The necessary speed that the spacecraft must have to enter a circular lunar parking orbit v_{om} is found from modifying equation 6.49 to

$$v_{om} = \sqrt{\frac{GM_m}{r_m + h_m}} \tag{6H.11}$$

where M_m is the mass of the moon, r_m is the radius of the moon, and h_m is the height of the spacecraft above the surface of the moon in its circular lunar parking orbit. The necessary change in speed to transfer from the Hohmann ellipse to the circular lunar parking orbit is obtained by subtracting equation 6H.11 from equation 6H.9. Thus the necessary Δv is

$$\Delta v_{II} = v_{TEa} - v_{om} \tag{6H.12}$$

The spacecraft engines are turned on to obtain this necessary change in speed. When the engines are shut off the spacecraft will have the speed v_{om}, and will stay in the circular lunar parking orbit until the astronauts are ready to descend to the lunar surface. The process is repeated for the return to earth.

The Hohmann transfer is the simplest of the transfer orbits and is also the orbit of minimum energy. However, it has the disadvantage of having a large flight time. In the very early stages of the Apollo program, the Hohmann transfer ellipse was considered for the lunar transfer orbit. However, because of its long flight time, it was discarded for a hyperbolic transfer orbit that had been perfected by the Jet Propulsion Laboratories in California on its *Ranger, Surveyor,* and *Lunar Orbiter* unmanned spacecrafts to the moon. The hyperbolic orbit requires a great deal more energy, but its flight time is relatively small. The procedure for a trip on a hyperbolic orbit is similar to the elliptical orbit, only another equation is necessary for the speed of the spacecraft in the hyperbolic orbit. The principle however is the same. Determine the current speed in the particular orbit, then determine the speed that is necessary for the other orbit. The difference between the two of them is the necessary Δv. The spacecraft engines are turned on until this value of Δv is obtained. A typical orbital picture for this type of transfer is shown in figure 3.

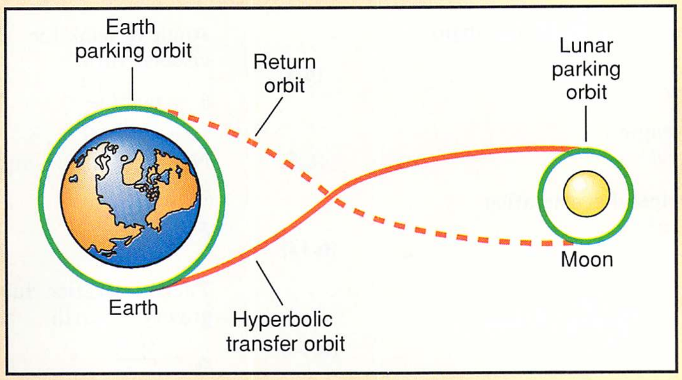


Figure 3
A hyperbolic transfer orbit.

Unmanned satellites have since traveled to Mars, Venus, Saturn, Jupiter, Uranus, and Neptune. And what about manned trips to these planets? On July 20, 1989, the twentieth anniversary of the first landing on the moon, the president of the United States, George Bush, announced to the world that the United States will begin planning a manned trip to the planet Mars and eventually to an exploration of our entire solar system. Man is thus getting ready to leave his cradle.

The Language of Physics

Uniform circular motion
Motion in a circle at constant speed. Because the velocity vector changes in direction with time, this type of motion is accelerated motion (p. 155).

Centripetal acceleration
When a body moves in uniform circular motion, the acceleration is called centripetal acceleration. The direction of the centripetal acceleration is toward the center of the circle (p. 156).

Radian
A unit that is used to measure an angle. It is defined as the ratio of the arc length subtended to the radius of the circle, where 2π radians equals 360° (p. 156).

Centripetal force
The force that is necessary to cause an object to move in a circle at constant speed. The centripetal force acts toward the center of the circle (p. 160).

Centrifugal force
The reaction force to the centripetal force. The reaction force does not act on the same body as the centripetal force. That is, if a string were tied to a rock and the rock were swung in a horizontal circle at constant speed, the centripetal force would act on the rock while the centrifugal force would act on the string (p. 161).

Centrifuge
A device for separating particles of different densities in a liquid. The centrifuge spins at a high speed. The more massive particles in the mixture will separate to the bottom of the test tube while the particles of smaller mass will separate to the top (p. 170).

Newton's law of universal gravitation
Between every two masses in the universe there is a force of attraction that is directly proportional to the product of their masses and inversely proportional to the square of the distance separating them (p. 171).

Kepler's laws of planetary motion
(1) The orbit of each planet is an ellipse with the sun at one focus. (2) The speed of the planet varies in such a way that the line joining the planet and the sun sweeps out equal areas in equal times. (3) The cube of the semimajor axes of the elliptical orbit is proportional to the square of the time for the planet to make a complete revolution about the sun (p. 178).

Geosynchronous satellite
A satellite whose orbital motion is synchronized with the rotation of the earth. In this way the satellite is always over the same point on the equator as the earth turns (p. 179).

Summary of Important Equations

Definition of angle in radians

$$\theta = \frac{s}{r} \tag{6.4}$$

Arc length

$$s = r\theta \tag{6.5}$$

Centripetal acceleration

$$a_c = \frac{v^2}{r} \tag{6.12}$$

Centripetal force

$$F_c = ma_c = \frac{mv^2}{r} \tag{6.14}$$

Angle of bank for circular turn

$$\theta = \tan^{-1}\frac{v^2}{rg} \tag{6.33}$$

Newton's law of universal gravitation

$$F = \frac{Gm_1m_2}{r^2} \tag{6.37}$$

The acceleration due to gravity on earth

$$g_e = \frac{Gm_e}{r^2} \tag{6.43}$$

The acceleration due to gravity on the moon

$$g_m = \frac{Gm_m}{r_m^2} \tag{6.46}$$

Speed of a satellite in a circular orbit

$$v = \sqrt{Gm_e/r} \tag{6.49}$$

Questions for Chapter 6

1. If a car is moving in uniform circular motion at a speed of 5.00 m/s and has a centripetal acceleration of 2.50 m/s^2, will the speed of the car increase at 2.50 m/s every second?
2. Does it make any sense to say that a car in uniform circular motion is moving with a velocity that is tangent to a circle and yet the acceleration is perpendicular to the tangent? Should not the acceleration be tangential because that is the direction that the car is moving?
3. If a car is moving in uniform circular motion, and the acceleration is toward the center of that circle, why does the car not move into the center of the circle?
4. Answer the student's question, "If an object moving in uniform circular motion is accelerated motion, why doesn't the speed change with time?"
5. Reply to the student's statement, "I know there is a centrifugal force acting on me when I move in circular motion in my car because I can feel the force pushing me against the side of the car."
6. † Is it possible to change to a noninertial coordinate system, say a coordinate system that is fixed to the rotating body, to study uniform circular motion? In this rotating coordinate system is there a centrifugal force?
7. If you take a pail of water and turn it upside down all the water will spill out. But if you take the pail of water, attach a rope to the handle, and turn it rapidly in a vertical circle the water will not spill out when it is upside down at the top of the path. Why is this?
8. † In high-performance jet aircraft the pilot must wear a pressure suit that exerts pressure on the abdomen and upper thighs of the pilot when the pilot pulls out of a steep dive. Why is this necessary?
9. If the force of gravity acting on a body is directly proportional to its mass, why does a massive body fall at the same rate as a less massive body?
10. Why does the earth bulge at the equator and not at the poles?
11. If the acceleration of gravity varies from place to place on the surface of the earth, how does this affect records made in the Olympics in such sports as shot put, javelin throwing, high jump, and the like?
12. What is wrong with applying Newton's second law in the form $F = ma$ to satellite motion? Does this same problem occur in the motion of an airplane?
13. † How can you use Kepler's second law to explain that the earth moves faster in its motion about the sun when it is closer to the sun?
14. Could you place a synchronous satellite in a polar orbit about the earth? At 45° latitude?
15. Explain how you can use a Hohmann transfer orbit to allow one satellite in an earth orbit to rendezvous with another satellite in a different earth orbit.
16. † A satellite is in a circular orbit. Explain what happens to the orbit if the engines are momentarily turned on to exert a thrust (a) in the direction of the velocity, (b) opposite to the velocity, (c) toward the earth, and (d) away from the earth.
17. A projectile fired close to the earth falls toward the earth and eventually crashes to the earth. The moon in its orbit about the earth is also falling toward the earth. Why doesn't it crash into the earth?
18. † The gravitational force on the earth caused by the sun is greater than the gravitational force on the earth caused by the moon. Why then does the moon have a greater effect on the tides than the sun?
19. † How was the universal gravitational constant G determined experimentally?
20. A string is tied to a rock and then the rock is put into motion in a vertical circle. Is this an example of uniform circular motion?

Problems for Chapter 6

6.3 Angles Measured in Radians

1. Express the following angles in radians: (a) 360°, (b) 270°, (c) 180°, (d) 90°, (e) 60°, (f) 30°, and (g) 1 rev.
2. Express the following angles in degrees: (a) 2π rad, (b) π rad, (c) 1 rad, and (d) 0.500 rad.
3. A record player turns at 33-1/3 rpm. What distance along the arc has a point on the edge moved in 1.00 min if the record has a diameter of 10.0 in.?

6.4 and 6.5 The Centripetal Acceleration and the Centripetal Force

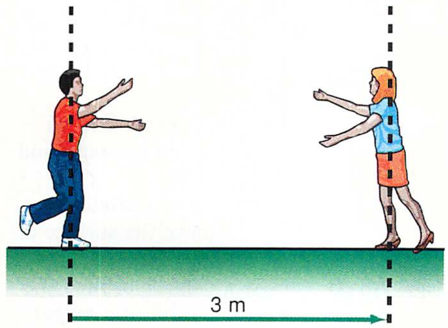
4. A 4.00-kg stone is whirled at the end of a 2.00-m rope in a horizontal circle at a speed of 15.0 m/s. Ignoring the gravitational effects (a) calculate the centripetal acceleration and (b) calculate the centripetal force.
5. An automatic washing machine, in the spin cycle, is spinning wet clothes at the outer edge at 8.00 m/s. The diameter of the drum is 0.450 m. Find the acceleration of a piece of clothing in this spin cycle.
6. A 3200-lb car moving at 60.0 mph goes around a curve of 1000-ft radius. What is the centripetal acceleration? What is the centripetal force on the car?
7. A 1500-kg car moving at 86.0 km/hr goes around a curve of 325-m radius. What is the centripetal acceleration? What is the centripetal force on the car?
8. An electron is moving at a speed of 2.00×10^6 m/s in a circle of radius 0.0500 m. What is the force on the electron?
9. Find the centripetal force on a 70.0-lb girl on a merry-go-round that turns through one revolution in 40.0 s. The radius of the merry-go-round is 10.0 ft.

6.7 Examples of Centripetal Force

10. A boy sits on the edge of a polished wooden disk. The disk has a radius of 3.00 m and the coefficient of friction between his pants and the disk is 0.300. What is the maximum speed of the disk at the moment the boy slides off?
11. A 3200-lb car begins to skid when traveling at 60.0 mph around a level curve of 500-ft radius. Find the centripetal acceleration and the coefficient of friction between the tires and the road.
12. A 1200-kg car begins to skid when traveling at 80.0 km/hr around a level curve of 125-m radius. Find the centripetal acceleration and the coefficient of friction between the tires and the road.
13. At what angle should a bobsled turn be banked if the sled, moving at 26.0 m/s, is to round a turn of radius 100 m?
14. A motorcyclist goes around a curve of 400-ft radius at a speed of 60.0 mph, without leaning into the turn. (a) What must the coefficient of friction between the tires and the road be in order to supply the necessary centripetal force? (b) If the road is iced and the motorcyclist can not depend on friction, at what angle from the vertical should the motorcyclist lean to supply the necessary centripetal force?
15. A motorcyclist goes around a curve of 100-m radius at a speed of 95.0 km/hr, without leaning into the turn. (a) What must the coefficient of friction between the tires and the road be in order to supply the necessary centripetal force? (b) If the road is iced and the motorcyclist can not depend on friction, at what angle from the vertical should the motorcyclist lean to supply the necessary centripetal force?
16. At what angle should a highway be banked for cars traveling at a speed of 60.0 mph, if the radius of the road is 1000 ft and no frictional forces are involved?
17. At what angle should a highway be banked for cars traveling at a speed of 100 km/hr, if the radius of the road is 400 m and no frictional forces are involved?
18. An airplane is flying in a circle with a speed of 200 knots. (A knot is a nautical mile per hour.) The aircraft weighs 2000 lb and is banked at an angle of 30.0°. Find the radius of the turn in feet.
19. An airplane is flying in a circle with a speed of 650 km/hr. At what angle with the horizon should a pilot make a turn of radius of 8.00 km such that a component of the lift of the aircraft supplies the necessary centripetal force for the turn?

6.8 Newton's Law of Universal Gravitation

20. Two large metal spheres are separated by a distance of 2.00 m from center to center. If each sphere has a mass of 5000 kg, what is the gravitational force between them?
21. A 5.00-kg mass is 1.00 m from a 10.0-kg mass. (a) What is the gravitational force that the 5.00-kg mass exerts on the 10.0-kg mass? (b) What is the gravitational force that the 10.0-kg mass exerts on the 5.00-kg mass? (c) If both masses are free to move, what will their initial acceleration be?
22. Three point masses of 10.0 kg, 20.0 kg, and 30.0 kg are located on a line at 10.0 cm, 50.0 cm, and 80.0 cm, respectively. Find the resultant gravitational force on (a) the 10.0-kg mass, (b) the 20.0-kg mass, and (c) the 30.0-kg mass.
23. Pete meets Eileen for the first time and is immediately attracted to her. If Pete has a mass of 75.0 kg and Eileen has a mass of 50.0 kg and they are separated by a distance of 3.00 m, is their attraction purely physical?



24. What is the gravitational force between a proton and an electron in a hydrogen atom if they are separated by a distance of 5.29×10^{-11} m?

6.11–6.13 The Acceleration Due to Gravity

25. What is the value of g at a distance from the center of the earth of (a) 1 earth radius, (b) 2 earth radii, (c) 10 earth radii, and (d) at the distance of the moon?

26. What is the weight of a body, in terms of its weight at the surface of the earth, at a distance from the center of the earth of (a) 1 earth radius, (b) 2 earth radii, (c) 10 earth radii, and (d) at the distance of the moon? How can an object in a satellite, at say 2 earth radii, be considered to be weightless?

27. Calculate the acceleration of gravity on the surface of Mars. What would a man who weighs 180 lb on earth weigh on Mars?

†28. It is the year 2020 and a base has been established on Mars. An enterprising businessman decides to buy coffee on earth at \$5.00/lb and sell it on Mars for \$10.00/lb. How much does he make or lose per lb when he sells it on Mars? Ignore the cost of transportation from earth to Mars.

29. The sun's radius is 110 times that of the earth, and its mass is 333,000 times as large. What would be the weight of a 1.00-kg object at the surface of the sun, assuming that it does not melt or evaporate there?

6.14 Satellite Motion

30. What is the velocity of the moon around the earth in a circular orbit? What is the time for one revolution?

31. Calculate the velocity of the earth in an approximate circular orbit about the sun. Calculate the time for one revolution.

32. A satellite orbits the earth 700 mi above the surface. Find its speed and its period of revolution.

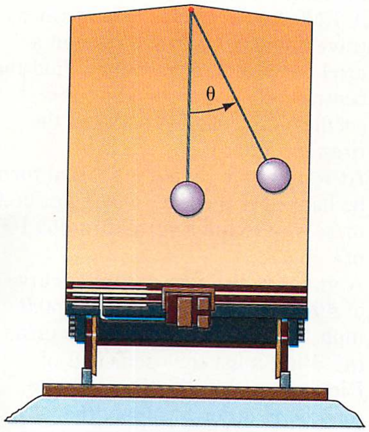
33. Calculate the speed of a satellite orbiting 100 km above the surface of Mars. What is its period?

†34. An Apollo space capsule orbited the moon in a circular orbit at a height of 112 km above the surface. The time for one complete orbit, the period T, was 120 min. Find the mass of the moon.

†35. A satellite orbits the earth in a circular orbit in 130 min. What is the distance of the satellite to the center of the earth? What is its height above the surface? What is its speed?

Additional Problems

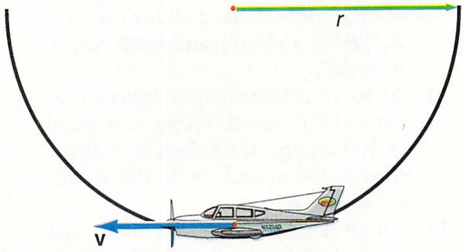
†36. A rock attached to a string hangs from the roof of a moving train. If the train is traveling at 50.0 mph around a level curve of 500-ft radius, find the angle that the string makes with the vertical.



37. Find the centripetal force due to the rotation of the earth acting on a 100 kg person at (a) the equator, (b) 45.0° north latitude, and (c) the north pole.

38. Find the resultant vector acceleration caused by the acceleration due to gravity and the centripetal acceleration for a person located at (a) the equator, (b) 45.0° north latitude, and (c) the north pole.

†39. A pilot who weighs 180 lb pulls out of a vertical dive at 400 mph along an arc of a circle of 5280-ft radius. Find the centripetal acceleration, centripetal force, and the net normal force on the pilot at the bottom of the dive.



40. A 90-kg pilot pulls out of a vertical dive at 685 km/hr along an arc of a circle of 1500-m radius. Find the centripetal acceleration, centripetal force, and the net force on the pilot at the bottom of the dive.

†41. What is the minimum speed of an airplane in making a vertical loop such that an object in the plane will not fall during the peak of the loop? The radius of the loop is 1000 ft.

†42. A rope is attached to a pail of water and the pail is then rotated in a vertical circle of 80.0-cm radius. What must the minimum speed of the pail of water be such that the water will not spill out?

43. A mass is attached to a string and is swung in a vertical circle. At a particular instant the mass is moving at a speed v, and its velocity vector makes an angle θ with the horizontal. Show that the normal component of the acceleration is given by

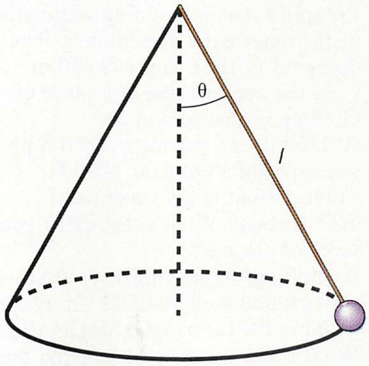
$$T + w\sin\theta = mv^2/r$$

and the tangential component of the acceleration is given by

$$a_T = -g\cos\theta$$

Hence show why this motion in a vertical circle is not uniform circular motion.

†44. A 10.0-N ball attached to a string 1.00 m long moves in a horizontal circle. The string makes an angle of 60.0° with the vertical. (a) Find the tension in the string. (b) Find the component of the tension that supplies the necessary centripetal force. (c) Find the speed of the ball.



45. A mass $m_A = 35.0$ g is on a smooth horizontal table. It is connected by a string that passes through the center of the table to a mass $m_B = 25.0$ g. At what uniform speed should m_A move in a circle of radius $r = 40.0$ cm such that mass m_B remains motionless?

†46. Three point masses of 30.0 kg, 50.0 kg, and 70.0 kg are located at the vertices of an equilateral triangle 1.00 m on a side. Find the resultant gravitational force on each mass.

†47. Four metal spheres are located at the corners of a square of sides of 0.300 m. If each sphere has a mass of 10.0 kg, find the force on the sphere in the lower right-hand corner.

48. What is the gravitational force between the earth and the moon? If a steel cable can withstand a force of 7.50×10^4 N/cm^2, what must the diameter of a steel cable be to sustain the equivalent force?

†49. At what speed would the earth have to rotate such that the centripetal force at the equator would be equal to the weight of a body there? If the earth rotated at this velocity, how long would a day be? If a 200-lb man stood on a weighing scale there, what would the scales read?

†50. What would the mass of the earth have to be in order that the gravitational force is inadequate to supply the necessary centripetal force to keep a person on the surface of the earth at the equator? What density would this correspond to? Compare this to the actual density of the earth.

†51. Compute the gravitational force of the sun on the earth. Then compute the gravitational force of the moon on the earth. Which do you think would have a greater effect on the tides, the sun or the moon? Which has the greatest effect?

†52. Find the force exerted on 1.00 kg of water by the moon when (a) the 1.00 kg is on the side nearest the moon and (b) when the 1.00 kg is on the side farthest from the moon. Would this account for tides?

†53. By how much does (a) the sun and (b) the moon change the value of g at the surface of the earth?

54. How much greater would the range of a projectile be on the moon than on the earth?

†55. Find the point between the earth and the moon where the gravitational forces of earth and moon are equal. Would this be a good place to put a satellite?

†56. An earth satellite is in a circular orbit 110 mi above the earth. The period, the time for one orbit, is 88.0 min. Determine the velocity of the satellite and the acceleration of gravity in the satellite at the satellite altitude.

†57. Show that Kepler's third law, which shows the relationship between the period of motion and the radius of the orbit, can be found for circular orbits by equating the centripetal force to the gravitational force, and obtaining

$$T^2 = \frac{4\pi^2 r^3}{Gm}$$

†58. Using Kepler's third law from problem 57, find the mass of the sun. If the radius of the sun is 7.00×10^8 m, find its density.

†59. The speed of the earth around the sun was found, using dynamical principles in the example 6.11 of section 6.14, to be 66,600 mph. Show that this result is consistent with a purely kinematical calculation of the speed of the earth about the sun.

†60. A better approximation for equation 6.52, the "burn time" for the rocket engines, can be obtained if the rate at which the rocket fuel burns, is a known constant. The rate at which the fuel burns is then given by $\Delta m/\Delta t = K$. Hence, the mass at any time during the burn will be given by $(m_0 - K\Delta t)$, where m_0 is the initial mass of the rocket ship before the engines are turned on. Show that for this approximation the time of burn becomes

$$\Delta t = \frac{m_0 \Delta v}{F + K\Delta v}$$

†61. If a spacecraft is to transfer from a 200 nautical mile earth parking orbit to an 80.0 nautical mile lunar parking orbit by a Hohmann transfer ellipse, find (a) the location of the center of mass of the earth-moon system, (b) the perigee distance of the transfer ellipse, (c) the apogee distance, (d) the semimajor axis of the ellipse, (e) the speed of the spacecraft in the earth circular parking orbit, (f) the speed necessary for insertion into the Hohmann transfer ellipse, (g) the necessary Δv for this insertion, (h) the speed of the spacecraft in a circular lunar parking orbit, (i) the speed of the spacecraft on the Hohmann transfer at time of lunar insertion, and (j) the necessary Δv for insertion into the lunar parking orbit.

Interactive Tutorials

62. Two masses $m_1 = 5.10 \times 10^{21}$ kg and $m_2 = 3.00 \times 10^{14}$ kg are separated by a distance $r = 4.30 \times 10^5$ m. Calculate their gravitational force of attraction.

63. Planet X has mass $m_p = 3.10 \times 10^{25}$ kg and a radius $r_p = 5.40 \times 10^7$ m. Calculate the acceleration due to gravity g at distances of 1–10 planet radii from the planet's surface.

64. Find the angle of bank for a car making a turn on a banked road.

65. Find the speed of a satellite in a circular orbit about its parent body.

66. You are to plan a trip to the planet Mars using the Hohmann transfer ellipse described in the "Have you ever wondered . . . ?" section. The spacecraft is to transfer from a 925-km earth circular parking orbit to a 185-km circular parking orbit around Mars. Find (a) the center of mass of the Earth-Sun-Mars system, (b) the perigee distance of the transfer ellipse, (c) the apogee distance of the transfer ellipse, (d) the semimajor axis of the ellipse, (e) the speed of the spacecraft in the earth parking orbit, (f) the speed necessary for insertion into the Hohmann transfer ellipse, (g) the necessary Δv for insertion into the transfer ellipse, (h) the necessary speed in the Mars circular parking orbit, (i) the speed of the spacecraft in the transfer ellipse at Mars, and (j) the necessary Δv for insertion into the Mars parking orbit.