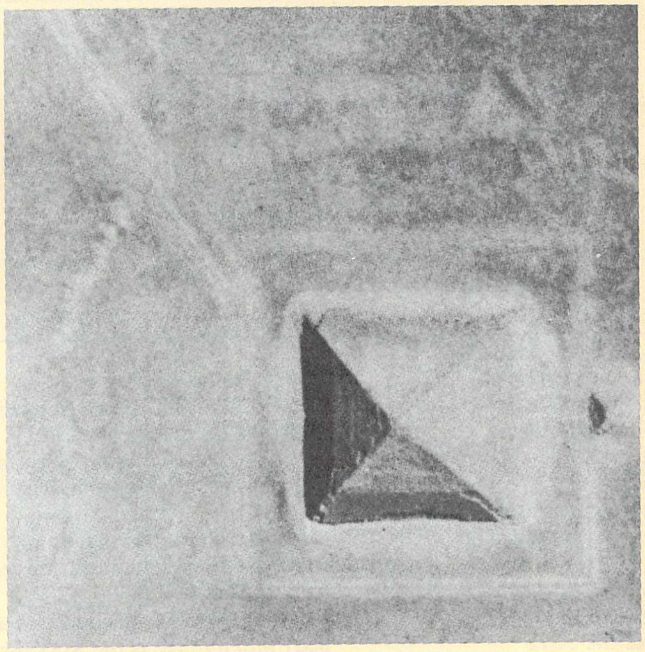


Figure 4
Aerial view of the pyramid of Dashur.

The inclined plane is called a simple machine. With it, we have amplified our ability to move a very heavy stone to the top of the hill. This amplification is called the *ideal mechanical advantage* (IMA) of the inclined plane and is defined as

$$\text{Ideal mechanical advantage} = \frac{\text{Force out}}{\text{Force in}} \tag{H7.6}$$

or

$$\text{IMA} = \frac{F_{out}}{F_{in}} \tag{H7.7}$$

The force that we get out of the machine, in this example, is the weight of the stone w_s, which ends up at the top of the incline, while the force into the machine is equal to the force F that is exerted on the stone in pushing it up the incline. Thus, the ideal mechanical advantage is

$$\text{IMA} = \frac{w_s}{F} \tag{H7.8}$$

Using equation H7.4 this becomes

$$\text{IMA} = \frac{w_s}{F} = \frac{L}{h} \tag{H7.9}$$

Hence if $L = 10h$, the IMA is

$$\text{IMA} = 10\frac{h}{h} = 10$$

and the amplification of the force is 10.

The angle θ of the inclined plane, found from the geometry of figure 3, is

$$\sin\theta = \frac{h}{L} \tag{H7.10}$$

Thus, by making θ very small, a slight incline, a very small force could be applied to move the very massive stones of the pyramid into position. The inclined plane does not give us something for nothing, however. The work done in lifting the stone or pushing the stone is the same. Hence, the smaller force F must be exerted for a very large distance L to do the same work as lifting the very massive stone to the relatively short height h. However, if we are limited by the force F that we can exert, as were the ancient Egyptians, then the inclined plane gives us a decided advantage. An aerial view of the pyramid of Dashur is shown in figure 4. Notice the ramp under the sands leading to the pyramid.[1]

1. This picture is taken from *Secrets of the Great Pyramids* by Peter Tompkins, Harper Colophon Books, 1978.

The Language of Physics

Energy
The ability of a body or system of bodies to perform work (p. 189).

System
An aggregate of two or more particles that is treated as an individual unit (p. 189).

Work
The product of the force acting on a body in the direction of the displacement, times the displacement of the body (p. 189).

Power
The time rate of doing work (p. 192).

Gravitational potential energy
The energy that a body possesses by virtue of its position in a gravitational field. The potential energy is equal to the work that must be done to put the body into that particular position (p. 194).

Kinetic energy
The energy that a body possesses by virtue of its motion. The kinetic energy is equal to the work that must be done to bring the body from rest into that state of motion (p. 196).

Closed system
An isolated system that is not affected by any external influences (p. 198).

Law of conservation of energy
In any closed system, the total energy of the system remains a constant. To say that energy is conserved means that the energy is a constant (p. 198).

Conservative system
A system in which the difference in energy is the same regardless of the path taken between two different positions. In a conservative system the total mechanical energy is conserved (p. 205).

Summary of Important Equations

Work done
$W = Fx$ (7.1)

Work done in general
$W = Fx \cos \theta$ (7.2)

Power
$P = W/t$ (7.3)

Power of moving system
$P = Fv$ (7.4)

Gravitational potential energy
$PE = mgh$ (7.7)

Kinetic energy
$KE = \frac{1}{2}mv^2$ (7.12)

Total mechanical energy
$E_{tot} = KE + PE$

Conservation of mechanical energy
$\Delta E = E_2 - E_1 = 0$ (7.23)

$E_2 = E_1 = \text{constant}$ (7.24)

Questions for Chapter 7

1. If the force acting on a body is perpendicular to the displacement, how much work is done in moving the body?
2. A person is carrying a heavy suitcase while walking along a horizontal corridor. Does the person do work (a) against gravity (b) against friction?
3. A car is moving at 90 km/hr when it is braked to a stop. Where does all the kinetic energy of the moving car go?

†4. A rowboat moves in a northerly direction upstream at 3 mph relative to the water. If the current moves south at 3 mph relative to the bank, is any work being done?

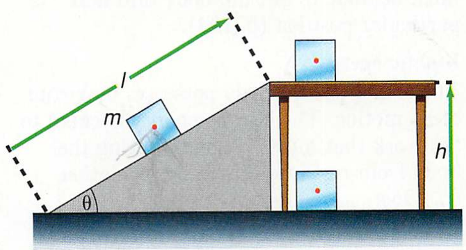
†5. For a person to lose weight, is it more effective to exercise or to cut down on the intake of food?

6. If you lift a body to a height h with a force that is greater than the weight of a body, where does the extra energy go?
7. Potential energy is energy that a body possesses by virtue of its position, while kinetic energy is energy that a body possesses by virtue of its speed. Could there be an energy that a body possesses by virtue of its acceleration? Discuss.
8. For a conservative system, what is $\Delta E/\Delta t$?
9. Describe the transformation of energy in a pendulum as it moves back and forth.
10. If positive work is done putting a body into motion, is the work done in bringing a moving body to rest negative work? Explain.

Problems for Chapter 7

7.2 Work

1. A 500-lb box is raised through a height of 15.0 ft. How much work is done in lifting the box at a constant velocity?
2. How much work is done if (a) a force of 150 N is used to lift a 10.0-kg mass to a height of 5.00 m and (b) a force of 150 N, parallel to the surface, is used to pull a 10.0-kg mass, 5.00 m on a horizontal surface?
3. A force of 8.00 N is used to pull a sled through a distance of 100 m. If the force makes an angle of 40.0° with the horizontal, how much work is done?
4. A person pushes a lawn mower with a force of 50.0 N at an angle of 35.0° below the horizontal. If the mower is moved through a distance of 25.0 m, how much work is done?
5. A consumer's gas bill indicates that they have used a total of 37 therms of gas for a 30-day period. Express this energy in joules. A therm is a unit of energy equal to 100,000 Btu and a Btu (British thermal unit) is a unit of energy equal to 778 ft lb.
6. A 670-kg man lifts a 200-kg mass to a height of 1.00 m above the floor and then carries it through a horizontal distance of 10.0 m. How much work is done (a) against gravity in lifting the mass, (b) against gravity in carrying it through the horizontal distance, and (c) against friction in carrying it through the horizontal distance?
7. Calculate the work done in (a) pushing a 4.00-kg block up a frictionless inclined plane 10.0 m long that makes an angle of 30.0° with the horizontal and (b) lifting the block vertically from the ground to the top of the plane, 5.00 m high. (c) Compare the force used in parts a and b.



8. A football player weighs 200 lb and does a chin-up by pulling himself up by his arms to an additional height of 30.0 in. above the floor. If he does a total of 20 chin-ups, how much work does he do?
9. A 110-kg football player does a chin-up by pulling himself up by his arms an additional height of 50.0 cm above the floor. If he does a total of 25 chin-ups, how much work does he do?

7.3 Power

10. A consumer's electric bill indicates that they have used a total of 793 kwh of electricity for a 30-day period. Express this energy in (a) joules and (b) ft lb. (c) What is the average power used per hour?
11. A 150-lb person climbs a rope at a constant velocity of 2.00 ft/s in a period of time of 10.0 s. (a) How much power does the person expend? (b) How much work is done?
12. You are designing an elevator that must be capable of lifting a load (elevator plus passengers) of 4000 lb to a height of 12 floors (120 ft) in

1 min. What horsepower motor should you require if half of the power is used to overcome friction?

13. A locomotive pulls a train at a velocity of 40.0 mph with a force of 15,000 lb. What power is exerted by the locomotive?

14. A locomotive pulls a train at a velocity of 88.0 km/hr with a force of 55,000 N. What power is exerted by the locomotive?

7.4 Gravitational Potential Energy

15. Find the potential energy of a 7.00-kg mass that is raised 2.00 m above the desk. If the desk is 1.00 m high, what is the potential energy of the mass with respect to the floor?

16. A 5.00-kg block is at the top of an inclined plane that is 4.00 m long and makes an angle of 35.0° with the horizontal. Find the potential energy of the block.

17. A 15.0-kg sledge hammer is 2.00 m high. How much work can it do when it falls to the ground?

18. A pile driver lifts a 500-lb hammer 10.0 ft before dropping it on a pile. If the pile is driven 4.00 in. into the ground when hit by the hammer, what is the average force exerted on the pile?

7.5 Kinetic Energy

19. What is the kinetic energy of the earth as it travels at a velocity of 30.0 km/s in its orbit about the sun?

20. Compute the kinetic energy of a 3200-lb auto traveling at (a) 15.0 mph, (b) 30.0 mph, and (c) 60.0 mph.

21. Compare the kinetic energy of a 1200-kg auto traveling at (a) 30.0 km/hr, (b) 60.0 km/hr, and (c) 120 km/hr.

22. If an electron in a hydrogen atom has a velocity of 2.19×10^6 m/s, what is its kinetic energy?

23. A 1000-lb airplane traveling at 150 mph is 3000 ft above the terrain. What is its kinetic energy and its potential energy?

24. A 700-kg airplane traveling at 320 km/hr is 1500 m above the terrain. What is its kinetic energy and its potential energy?

25. A 10.0-g bullet, traveling at a velocity of 900 m/s hits and is embedded 2.00 cm into a large piece of oak wood that is fixed at rest. What is the kinetic energy of the bullet? What is the average force stopping the bullet?

26. A little league baseball player throws a baseball (0.15 kg) at a speed of 8.94 m/s. (a) How much work must be done to catch this baseball? (b) If the catcher moves his glove backward by 2.00 cm while catching the ball, what is the average force exerted on his glove by the ball? (c) What is the average force if the distance is 20.0 cm? Is there an advantage in moving the glove backward?

7.6 The Conservation of Energy

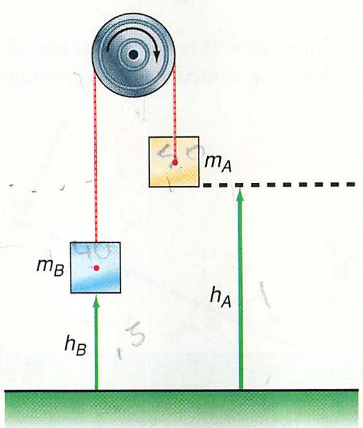
27. A 2.00-kg block is pushed along a horizontal frictionless table a distance of 3.00 m, by a horizontal force of 12.0 N. Find (a) how much work is done by the force, (b) the final kinetic energy of the block, and (c) the final velocity of the block. (d) Using Newton's second law, find the acceleration and then the final velocity.

28. A 2.75-kg block is placed at the top of a 40.0° frictionless inclined plane that is 40.0 cm high. Find (a) the work done in lifting the block to the top of the plane, (b) the potential energy at the top of the plane, (c) the kinetic energy when the block slides down to the bottom of the plane, (d) the velocity of the block at the bottom of the plane, and (e) the work done in sliding down the plane.

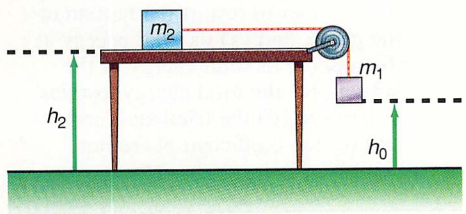
29. A projectile is fired vertically with an initial velocity of 200 ft/s. Using the law of conservation of energy, find how high the projectile rises.

30. A 3.00-kg block is lifted vertically through a height of 6.00 m by a force of 40.0 N. Find (a) the work done in lifting the block, (b) the potential energy of the block at 6.00 m, (c) the kinetic energy of the block at 6.00 m, and (d) the velocity of the block at 6.00 m.

31. Apply the law of conservation of energy to an Atwood's machine and find the velocity of block A as it hits the ground. $m_B = 40.0$ g, $m_A = 50.0$ g, $h_B = 0.500$ m, and $h_A = 1.00$ m.



†32. Determine the velocity of block 2 when the height of block 1 is equal to $h_0/4$. $m_2 = 35.0$ g, $m_1 = 20.0$ g, $h_0 = 1.50$ m, and $h_2 = 2.00$ m.



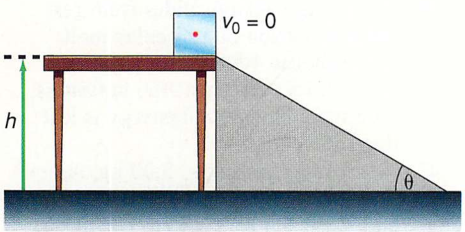
33. A 250-g bob is attached to a string 1.00 m long to make a pendulum. If the pendulum bob is pulled to the right, such that the string makes an angle of 15.0° with the vertical, what is (a) the maximum potential energy, (b) the maximum kinetic energy, and (c) the maximum velocity of the bob and where does it occur?

34. A 45.0-kg girl is on a swing that is 2.00 m long. If the swing is pulled to the right, such that the rope makes an angle of 30.0° with the vertical, what is (a) the maximum potential energy of the girl, (b) her maximum kinetic energy, and (c) the maximum velocity of the swing and where does it occur?

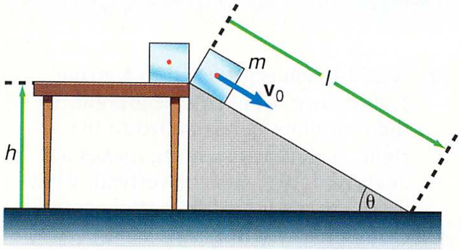
7.7 Further Analysis of the Conservation of Energy

35. A 3.56-kg mass moving at a speed of 3.25 m/s enters a region where the coefficient of kinetic friction is 0.500. How far will the block move before it comes to rest?

36. A 5.00-kg mass is placed at the top of a 35.0° rough inclined plane that is 30.0 cm high. The coefficient of kinetic friction between the mass and the plane is 0.400. Find (a) the potential energy at the top of the plane, (b) the work done against friction as it slides down the plane, (c) the kinetic energy of the mass at the bottom of the plane, and (d) the velocity of the mass at the bottom of the plane.



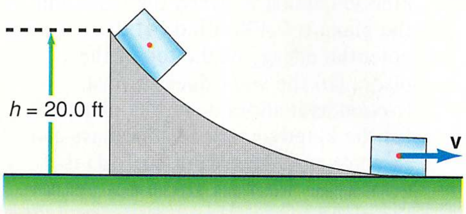
37. A 100-g block is pushed down a rough inclined plane with an initial velocity of 1.50 m/s. The plane is 2.00 m long and makes an angle of 35.0° with the horizontal. If the block comes to rest at the bottom of the plane, find (a) its total energy at the top, (b) its total energy at the bottom, (c) the total energy lost due to friction, (d) the frictional force, and (e) the coefficient of friction.



38. A 1.00-kg block is pushed along a rough horizontal floor with a horizontal force of 5.00 N for a distance of 5.00 m. If the block is moving at a constant velocity of 4.00 m/s, find (a) the work done on the block by the force, (b) the kinetic energy of the block, and (c) the energy lost to friction.

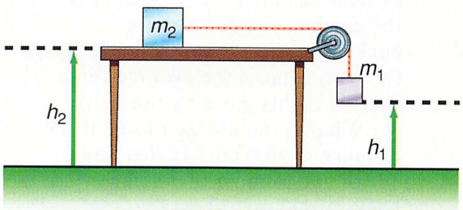
39. A 500-lb box is pushed along a rough floor by a horizontal force. The block moves at constant velocity for a distance of 15.0 ft. If the coefficient of friction between the box and the floor is 0.30, how much work is done in moving the box?

40. A 10.0-lb package slides from rest down a portion of a circular mail chute that is 20.0 ft above the ground. Its velocity at the bottom is 20.0 ft/s. How much energy is lost due to friction?

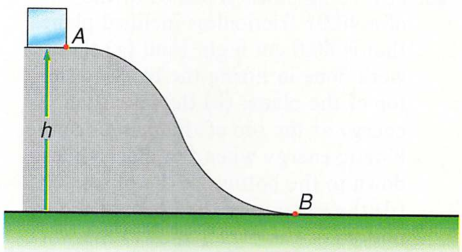


41. A 6.68-kg package slides from rest down a portion of a circular mail chute that is 4.58 m above the ground. Its velocity at the bottom is 7.63 m/s. How much energy is lost due to friction?

42. In the diagram m_2 = 3.00 kg, m_1 = 5.00 kg, h_2 = 1.00 m, h_1 = 0.750 m, and μ_k = 0.400. Find (a) the initial total energy of the system, (b) the work done against friction as m_2 slides on the rough surface, (c) the velocity v_1 of mass m_1 as it hits the ground, and (d) the kinetic energy of m_1 as it hits the ground.



†43. A 5.00-kg body is placed at the top of the track, position *A*, 2.00 m above the base of the track, as shown in the diagram. (a) Find the total energy of the block. (b) The block is allowed to slide from rest down the frictionless track to the position *B*. Find the velocity of the body at *B*. (c) The block then moves over the level rough surface of μ_k = 0.300. How far will the block move before coming to rest?



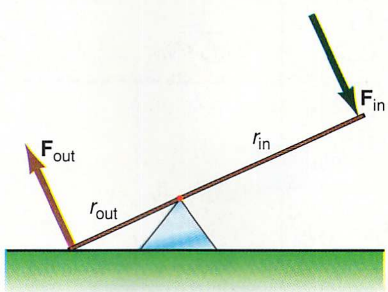
44. A 0.500-kg ball is dropped from a height of 3.00 m. Upon hitting the ground it rebounds to a height of 1.50 m. (a) How much mechanical energy is lost in the rebound, and what happens to this energy? (b) What is the velocity just before and just after hitting the ground?

Additional Problems

†45. The concept of work can be used to describe the action of a lever. Using the principle of work in equals work out, show that

$$F_{out} = \frac{r_{in}}{r_{out}} F_{in}$$

Show how this can be expressed in terms of a mechanical advantage.



†46. Show how the inclined plane can be considered as a simple machine by comparing the work done in sliding an object up the plane with the work done in lifting the block to the top of the plane. How does the inclined plane supply a mechanical advantage?

47. A force acting on a 300-g mass causes it to move at a constant speed over a rough surface. The coefficient of kinetic friction is 0.350. Find the work required to move the mass a distance of 2.00 m.

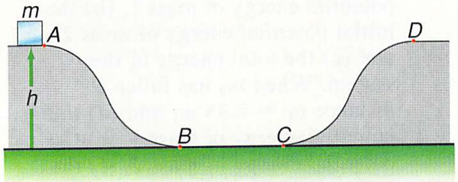
48. A 5.00-kg projectile is fired at an angle of 58.0° above the horizontal with the initial velocity of 30.0 m/s. Find (a) the total energy of the projectile, (b) the total energy in the vertical direction, (c) the total energy in the horizontal direction, (d) the total energy at the top of the trajectory, (e) the potential energy at the top of the trajectory, (f) the maximum height of the projectile, (g) the kinetic energy at the top of the trajectory, and (h) the velocity of the projectile as it hits the ground.

49. It takes 20,000 W to keep a 1600-kg car moving at a constant speed of 60.0 km/hr on a level road. How much power is required to keep the car moving at the same speed up a hill inclined at an angle of 22.0° with the horizontal?

50. John consumes 5000 kcal/day. His metabolic efficiency is 70.0%. If his normal activity utilizes 2000 kcal/day, how many hours will John have to exercise to work off the excess calories by (a) walking, which uses 3.80 kcal/hr; (b) swimming, which uses 8.00 kcal/hr; and (c) running, which uses 11.0 kcal/hr?

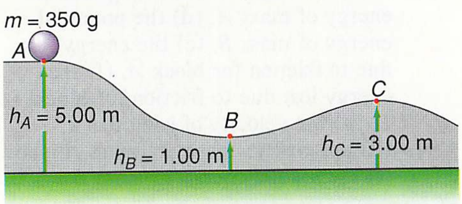
51. A 2.50-kg mass is at rest at the bottom of a 5.00-m-long rough inclined plane that makes an angle of 25.0° with the horizontal. When a constant force is applied up the plane and parallel to it, it causes the mass to arrive at the top of the incline at a speed of 0.855 m/s. Find (a) the total energy of the mass when it is at the top of the incline, (b) the work done against friction, and (c) the magnitude of the applied force. The coefficient of friction between the mass and the plane is 0.350.

†52. A 2.00-kg block is placed at the position *A* on the track that is 3.00 m above the ground. Paths *A-B* and *C-D* of the track are frictionless, while section *B-C* is rough with a coefficient of kinetic friction of 0.350 and a length of 1.50 m. Find (a) the total energy of the block at *A*, (b) the velocity of the block at *B*, (c) the energy lost along path *B-C*, and (d) how high the block rises along path *C-D*.



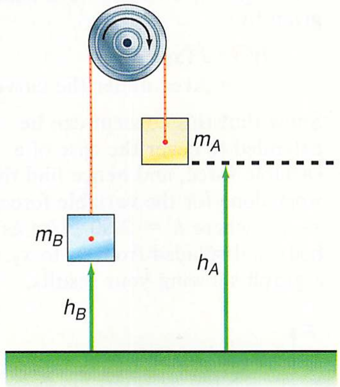
53. A mass $m = 3.50$ kg is launched with an initial velocity $v_0 = 1.50$ m/s from the position *A* at a height $h = 3.80$ m above the reference plane in the diagram for problem 52. Paths *A-B* and *C-D* of the track are frictionless, while path *B-C* is rough with a coefficient of kinetic friction of 0.300 and a length of 3.00 m. Find (a) the number of oscillations the block makes before coming to rest along the path *B-C* and (b) where the block comes to rest on path *B-C*.

54. A ball starts from rest at position *A* at the top of the track. Find (a) the total energy at *A*, (b) the total energy at *B*, (c) the velocity of the ball at *B*, and (d) the velocity of the ball at *C*.

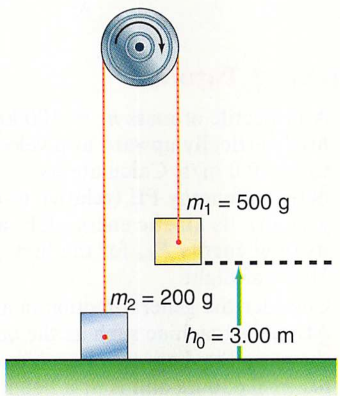


55. A 20.0-kg mass is at rest on a rough horizontal surface. It is then accelerated by a net constant force of 8.6 N. After the mass has moved 1.5 m from rest, the force is removed and the mass comes to rest in 2.00 m. Using energy methods find the coefficient of kinetic friction.

56. In an Atwood's machine $m_B = 30.0$ g, $m_A = 50.0$ g, $h_B = 0.400$ m, and $h_A = 0.800$ m. The machine starts from rest and mass m_A acquires a velocity of 1.25 m/s as it strikes the ground. Find the energy lost due to friction in the bearings of the pulley.



†57. What is the total energy of the Atwood's machine in the position shown in the diagram? If the blocks are released and m_1 falls through a distance of 1.00 m, what is the kinetic and potential energy of each block, and what are their velocities?



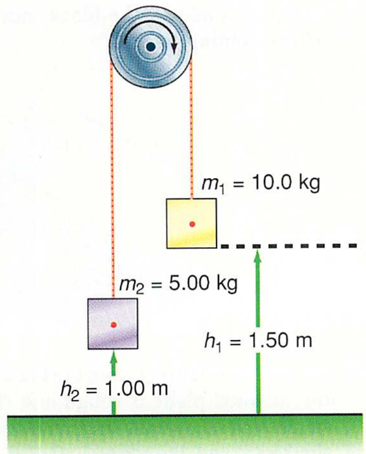
†58. The gravitational potential energy of a mass *m* with respect to infinity is given by

$$\text{PE} = -\frac{Gm_E m}{r}$$

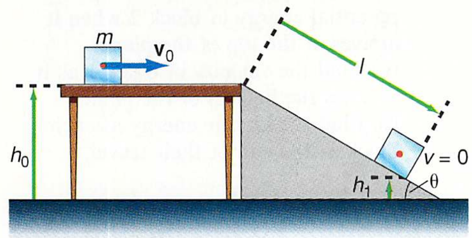
where *G* is the universal gravitational constant, m_E is the mass of the earth, and *r* is the distance from the center of the earth to the mass *m*. Find the escape velocity of a spaceship from the earth. (The escape velocity is the necessary velocity to remove a body from the gravitational attraction of the earth.)

†59. Modify problem 58 and find the escape velocity for (a) the moon, (b) Mars, and (c) Jupiter.

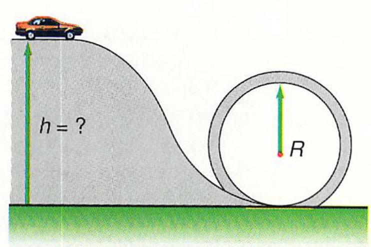
†60. The entire Atwood's machine shown is allowed to go into free-fall. Find the velocity of m_1 and m_2 when the entire system has fallen 1.00 m.



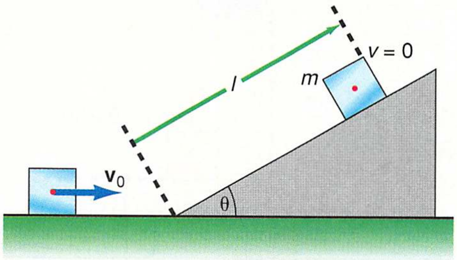
†61. A 1.50-kg block moves along a smooth horizontal surface at 2.00 m/s. The horizontal surface is at a height h_0 above the ground. The block then slides down a rough hill, 20.0 m long, that makes an angle of 30.0° with the horizontal. The coefficient of kinetic friction between the block and the hill is 0.600. How far down the hill will the block move before coming to rest?



†62. At what point above the ground must a car be released such that when it rolls down the track and into the circular loop it will be going fast enough to make it completely around the loop? The radius of the circular loop is *R*.

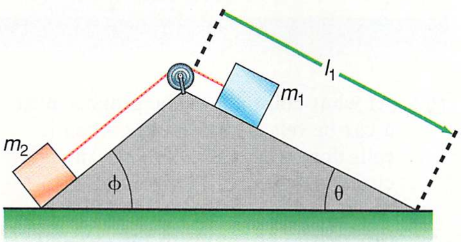


†63. A 1.50-kg block moves along a smooth horizontal surface at 2.00 m/s. It then encounters a smooth inclined plane that makes an angle of 53.0° with the horizontal. How far up the incline will the block move before coming to rest?



†64. Repeat problem 63, but in this case the inclined plane is rough and the coefficient of kinetic friction between the block and the plane is 0.400.

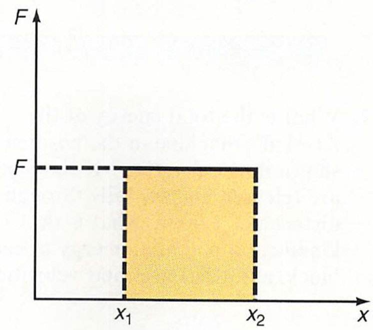
†65. In the diagram mass m_1 is located at the top of a rough inclined plane that has a length $l_1 = 0.500$ m. $m_1 = 0.500$ kg, $m_2 = 0.200$ kg, $\mu_{k_1} = 0.500$, $\mu_{k_2} = 0.300$, $\theta = 50.0°$, and $\phi = 50.0°$. (a) Find the total energy of the system in the position shown. (b) The system is released from rest. Find the work done for block 1 to overcome friction as it slides down the plane. (c) Find the work done for block 2 to overcome friction as it slides up the plane. (d) Find the potential energy of block 2 when it arrives at the top of the plane. (e) Find the velocity of block 1 as it reaches the bottom of the plane. (f) Find the kinetic energy of each block at the end of their travel.



†66. If a constant force acting on a body is plotted against the displacement of the body from x_1 to x_2, as shown in the diagram, then the work done is given by

$$W = F(x_2 - x_1)$$
$$= \text{Area under the curve}$$

Show that this concept can be extended to cover the case of a variable force, and hence find the work done for the variable force, $F = kx$, where $k = 2.00$ N/m as the body is displaced from x_1 to x_2. Draw a graph showing your results.



Interactive Tutorials

67. A projectile of mass $m = 100$ kg is fired vertically upward at a velocity $v_0 = 50.0$ m/s. Calculate its potential energy PE (relative to the ground), its kinetic energy KE, and its total energy E_{tot} for the first 10.0 s of flight.

68. Consider the general motion in an Atwood's machine such as the one shown in the diagram of problem 31; $m_A = 0.650$ kg and is at a height $h_A = 2.55$ m above the reference plane and mass $m_B = 0.420$ kg is at a height $h_B = 0.400$ m. If the system starts from rest, find (a) the initial potential energy of mass A, (b) the initial potential energy of mass B, and (c) the total energy of the system. When m_A has fallen a distance $y_A = 0.75$ m, find (d) the potential energy of mass A, (e) the potential energy of mass B, (f) the speed of each mass at that point, (g) the kinetic energy of mass A, and (h) the kinetic energy of mass B. (i) When mass A hits the ground, find the speed of each mass.

69. Consider the general motion in the combined system shown in the diagram of problem 42; $m_1 = 0.750$ kg and is at a height $h_1 = 1.85$ m above the reference plane and mass $m_2 = 0.285$ kg is at a height $h_2 = 2.25$ m, $\mu_k = 0.450$. If the system starts from rest, find (a) the initial potential energy of mass 1, (b) the initial potential energy of mass 2, and (c) the total energy of the system. When m_1 has fallen a distance $y_1 = 0.35$ m, find (d) the potential energy of mass 1, (e) the potential energy of mass 2, (f) the energy lost due to friction as mass 2 slides on the rough surface, (g) the speed of each mass at that point, (h) the kinetic energy of mass 1, and (i) the kinetic energy of mass 2. (j) When mass 1 hits the ground, find the speed of each mass.

70. Consider the general case of motion shown in the diagram with mass m_A initially located at the top of a rough inclined plane of length l_A, and mass m_B is at the bottom of the second plane; x_A is the distance from the mass A to the bottom of the plane. Let $m_A = 0.750$ kg, $m_B = 0.250$ kg, $l_A = 0.550$ m, $\theta = 40.0°$, $\phi = 30.0°$, $\mu_{k_A} = 0.400$, $\mu_{k_B} = 0.300$, and $x_A = 0.200$ m. When $x_A = 0.200$ m, find (a) the initial total energy of the system, (b) the distance block B has moved, (c) the potential energy of mass A, (d) the potential energy of mass B, (e) the energy lost due to friction for block A, (f) the energy lost due to friction for block B, (g) the velocity of each block, (h) the kinetic energy of mass A, and (i) the kinetic energy of mass B.