The Language of Physics

Angular displacement
The angle that a body rotates through while in rotational motion (p. 241).

Angular velocity
The change in the angular displacement of a rotating body about the axis of rotation with time (p. 242).

Angular acceleration
The change in the angular velocity of a rotating body with time (p. 243).

Kinematic equations for rotational motion
A set of equations that give the angular displacement and angular velocity of a rotating body at any instant of time, and the angular velocity at a particular angular displacement, if the angular acceleration of the body is constant (p. 244).

Kinetic energy of rotation
The energy that a body possesses by virtue of its rotational motion (p. 248).

Moment of inertia
The measure of the resistance of a body to a change in its rotational motion. It is the rotational analogue of mass, which is a measure of the resistance of a body to a change in its translational motion. The larger the moment of inertia of a body the more difficult it is to put that body into rotational motion (p. 248).

Newton's second law for rotational motion
When an unbalanced external torque acts on a body, it gives that body an angular acceleration. The angular acceleration is directly proportional to the torque and inversely proportional to the moment of inertia (p. 250).

Newton's first law for rotational motion
A body in motion at a constant angular velocity will continue in motion at that same constant angular velocity unless acted upon by some unbalanced external torque (p. 251).

Newton's third law of rotational motion
If body A and body B have the same axis of rotation, and if body A exerts a torque on body B, then body B exerts an equal but opposite torque on body A (p. 251).

Angular momentum
The product of the moment of inertia of a rotating body and its angular velocity (p. 259).

Law of conservation of angular momentum
If the total external torque acting on a system is zero, then there is no change in the angular momentum of the system, and the final angular momentum is equal to the initial angular momentum (p. 260).

Summary of Important Equations

Angular velocity
$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\theta}{t} \quad (9.1)$$

Angular acceleration
$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega - \omega_0}{t} \quad (9.3)$$

Kinematic equations
$$\omega = \omega_0 + \alpha t \quad (9.4)$$
$$\theta = \omega_0 t + \tfrac{1}{2}\alpha t^2 \quad (9.9)$$
$$\omega^2 = \omega_0^2 + 2\alpha\theta \quad (9.10)$$

Relations between translational and rotational variables
$$s = r\theta \quad (6.5)$$
$$v = r\omega \quad (9.2)$$
$$a = r\alpha \quad (9.5)$$

Centripetal acceleration
$$a_c = \omega^2 r \quad (9.11)$$

Kinetic energy of rotation
$$KE_{rot} = \tfrac{1}{2}I\omega^2 \quad (9.15)$$

Moment of inertia
$$I = \sum_{i=1}^{n} m_i r_i^2 \quad (9.16)$$

Moment of inertia for a single mass
$$I = mr^2 \quad (9.17)$$

Newton's second law for rotational motion
$$\tau = I\alpha \quad (9.22)$$

Angular momentum
$$L = I\omega \quad (9.48)$$

Newton's second law in terms of momentum
$$\tau = \frac{\Delta L}{\Delta t} \quad (9.49)$$

Law of conservation of angular momentum (no external torques)
$$L_f = L_i \quad (9.52)$$

Work done in rotational motion
$$W = \tau\theta \quad (9.72)$$

Power expended in rotational motion
$$P = \tau\omega \quad (9.74)$$

Questions for Chapter 9

1. Discuss the similarity between the equations for translational motion and the equations for rotational motion.
2. When moving in circular motion at a constant angular velocity, why does the body at the greatest distance from the axis of rotation move faster than the body closest to the axis of rotation?
3. It is easy to observe the angular velocity of the second hand of a clock. Why is it more difficult to observe the angular velocity of the minute and hour hands of the clock?

†4. If a cylinder, a ball, and a ring are placed at the top of an inclined plane and then allowed to roll down the plane, in what order will they arrive at the bottom of the plane? Why?

†5. How would you go about approximating the rotational kinetic energy of our galaxy?

6. Which would be more difficult to put into rotational motion, a large sphere or a small sphere? Why?
7. Why must the axis of rotation be specified when giving the moment of inertia of an object?

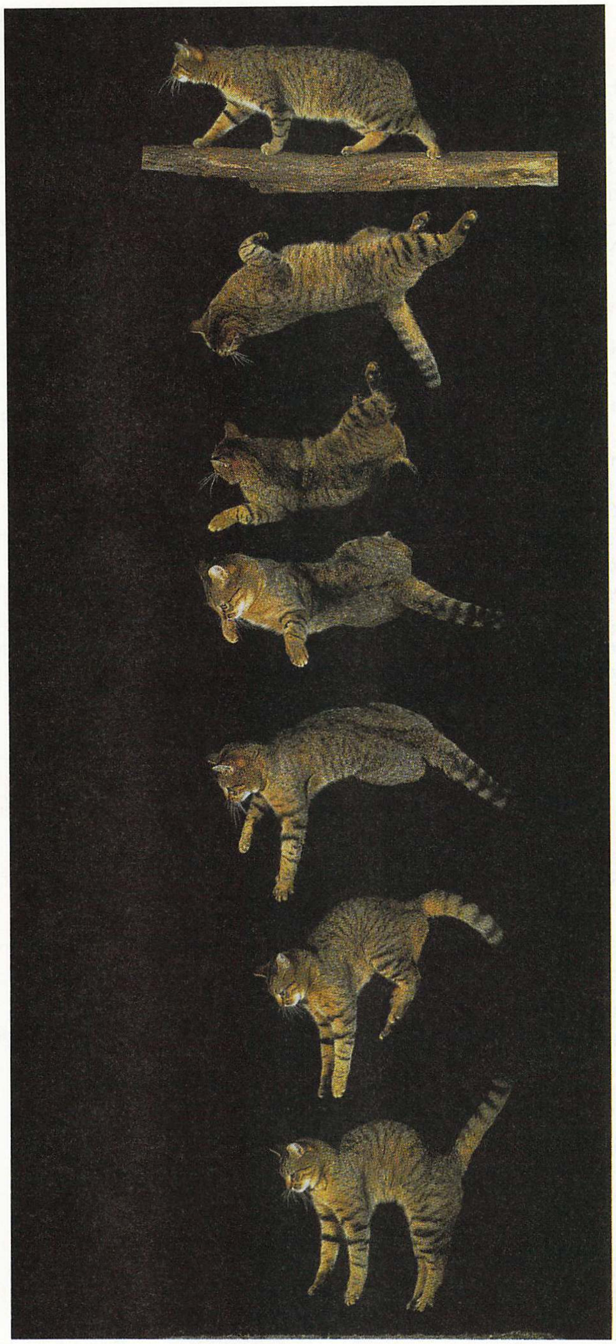
†8. If two balls collide such that the force transmitted lies along a line connecting the center of mass of each body, can either ball be put into rotational motion? If the balls collide in a glancing collision in which there is also friction between the two surfaces as they collide, can either ball be put into rotational motion? Draw a diagram of the collision in both cases and discuss both possibilities.

†9. As long as there are no external torques acting on the earth, the earth will continue to spin forever at its present angular velocity. Discuss the possibility of small perturbative torques that might act on the earth and what effect they might have.

†10. Can the angular displacement, angular velocity, and angular acceleration be treated as vectors? Consider a rotation of your book through an angular displacement of 90° about the x-axis, then a rotation through an angular displacement of 90° about the y-axis, and finally a rotation through an angular displacement of 90° about the z-axis. Would you get the same result if you changed the order of the rotations to the y-, x-, and then z-axis? What happens if the rotations are infinitesimal?

†11. If the instantaneous angular velocity can be considered as a vector, should the angular momentum also be considered as a vector? If so, what direction would it have? What would the change in the direction of the angular momentum look like?

†12. It is said that if you throw a cat, upside down, into the air, it will always land on its feet. Discuss this possibility from the point of view of the cat moving his legs and tail and thus changing his moment of inertia and hence his angular velocity.



Problems for Chapter 9

9.2 Rotational Kinematics

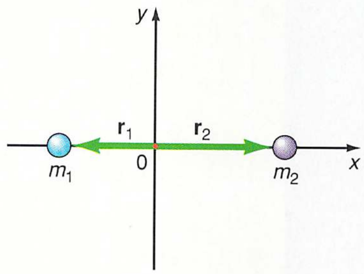
1. Express the following angular velocities of a phonograph turntable in terms of rad/s. (a) 33 1/3 rpm (revolutions per minute), (b) 45 rpm, and (c) 78 rpm.
2. Determine the angular velocity of the following hands of a clock: (a) the second hand, (b) the minute hand, and (c) the hour hand.
3. A cylinder 15.0 cm in diameter rotates at 1000 rpm. (a) What is its angular velocity in rad/s? (b) What is the tangential velocity of a point on the rim of the cylinder?
4. A circular saw blade rotating at 3600 rpm is reduced to 3450 rpm in 2.00 s. What is the angular acceleration of the blade?
5. A circular saw blade rotating at 3600 rpm is braked to a stop in 6 s. What is the angular acceleration? How many revolutions did the blade make before coming to a stop?
6. A wheel 50.0 cm in diameter is rotating at an initial angular velocity of 6.00 rad/s. It is given an acceleration of 2.00 rad/s². Find (a) the angular velocity at 5.00 s, (b) the angular displacement at 5.00 s, (c) the tangential velocity of a point on the rim at 5.00 s, (d) the tangential acceleration of a point on the rim, (e) the centripetal acceleration of a point on the rim, and (f) the resulting acceleration of a point on the rim.

9.3 The Kinetic Energy of Rotation

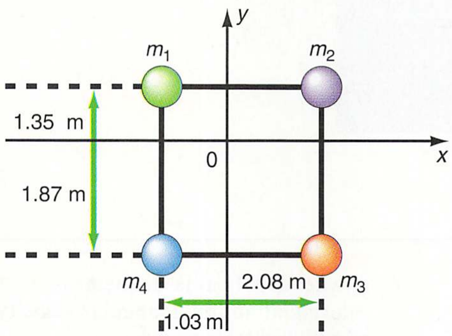
7. Find the kinetic energy of a 2.00-kg cylinder, 25.0 cm in diameter, if it is rotating about its longitudinal axis at an angular velocity of 0.550 rad/s.
8. A 3.00-kg ball, 15.0 cm in diameter, rotates at an angular velocity of 3.45 rad/s. Find its kinetic energy.

9.4 The Moment of Inertia

9. Calculate the moment of inertia of a 0.500-kg meterstick about an axis through its center, and perpendicular to its length.
10. Compute the moment of inertia through its center of a 16.0-lb bowling ball of radius 4.00 in.
11. Find the moment of inertia for the system of point masses shown for (a) rotation about the y-axis and (b) for rotation about the x-axis. Given are $m_1 = 2.00$ kg, $m_2 = 3.50$ kg, $r_1 = 0.750$ m, and $r_2 = 0.873$ m.

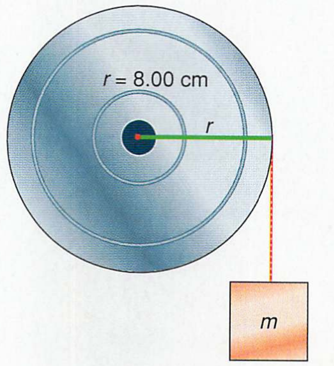


†12. Find the moment of inertia for the system shown for rotation about (a) the y-axis, (b) the x-axis, and (c) an axis going through masses m_2 and m_4. Assume $m_1 = 0.532$ kg, $m_2 = 0.425$ kg, $m_3 = 0.879$ kg, and $m_4 = 0.235$ kg.

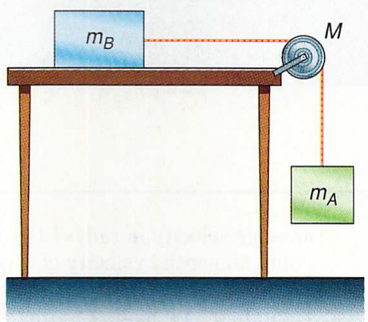


9.5 Newton's Laws for Rotational Motion and 9.6 Rotational Dynamics

13. A solid wheel of mass 5.00 kg and radius 0.350 m is set in motion by a constant force of 6.00 N applied tangentially. Determine the angular acceleration of the wheel.
14. A torque of 5.00 m N is applied to a body. Of this torque, 2.00 m N of it is used to overcome friction in the bearings. The body has a resultant angular acceleration of 5.00 rad/s^2. (a) When the applied torque is removed, what is the angular acceleration of the body? (b) If the angular velocity of the body was 100 rad/s when the applied torque was removed how long will it take the body to come to rest?
15. A mass of 200 g is attached to a wheel by a string wrapped around the wheel. The wheel has a mass of 1.00 kg. Find the acceleration of the mass. Assume that the moment of inertia of the wheel is the same as a disk.



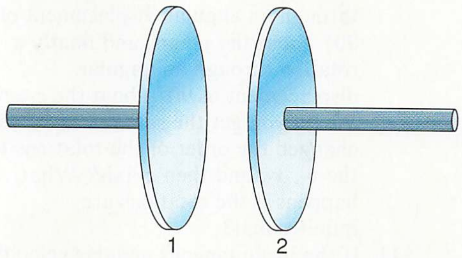
16. A mass m_A of 10.0 kg is attached to another mass m_B of 4.00 kg by a string that passes over a pulley of mass $M = 1.00$ kg. The coefficient of kinetic friction between block B and the table is 0.400. Find (a) the acceleration of each block of the system, (b) the tensions in the cords, and (c) the velocity of block A as it hits the floor 0.800 m below its starting point.



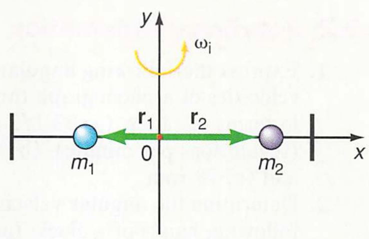
17. A mass $m_A = 100$ g, and another mass $m_B = 200$ g are attached to an Atwood's machine that has a pulley mass $M = 1.00$ kg. (a) Find the acceleration of each block of the system. (b) Find the velocity of mass B as it hits the floor 1.50 meters below its starting point.

9.7 Angular Momentum and Its Conservation

†18. A 75-kg student stands at the edge of a large disk of 150-kg mass that is rotating freely at an angular velocity of 0.800 rad/s. The disk has a radius of $R = 3.00$ m. (a) Find the initial moment of inertia of the disk and student and its kinetic energy. The student now walks toward the center of the disk. Find the moment of inertia, the angular velocity, and the kinetic energy when the student is at (b) $3R/4$, (c) $R/2$, and (d) $R/4$.
19. Two disks are to be made into an idealized clutch. Disk 1 has a mass of 3.00 kg and a radius of 20.0 cm, while disk 2 has a mass of 1.00 kg and a radius of 20.0 cm. If disk 2 is originally at rest and disk 1 is rotating at 2000 rpm, what is the final angular velocity of the coupled disks?



†20. Two beads are fixed on a thin long wire on the x-axis at $r_1 = 0.700$ m and $r_2 = 0.800$ m, as shown in the diagram. Assume $m_1 = 85.0$ g and $m_2 = 63.0$ g. The combination is spinning about the y-axis at an angular velocity of 4.00 rad/s. A catch is then released allowing the beads to move freely to the stops at the end of the wire, which is 1.00 m from the origin. Find (a) the initial moment of inertia of the system, (b) the initial angular momentum of the system, (c) the initial kinetic energy of the system, (d) the final angular momentum of the system, (e) the final angular velocity of the system, and (f) the final kinetic energy of the system.



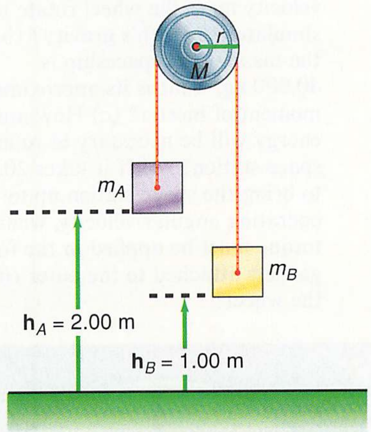
9.8 Combined Translational and Rotational Motion Treated by the Law of Conservation of Energy

21. Find the velocity of (a) a cylinder and (b) a ring at the bottom of an inclined plane that is 2.00 m high. The cylinder and ring start from rest and roll down the plane.

22. Compute the velocity of a cylinder at the bottom of a plane 1.5 m high if (a) it slides without rotating on a frictionless plane and (b) it rotates on a rough plane.

23. A 1.50-kg ball, 10.0 cm in radius, is rolling on a table at a velocity of 0.500 m/s. (a) What is its angular velocity about its center of mass? (b) What is the translational kinetic energy of its center of mass? (c) What is its rotational kinetic energy about its center of mass? (d) What is its total kinetic energy?

24. Using the law of conservation of energy for the Atwood's machine shown, find the velocity of m_A at the ground, if m_A = 20.0 g, m_B = 10.0 g, M = 1.00 kg, and r = 15.0 cm.



9.9 Work in Rotational Motion

25. A constant force of 2.50 N acts tangentially on a cylinder of 12.5-cm radius and the cylinder rotates through an angle of 5.00 rev. How much work is done in rotating the cylinder?

26. An engine operating at 1800 rpm develops 200 hp, what is the torque developed?

Additional Problems

27. Determine (a) the angular velocity of the earth, (b) its moment of inertia, and (c) its kinetic energy of rotation. (d) Compare this with its kinetic energy of translation. (e) Find the angular momentum of the earth.

28. The earth rotates once in a day. If the earth could collapse into a smaller sphere, what would be the radius of that sphere that would give a point on the equator a linear velocity equal to the velocity of light $c = 3.00 \times 10^8$ m/s? Use the initial angular velocity of the earth and the moment of inertia determined in problem 27.

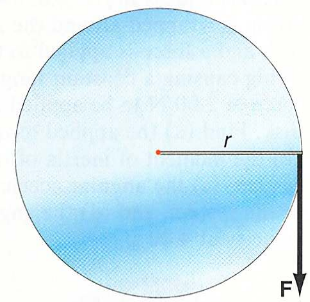
29. A disk of 10.0-cm radius, having a mass of 100 g, is set into motion by a constant tangential force of 2.00 N. Determine (a) the moment of inertia of the disk, (b) the torque applied to the disk, (c) the angular acceleration of the disk, (d) the angular velocity at 2.00 s, (e) the angular displacement at 2.00 s, (f) the kinetic energy at 2.00 s, and (g) the angular momentum at 2.00 s.

30. A 3.50-kg solid disk of 25.5 cm diameter has a cylindrical hole of 3.00-cm radius cut into it. The hole is 1.00 cm in from the edge of the solid disk. Find (a) the initial moment of inertia of the disk about an axis perpendicular to the disk before the hole was cut into it and (b) the moment of inertia of the solid disk with the hole in it. State the assumptions you use in solving the problem.

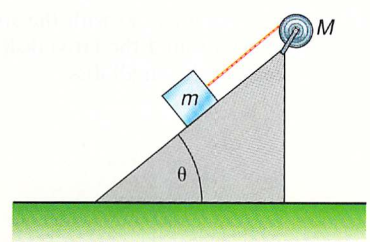
†31. Due to slight effects caused by tidal friction between the water and the land and the nonsphericity of the sun, there is a slight angular deceleration of the earth. The length of a day will increase by approximately 1.5×10^{-3} s in a century. (a) What will be the angular velocity of the earth after one century? (b) What will be the change in the angular velocity of the earth per century? (c) As a first approximation, is it reasonable to assume that there are no external torques acting on the earth and the angular velocity of the earth is a constant?

32. A string of length 1.50 m with a small bob at one end is connected to a horizontal disk of negligible radius at the other end. The disk is put into rotational motion and is now rotating at an angular velocity ω = 5.00 rad/s. Find the angle that the string makes with the vertical.

33. A constant force of 5.00 N acts on a disk of 3.00-kg mass and diameter of 50.0 cm for 10.0 s. Determine (a) the angular acceleration, (b) the angular velocity after 10.0 s, and (c) the kinetic energy after 10.0 s. (d) Compute the work done to cause the disk to rotate and compare with your answer to part c.

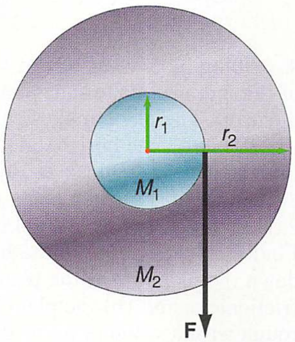


†34. A 5.00-kg block is at rest at the top of the inclined plane shown in the diagram. The plane makes an angle of 32.5° with the horizontal. A string is attached to the block and tied around the disk, which has a mass of 2.00 kg and a radius of 8.00 cm. Find the acceleration of the block down the plane if (a) the plane is frictionless, and (b) the plane is rough with a value of μ_k = 0.54.



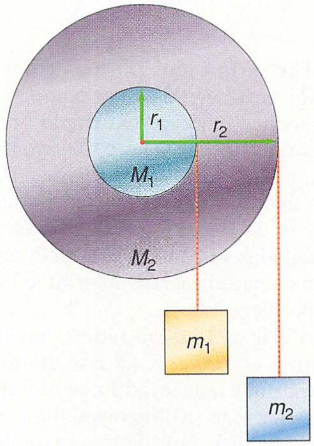
35. A large cylinder has a radius of 12.5 cm and it is pressed against a smaller cylinder of radius 4.50 cm such that the two axes of the cylinders are parallel. When the larger cylinder rotates about its axis, it causes the smaller cylinder to rotate about its axis. The larger cylinder accelerates from rest to a constant angular velocity of 20 rad/s. Find (a) the tangential velocity of a point on the surface of the large cylinder, (b) the tangential velocity of a point on the surface of the smaller cylinder, and (c) the angular velocity of the smaller cylinder. Can you think of this setup as a kind of mechanical advantage?

†36. A small disk of $r_1 = 5.00$-cm radius is attached to a larger disk of $r_2 = 15.00$-cm radius such that they have a common axis of rotation, as shown in the diagram. The small disk has a mass $M_1 = 0.250$ kg and the large disk has a mass $M_2 = 0.850$ kg. A string is wrapped around the small disk and a force is applied to the string causing a constant tangential force of 2.00 N to be applied to the disk. Find (a) the applied torque, (b) the moment of inertia of the system, (c) the angular acceleration of the system, and (d) the angular velocity at 4.00 s.



†37. Repeat problem 36 with the string wrapped around the large disk instead of the small disk.

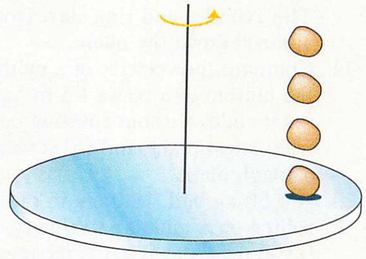
†38. A small disk of mass $M_1 = 50.0$ g is connected to a larger disk of mass $M_2 = 200.0$ g such that they have a common axis of rotation. The small disk has a radius $r_1 = 10.0$ cm, while the large disk has a radius of $r_2 = 30.0$ cm. A mass $m_1 = 25.0$ g is connected to a string that is wrapped around the small disk, while a mass $m_2 = 35.0$ g is connected to a string and wrapped around the large disk, as shown in the diagram. Find (a) the moment of inertia of each disk, (b) the moment of inertia of the combined disks, (c) the net torque acting on the disks, (d) the angular acceleration of the disks, (e) the angular velocity of the disks at 4.00 s, (f) the kinetic energy of the disks at 4.00 s, and (g) the angular momentum of the disks at 4.00 s.



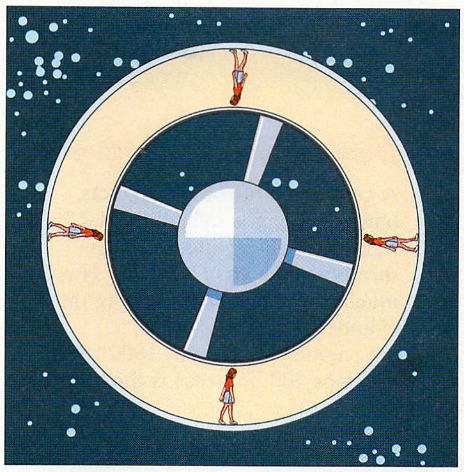
†39. One end of a string is wrapped around a pulley and the other end is connected to the ceiling, which is 3.00 m above the floor. The mass of the pulley is 200 g and has a radius of 10.0 cm. The pulley is released from rest and is allowed to fall. Find (a) the initial total energy of the system, and (b) the velocity of the pulley just before it hits the floor.

40. This is essentially the same problem as problem 39 but is to be treated by Newton's second law for rotational motion. Find the angular acceleration of the cylinder and the tension in the string.

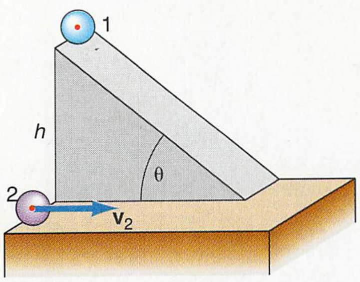
†41. A 1.5-kg disk of 0.500-m radius is rotating freely at an angular velocity of 2.00 rad/s. Small 5-g balls of clay are dropped onto the disk at 3/4 of the radius at a rate of 4 per second. Find the angular velocity of the disk at 10.0 s.



†42. A space station is to be built in orbit in the shape of a large wheel of outside radius 100.0 m and inside radius of 97.0 m. The satellite is to rotate such that it will have a centripetal acceleration exactly equal to the acceleration of gravity g on earth. The astronauts will then be able to walk about and work on the rim of the wheel in an environment similar to earth. (a) At what angular velocity must the wheel rotate to simulate the earth's gravity? (b) If the mass of the spaceship is 40,000 kg, what is its approximate moment of inertia? (c) How much energy will be necessary to rotate the space station? (d) If it takes 20.0 rev to bring the space station up to its operating angular velocity, what torque must be applied in the form of gas jets attached to the outer rim of the wheel?



†43. At the instant that ball 1 is released from rest at the top of a rough inclined plane a second ball (2) moves past it on the horizontal surface below at a constant velocity of 2.30 m/s. The plane makes an angle $\theta = 35.0°$ with the horizontal and the height of the plane is 0.500 m. Using Newton's second law for combined translational and rotational motion find (a) the acceleration a of ball 1 down the plane, (b) the velocity of ball 1 at the base of the incline, (c) the time it takes for ball 1 to reach the bottom of the plane, (d) the distance that ball 2 has moved in this time, and (e) at what horizontal distance from the base of the incline will ball 1 overtake ball 2.



Interactive Tutorials

44. A cylinder of mass $m = 4.00$ kg and radius $r = 2.00$ m is rotating at an angular velocity $\omega = 3600$ rpm. Calculate (a) its angular velocity ω in rad/s, (b) its moment of inertia I, (c) its rotational kinetic energy KE_{rot}, and (d) its angular momentum L.

45. A mass $m = 2.00$ kg is attached by a string that is wrapped around a frictionless solid cylinder of mass $M = 8.00$ kg and radius $R = 0.700$ m that is free to rotate. Calculate (a) the acceleration a of the mass m and (b) the tension T in the string.

46. A cylinder of mass $m = 2.35$ kg and radius $r = 0.345$ m is initially rotating at an angular velocity $\omega_0 = 1.55$ rad/s when a constant force $F = 9.25$ N is applied tangentially to the cylinder as in figure 9.8. Find (a) the moment of inertia I of the cylinder, (b) the torque τ acting on the cylinder, (c) the angular acceleration α of the cylinder, (d) the angular velocity ω of the cylinder at $t = 4.55$ s, and (e) the angular displacement θ at $t = 4.55$ s.

47. The moment of inertia of a continuous mass distribution. A meterstick, $m = 0.149$ kg, lies on the x-axis with the zero of the meterstick at the origin of the coordinate system. Determine the moment of inertia of the meterstick about an axis that passes through the zero of the meterstick and perpendicular to it. Assume that the meterstick can be divided into $N = 10$ equal parts.

48. This is a generalization of Interactive Tutorial problem 76 of chapter 4 but it also takes the rotational motion of the pulley into account. Derive the formula for the magnitude of the acceleration of the system shown in the diagram for problem 61 of chapter 4. The pulley has a mass M and the radius R. As a general case assume that the coefficient of kinetic friction between block A and the surface is μ_{kA} and between block B and the surface is μ_{kB}. Solve for all the special cases that you can think of. In all the cases, consider different values for the mass M of the pulley and see the effect it has on the results of the problem.

49. Consider the general motion in an Atwood's machine such as the one shown in figure 9.18. Mass $m_A = 0.650$ kg and is at a height $h_A = 2.55$ m above the reference plane and mass $m_B = 0.420$ kg is at a height $h_B = 0.400$ m. The pulley has a mass of $M = 2.00$ kg and a radius $R = 0.100$ m. If the system starts from rest, find (a) the initial potential energy of mass A, (b) the initial potential energy of mass B, and (c) the total energy of the system. When mass m_A has fallen a distance $y_A = 0.750$ m, find (d) the potential energy of mass A, (e) the potential energy of mass B, (f) the speed of each mass at that point, (g) the kinetic energy of mass A, (h) the kinetic energy of mass B, (i) the moment of inertia of the pulley (assume it to be a disk), (j) the angular velocity ω of the pulley, and (k) the rotational kinetic energy of the pulley. (l) When mass A hits the ground, find the speed of each mass and the angular velocity of the pulley.

50. Consider the general motion in the combined system shown in the diagram of problem 16. Mass $m_A = 0.750$ kg and is at a height $h_A = 1.85$ m above the reference plane and mass $m_B = 0.285$ kg is at a height $h_B = 2.25$ m, $\mu_k = 0.450$. The pulley has a mass $M = 1.85$ kg and a radius $R = 0.0800$ m. If the system starts from rest, find (a) the initial potential energy of mass A, (b) the initial potential energy of mass B, and (c) the total energy of the system. When m_A has fallen a distance $y_A = 0.35$ m, find (d) the potential energy of mass A, (e) the potential energy of mass B, (f) the energy lost due to friction as mass B slides on the rough surface, (g) the speed of each mass at that point, (h) the kinetic energy of mass A, (i) the kinetic energy of mass B, (j) the moment of inertia of the pulley (assumed to be a disk), (k) the angular velocity ω of the pulley, and (l) the rotational kinetic energy of the pulley. (m) When mass A hits the ground, find the speed of each mass.

51. A disk of mass $M = 3.55$ kg and a radius $R = 1.25$ m is rotating freely at an initial angular velocity $\omega_i = 1.45$ rad/s. Small balls of clay of mass $m_b = 0.025$ kg are dropped onto the rotating disk at the radius $r = 0.85$ m at the rate of $n = 5$ ball/s. Find (a) the initial moment of inertia of the disk, (b) the initial angular momentum of the disk, and (c) the angular velocity ω at $t = 6.00$s. (d) Plot the angular velocity ω as a function of the number of balls dropped.