the magnetic field at the position *A* of a long straight wire carrying a current of 10.0 A.

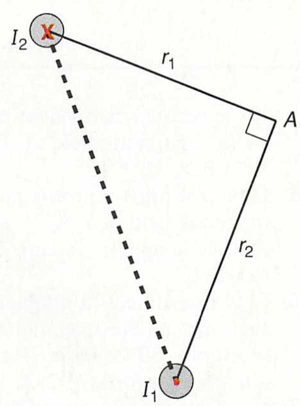


25. Two long parallel wires each carry a current of 5.00 A. If the wires are 15.00 cm apart, find the magnetic field midway between them if (a) the currents are in the same direction and (b) the currents are in the opposite direction.

26. Two parallel wires 10.0 cm apart carry currents of 10.0 A each. Find the magnetic field 5.00 cm to the left of wire 1, 5.00 cm to the right of wire 1, and 15.00 cm to the right of wire 1, if (a) the currents are in the same direction and (b) the currents are in the opposite directions.



27. Find the magnetic field at point *A* in the diagram if I_1 (15.0 A) is a current in a wire coming out of the page and I_2 (10.0 A) is a current in a wire going into the page. The distances to point *A* are $r_1 = 5.00$ cm and $r_2 = 10.00$ cm.



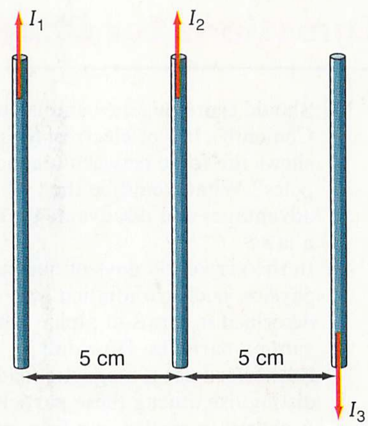
28. A solenoid is 20.0 cm long and carries 500 turns of wire. If the current in the solenoid is 2.00 A, find the magnetic field inside the solenoid.

29. You are asked to design a solenoid that will give a magnetic field of 0.100 T, yet the current must not exceed 10.0 A. Find the number of turns per unit length that the solenoid should have.

22.7 Force between Parallel, Current-Carrying Conductors—The Definition of the Ampere

30. Two parallel wires 15.0 cm apart carry currents of 10.0 A in the same direction. If the wires are 25.0 cm long, (a) find the magnitude and direction of the force on each wire. (b) If the direction of one current is reversed, find the force on each wire.

31. Wire 1 carries a current of 5.00 A; wire 2, 3.00 A; and wire 3, 7.00 A. Find the total force per unit length on wire 2.



32. Two horizontal parallel wires are placed one above the other. Find the equal currents in the two parallel wires, whose repulsive force will just balance the gravitational force on the top wire if the top wire has a mass of 5.00 g, is 20.0 cm long, and is 4.00 cm above the bottom wire.

22.8 Torque on a Current Loop in an External Magnetic Field—The Magnetic Dipole Moment

33. A coil of wire of 10.0-cm radius carries a current of 5.00 A. Find its magnetic dipole moment.

34. A coil of wire has a magnetic dipole moment of 25.0 A m^2. It is placed perpendicular to the horizontal magnetic field of the earth of 20.0×10^{-6} T. What torque will act on the coil?

35. A coil of wire 30.0 cm in diameter is placed at an angle of 60.0° in a magnetic field of 2.50×10^{-2} T and experiences a torque of 3.25×10^{-3} m N. Find (a) the magnetic dipole moment of the coil and (b) the current in the coil.

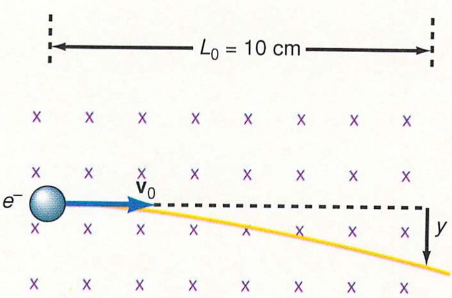
36. A rectangular galvanometer coil 2.00 cm by 1.50 cm has 10 turns of wire. If the current through the coil is 3.00 mA, find its magnetic dipole moment. The coil is placed in a magnetic field of 0.300 T. Find the torque on the coil when the magnetic dipole moment makes an angle of 30.0° with the magnetic field.

37. A coil of 3.00-cm radius, carrying a current of 2.00 A, is placed within a solenoid that carries a current of 3.00 A. If the solenoid has 5000 turns per meter, find the torque on the coil.

†38. A galvanometer coil, 5.00 cm^2 in area, is placed in a magnetic field of 1.00×10^{-3} T. If the coil deflects 40.0° when it carries its maximum current of 200 mA, find the torsion constant of the spring in the galvanometer.

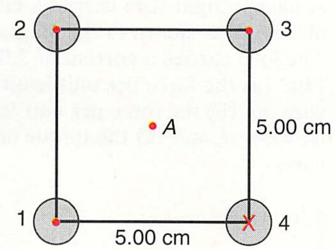
Additional Problems

†39. An electron moving at an initial velocity of 6.00×10^5 m/s enters a uniform magnetic field of 2.50×10^{-6} T directed into the paper. The length of the magnetic field is 10.0 cm, as shown in the diagram. Find (a) the force on the electron, (b) the magnitude and direction of the acceleration of the electron, (c) the time that the electron remains in the field, and (d) the amount of deflection of the electron as it leaves the magnetic field.



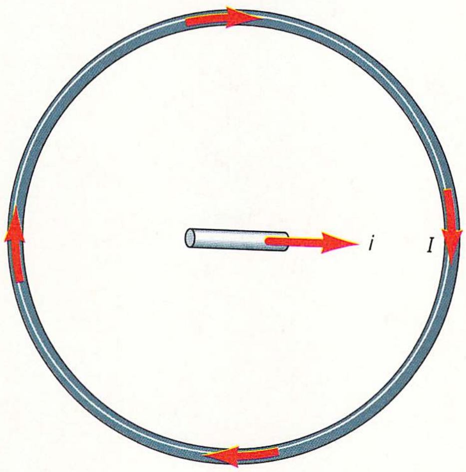
40. An electron moves at a speed of 4.00×10^5 m/s parallel to a long straight wire carrying a 12.0-A current. The velocity of the electron is in the same direction as the current in the wire. If the electron is 5.00 cm from the current, find the magnitude and direction of the force on the electron due to the magnetic field of the current.

41. The diagram shows the end view of four wires each carrying a current I = 10.0 A. Wires 1, 2, and 3 have currents coming out of the page, whereas wire 4 has a current going into the page. Find the magnetic field at the center of the square A.



42. A 5000-turn solenoid carries a 2.45-A current. A proton is located at the center of the coil, with a velocity of 7.50×10^4 m/s making an angle of 30.0° with the axis of the solenoid. Determine the magnitude of the force on the proton due to the magnetic field of the solenoid.

43. A small element of current is located at the center of a flat coil, as shown in the diagram. The coil has 50 turns, it is 50.0 cm in radius, and it carries a current of I = 350 mA in the clockwise direction. The element of current is 2.00 mm long and it carries a current of I = 10.0 mA to the right. Compute the magnitude and direction of the force on the element of current due to the magnetic field of the coil.



44. You are asked to design a circular coil that will have a value of 3.00×10^{-2} T at its center. Find the ratio of the current in the coil to the radius of the coil that will give this value of B. Pick a reasonable value for the combination of I and r such that the current is not too large nor the radius too small. Would it be desirable to introduce more than one loop of wire? What would be a better combination of I, r, and N?

45. An electron, located halfway between two long straight wires, moves at 2.00×10^6 m/s to the left. The top wire carries a 5.00-A current out of the paper, the bottom wire carries a current of 2.00 A into the paper, and the wires are 6.00 cm apart. Find the magnitude and direction of the force on the electron due to the magnetic fields of the current in the wires.

†46. In the Bohr model of the hydrogen atom, a negative electron orbits about a positive proton at a radius of 5.29×10^{-11} m and at a speed of 2.19×10^6 m/s. (a) How long does it take for the electron to revolve around the proton? (b) From the definition of current show that the orbiting electron constitutes a current and determine its magnitude. (c) Since the orbiting electron looks like a current loop, determine the magnetic field at the location of the proton caused by the orbiting electron.

†47. Everyone knows that the earth revolves around the sun. Yet, in our everyday experience the sun is seen to rise in the eastern sky and set in the western sky, as though the sun revolved around the earth. The observed motion seems to depend on the frame of reference of the observer. When the frame of reference is placed on the earth, the sun appears to revolve around the earth. In a similar way, if the frame of reference is placed on the orbiting electron in the Bohr theory of the atom, it appears as though the proton is moving in a circular orbit about the electron. Find the value of the magnetic field at the position of the electron, caused by the proton.

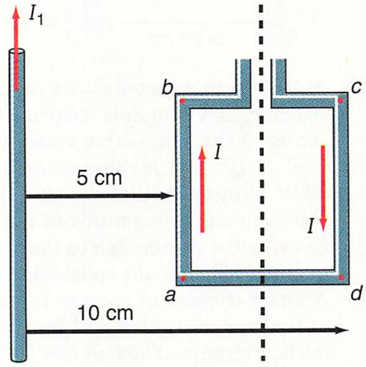
48. Find the magnetic fields at the center of the following two coils: (a) a 100-turn coil 20.0 cm long with 0.200-cm diameter, carrying a current of 2.00 A, and (b) a 100-turn coil 20.0 cm in diameter and 0.200 cm thick, carrying a current of 2.00 A.

†49. You are asked to design a solenoid by wrapping insulated copper wire around the hollow cardboard core of an empty roll of paper towels. The completed solenoid will then be connected to a 12.0-V battery and the maximum current that can flow in the circuit is 10.0 A. (a) What is the minimum value of the resistance of this solenoid? (b) If the wire used is #22 S&W gauge copper wire, which has a diameter of 6.44×10^{-4} m, what is the minimum length of this wire? (c) With the above restrictions on I and ℓ, how many turns of wire can you have if the diameter of the solenoid is 4.00 cm? (d) If the length of the cardboard core is 28.0 cm, and with all of the above restrictions, find the value of B inside the solenoid. (e) In this design, what factors might be changed to increase the value of B?

†50. A solenoid is filled with iron, which has a permeability of $\mu = 4.80 \times 10^{-3}$ T m/A. The solenoid has 200 turns of wire, is 30.0 cm long, and carries a current of 5.00 A. Find the value of B inside the solenoid. (*Hint:* just as the electric field in a vacuum was characterized by the permittivity of free space ϵ_0, and an electric field in a different medium was characterized by ϵ, the permittivity of the medium the magnetic field in a vacuum is characterized by the permeability μ_0, while the magnetic field in a medium is characterized by the permeability of that medium μ.)

†51. You are to design a galvanometer that has an internal resistance $R_g = 20.0\ \Omega$ and gives full-scale deflection for a current $I_g = 20.0$ mA. (a) Find the length of #22 B&S gauge copper wire necessary to give this resistance. (b) If this length of wire is to be made into a 500 turn loop, find the radius of the loop. (c) Find the area A of this loop. (d) Find the magnetic dipole moment of this loop. (e) If this loop is placed in a magnetic field of 2.50×10^{-2} T, find the maximum torque on the coil. (f) If the galvanometer needle is to rotate through an angle of $\pi/2$ rad, find the torsion constant of the spring that will supply the countertorque.

52. A long straight wire carries a current of 10.0 A as shown in the diagram. The loop carries a current of 2.00 A. Find (a) the force per unit length on wire *ab,* (b) the force per unit length on wire *cd,* and (c) the torque on the loop.

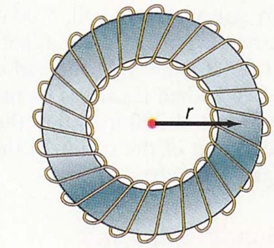


†53. The coil in a DC motor is called an *armature.* The armature, 15.0 cm by 10.0 cm, with 100 turns of wire, carries a current of 3.50 A in a magnetic field of 0.250 T. Find the maximum torque exerted by the motor. If the armature rotates at 1000 rpm, what is the power rating of the motor?

†54. A *toroid* is essentially a solenoid that is bent into a circle. As seen in the diagram, it looks like a doughnut. Use Ampère's law to show that the magnetic field within the toroid is given by

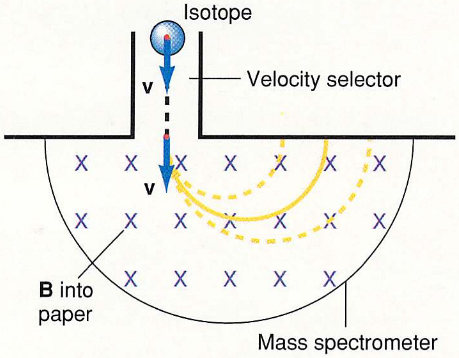
$$B = \frac{\mu_0 N I}{2\pi r}$$

where N is the number of turns of wire and r is the distance from the center of the toroid to the position where the magnetic field is desired. Note that the magnitude of the magnetic field varies with r.



Toroid

55. Every chemical element has isotopes. An *isotope* has the same number of protons as the element but a different number of neutrons. Hence an isotope has a different mass than the parent element. The different masses of the isotopes can be found with a mass spectrometer, as shown in the diagram. Ions of the isotope are fired into the velocity selector, and those of velocity v enter into the mass spectrometer where there is a magnetic field B into the paper. The path of the ions is bent into a circular path of radius R. (a) Find the equation for the mass of any isotope. (b) The chemical element oxygen has eight protons and eight neutrons. It has two stable isotopes, one with nine neutrons and the other with ten neutrons. Find the radii of the path of the isotopes in the spectrometer in terms of the radius of the oxygen element.



Interactive Tutorials

56. Force on a charge in a magnetic field. An electron is fired into a uniform magnetic field, $B = 0.255$ T, at a speed $v = 185$ m/s and at an angle $\theta = 43.5°$ to the direction of the magnetic field **B**. Find (a) the force F acting on the charge q and (b) the acceleration a of the charge.

57. Force on a current-carrying wire in a magnetic field. A length of wire $L = 42.0$ cm carries a current $I = 15.0$ A. The wire is placed in a uniform magnetic field, $B = 0.365$ T. If the wire makes an angle $\theta = 55.5°$ with the direction of the magnetic field **B**, find the magnitude of the force **F** on the wire.

58. The magnetic field at the center of a circular current loop. Find the magnetic field at the center of a circular current loop of $N = 10$ turns with a radius $r = 5.00$ cm that carries a current $I = 10.0$ A.

59. The magnetic field of a long straight wire. Find the magnetic field at a distance $r = 10.0$ cm from a long straight wire that carries a current $I = 12.5$ A.

60. The magnetic field inside a solenoid. Find the magnetic field inside a solenoid that has a length $L = 25.0$ cm and is composed of $N = 1000$ turns of wire. The current in the solenoid is $I = 7.50$ A.

61. The torque on a current loop in an external magnetic field. A circular coil, consisting of $N = 15$ turns of wire, having a diameter $d = 12.5$ cm, carries a current $I = 8.00$ A. The coil is placed in a uniform magnetic field $B = 0.385$ T. Find (a) the magnetic dipole moment m of the coil and (b) the maximum torque τ_{max}. (c) Plot the torque acting on the coil as a function of the angle θ that the magnetic dipole moment of the coil makes with the magnetic field.