where k_E is given by equation 11.49. Hence, the combination of springs in series executes simple harmonic motion and the period of that motion, given by equation 11.22, is

$$T = 2\pi\sqrt{\frac{m}{k_E}} = 2\pi\sqrt{m\left(\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}\right)} \tag{11.52}$$

Example 11.7

Springs in parallel. Three springs with force constants $k_1 = 10.0$ N/m, $k_2 = 12.5$ N/m, and $k_3 = 15.0$ N/m are connected in parallel to a mass of 0.500 kg. The mass is then pulled to the right and released. Find the period of the motion.

Solution

The period of the motion, found from equation 11.43, is

$$T = 2\pi\sqrt{\frac{m}{k_1 + k_2 + k_3}}$$

$$= 2\pi\sqrt{\frac{0.500\ \text{kg}}{10.0\ \text{N/m} + 12.5\ \text{N/m} + 15.0\ \text{N/m}}}$$

$$= 0.726\ \text{s}$$

Example 11.8

Springs in series. The same three springs as in example 11.7 are now connected in series. Find the period of the motion.

Solution

The period, found from equation 11.52, is

$$T = 2\pi\sqrt{m\left(\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}\right)}$$

$$= 2\pi\sqrt{(0.500\ \text{kg})\left(\frac{1}{10.0\ \text{N/m}} + \frac{1}{12.5\ \text{N/m}} + \frac{1}{15.0\ \text{N/m}}\right)}$$

$$= 1.56\ \text{s}$$

The Language of Physics

Periodic motion
Motion that repeats itself in equal intervals of time (p. 297).

Displacement
The distance a vibrating body moves from its equilibrium position (p. 297).

Simple harmonic motion
Periodic motion in which the acceleration of a body is directly proportional to its displacement from the equilibrium position but in the opposite direction. Because the acceleration is directly proportional to the displacement, the acceleration of the body is not constant. The kinematic equations developed in chapter 3 are no longer valid to describe this type of motion (p. 298).

Amplitude
The maximum displacement of the vibrating body (p. 298).

Cycle
One complete oscillation or vibratory motion (p. 298).

Period
The time for the vibrating body to complete one cycle (p. 298).

Frequency
The number of complete cycles or oscillations in unit time. The frequency is the reciprocal of the period (p. 299).

Reference circle
A body executing uniform circular motion does so in a circle. The projection of the position of the rotating body onto the x- or y-axis is equivalent to simple harmonic motion along that axis. Thus, vibratory motion is related to motion in a circle, the reference circle (p. 300).

Angular velocity
The angular velocity of the uniform circular motion in the reference circle is related to the frequency of the vibrating system. Hence, the angular velocity is called the angular frequency of the vibrating system (p. 300).

Potential energy of a spring
The energy that a body possesses by virtue of its configuration. A compressed spring has potential energy because it has the ability to do work as it expands to its equilibrium configuration. A stretched spring can also do work as it returns to its equilibrium configuration (p. 306).

Simple pendulum
A bob that is attached to a string and allowed to oscillate to and fro under the action of gravity. If the angle of the pendulum is small the pendulum will oscillate in simple harmonic motion (p. 309).

Summary of Important Equations

Restoring force in a spring
$$F_R = -kx \quad (11.1)$$

Defining relation for simple harmonic motion
$$a = -\frac{k}{m}x \quad (11.2)$$

Frequency
$$f = \frac{1}{T} \quad (11.3)$$

Displacement in simple harmonic motion
$$x = A \cos \omega t \quad (11.6)$$

Angular frequency
$$\omega = 2\pi f \quad (11.9)$$

Velocity as a function of time in simple harmonic motion
$$v = -\omega A \sin \omega t \quad (11.13)$$

Velocity as a function of displacement
$$v = \pm\omega\sqrt{A^2 - x^2} \quad (11.15)$$

Acceleration as a function of time
$$a = -\omega^2 A \cos \omega t \quad (11.17)$$

Angular frequency of a spring
$$\omega = \sqrt{\frac{k}{m}} \quad (11.20)$$

Frequency in simple harmonic motion
$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad (11.21)$$

Period in simple harmonic motion
$$T = 2\pi\sqrt{\frac{m}{k}} \quad (11.22)$$

Potential energy of a spring
$$PE = \tfrac{1}{2}kx^2 \quad (11.24)$$

Conservation of energy for a vibrating spring
$$\tfrac{1}{2}kA^2 = \tfrac{1}{2}kx^2 + \tfrac{1}{2}mv^2 \quad (11.27)$$

Period of motion of a simple pendulum
$$T_p = 2\pi\sqrt{\frac{l}{g}} \quad (11.37)$$

Equivalent spring constant for springs in parallel
$$k_E = k_1 + k_2 + k_3 \quad (11.41)$$

Period of motion for springs in parallel
$$T = 2\pi\sqrt{\frac{m}{k_1 + k_2 + k_3}} \quad (11.43)$$

Equivalent spring constant for springs in series
$$\frac{1}{k_E} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} \quad (11.49)$$

Period of motion for springs in series
$$T = 2\pi\sqrt{m\left(\frac{1}{k_1}+\frac{1}{k_2}+\frac{1}{k_3}\right)} \quad (11.52)$$

Questions for Chapter 11

1. Can the periodic motion of the earth be considered to be an example of simple harmonic motion?
2. Can the kinematic equations derived in chapter 3 be used to describe simple harmonic motion?
3. In the simple harmonic motion of a mass attached to a spring, the velocity of the mass is equal to zero when the acceleration has its maximum value. How is this possible? Can you think of other examples in which a body has zero velocity with a nonzero acceleration?
4. What is the characteristic of the restoring force that makes simple harmonic motion possible?
5. Discuss the significance of the reference circle in the analysis of simple harmonic motion.
6. How can a mass that is undergoing a one-dimensional translational simple harmonic motion have anything to do with an angular velocity or an angular frequency, which is a characteristic of two or more dimensions?
7. How is the angular frequency related to the physical characteristics of the spring and the vibrating mass in simple harmonic motion?

†8. In the entire derivation of the equations for simple harmonic motion we have assumed that the springs are massless and friction can be neglected. Discuss these assumptions. Describe qualitatively what you would expect to happen to the motion if the springs are not small enough to be considered massless.

†9. Describe how a geological survey for iron might be undertaken on the moon using a simple pendulum.

†10. How could a simple pendulum be used to make an accelerometer?

†11. Discuss the assumption that the displacement of each spring is the same when the springs are in parallel. Under what conditions is this assumption valid and when would it be invalid?

Problems for Chapter 11

11.2 Simple Harmonic Motion and *11.3* Analysis of Simple Harmonic Motion

1. A mass of 0.200 kg is attached to a spring of spring constant 30.0 N/m. If the mass executes simple harmonic motion, what will be its frequency?

2. A 30.0-g mass is attached to a vertical spring and it stretches 10.0 cm. It is then stretched an additional 5.00 cm and released. Find its period of motion and its frequency.

3. A 0.200-kg mass on a spring executes simple harmonic motion at a frequency f. What mass is necessary for the system to vibrate at a frequency of $2f$?

4. A simple harmonic oscillator has a frequency of 2.00 Hz and an amplitude of 10.0 cm. What is its maximum acceleration? What is its acceleration at $t = 0.25$ s?

5. A ball attached to a string travels in uniform circular motion in a horizontal circle of 50.0 cm radius in 1.00 s. Sunlight shining on the ball throws its shadow on a wall. Find the velocity of the shadow at (a) the end of its path and (b) the center of its path.

6. A 50.0-g mass is attached to a spring of force constant 10.0 N/m. The spring is stretched 20.0 cm and then released. Find the displacement, velocity, and acceleration of the mass at 0.200 s.

7. A 25.0-g mass is attached to a vertical spring and it stretches 15.0 cm. It is then stretched an additional 10.0 cm and then released. What is the maximum velocity of the mass? What is its maximum acceleration?

8. The displacement of a body in simple harmonic motion is given by $x = (0.15\text{ m})\cos[(5.00\text{ rad/s})t]$. Find (a) the amplitude of the motion, (b) the angular frequency, (c) the frequency, (d) the period, and (e) the displacement at 3.00 s.

9. A 500-g mass is hung from a coiled spring and it stretches 10.0 cm. It is then stretched an additional 15.0 cm and released. Find (a) the frequency of vibration, (b) the period, and (c) the velocity and acceleration at a displacement of 10.0 cm.

10. A mass of 0.200 kg is placed on a vertical spring and the spring stretches by 15.0 cm. It is then pulled down an additional 10.0 cm and then released. Find (a) the spring constant, (b) the angular frequency, (c) the frequency, (d) the period, (e) the maximum velocity of the mass, (f) the maximum acceleration of the mass, (g) the maximum restoring force, and (h) the equation of the displacement, velocity, and acceleration at any time t.

11.5 Conservation of Energy and the Vibrating Spring

11. A simple harmonic oscillator has a spring constant of 5.00 N/m. If the amplitude of the motion is 15.0 cm, find the total energy of the oscillator.

12. A body is executing simple harmonic motion. At what displacement is the potential energy equal to the kinetic energy?

13. A 20.0-g mass is attached to a horizontal spring on a smooth table. The spring constant is 3.00 N/m. The spring is then stretched 15.0 cm and then released. What is the total energy of the motion? What is the potential and kinetic energy when $x = 5.00$ cm?

14. A body is executing simple harmonic motion. At what displacement is the speed v equal to one-half the maximum speed?

11.6 The Simple Pendulum

15. Find the period and frequency of a simple pendulum 0.75 m long.

16. If a pendulum has a length L and a period T, what will be the period when (a) L is doubled and (b) L is halved?

17. Find the frequency of a child's swing whose ropes have a length of 3.25 m.

18. What is the period of a 0.500-m pendulum on the moon where $g_m = (1/6)g_e$?

19. What is the period of a pendulum 0.750 m long on a spaceship (a) accelerating at 4.90 m/s² and (b) moving at constant velocity?

20. What is the period of a pendulum in free-fall?

21. A pendulum has a period of 0.750 s at the equator at sea level. The pendulum is carried to another place on the earth and the period is now found to be 0.748 s. Find the acceleration due to gravity at this location.

11.7 Springs in Parallel and in Series

†22. Springs with spring constants of 5.00 N/m and 10.0 N/m are connected in parallel to a 5.00-kg mass. Find (a) the equivalent spring constant and (b) the period of the motion.

†23. Springs with spring constants 5.00 N/m and 10.0 N/m are connected in series to a 5.00-kg mass. Find (a) the equivalent spring constant and (b) the period of the motion.

Additional Problems

24. A 500-g mass is attached to a vertical spring of spring constant 30.0 N/m. How far should the spring be stretched in order to give the mass an upward acceleration of 3.00 m/s²?

25. A ball is caused to move in a horizontal circle of 40.0-cm radius in uniform circular motion at a speed of 25.0 cm/s. Its projection on the wall moves in simple harmonic motion. Find the velocity and acceleration of the shadow of the ball at (a) the end of its motion, (b) the center of its motion, and (c) halfway between the center and the end of the motion.

†26. The motion of the piston in the engine of an automobile is approximately simple harmonic. If the stroke of the piston (twice the amplitude) is equal to 8.00 in. and the engine turns at 1800 rpm, find (a) the acceleration at $x = A$ and (b) the speed of the piston at the midpoint of the stroke.

†27. A 535-g mass is dropped from a height of 25.0 cm above an uncompressed spring of $k = 20.0$ N/m. By how much will the spring be compressed?

28. A simple pendulum is used to operate an electrical device. When the pendulum bob sweeps through the midpoint of its swing, it causes an electrical spark to be given off. Find the length of the pendulum that will give a spark rate of 30.0 sparks per minute.

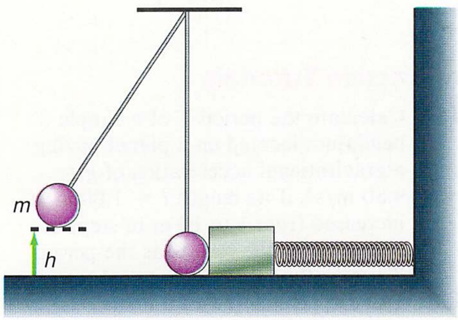
†29. The general solution for the period of a simple pendulum, without making the assumption of small angles of swing, is given by

$$T = 2\pi\sqrt{\frac{l}{g}}\left[1 + \frac{(\frac{1}{2})^2\sin^2\theta}{2} + \frac{(\frac{1}{2})^2(\frac{3}{4})^2\sin^4\theta}{2} + \ldots\right]$$

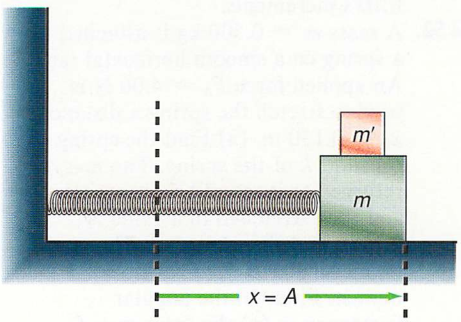
Find the period of a 1.00-m pendulum for $\theta = 10.0°$, $30.0°$, and $50.0°$ and compare with the period obtained with the small angle approximation. Determine the percentage error in each case by using the small angle approximation.

30. A pendulum clock on the earth has a period of 1.00 s. Will this clock run slow or fast, and by how much if taken to (a) Mars, (b) Moon, and (c) Venus?

†31. A pendulum bob, 355 g, is raised to a height of 12.5 cm before it is released. At the bottom of its path it makes a perfectly elastic collision with a 500-g mass that is connected to a horizontal spring of spring constant 15.8 N/m, that is at rest on a smooth surface. How far will the spring be compressed?



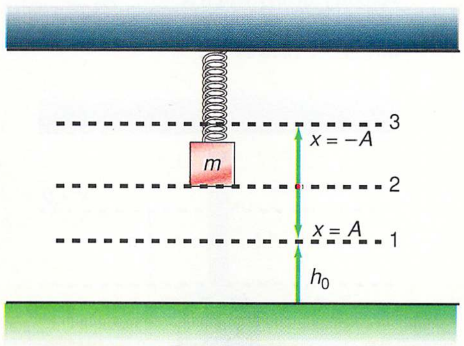
†32. A 500-g block is in simple harmonic motion as shown in the diagram. A mass m' is added to the top of the block when the block is at its maximum extension. How much mass should be added to change the frequency by 25%?



†33. A pendulum clock keeps correct time at a location at sea level where the acceleration of gravity is equal to 9.80 m/s². The clock is then taken up to the top of a mountain and the clock loses 3.00 s per day. How high is the mountain?

†34. Three people, who together weigh 422 lb, get into a car and the car is observed to move 2.00 in. closer to the ground. What is the spring constant of the car springs?

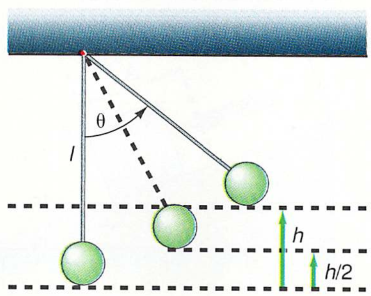
†35. In the accompanying diagram, the mass m is pulled down a distance of 9.50 cm from its equilibrium position and is then released. The mass then executes simple harmonic motion. Find (a) the total potential energy of the mass with respect to the ground when the mass is located at positions 1, 2, and 3; (b) the total energy of the mass at positions 1, 2, and 3; and (c) the speed of the mass at position 2. Assume $m = 55.6$ g, $k = 25.0$ N/m, $h_0 = 50.0$ cm.



†36. A 20.0-g ball rests on top of a vertical spring gun whose spring constant is 20 N/m. The spring is compressed 10.0 cm and the gun is then fired. Find how high the ball rises in its vertical trajectory.

†37. A toy spring gun is used to fire a ball as a projectile. Find the value of the spring constant, such that when the spring is compressed 10.0 cm, and the gun is fired at an angle of 62.5°, the range of the projectile will be 1.50 m. The mass of the ball is 25.2 g.

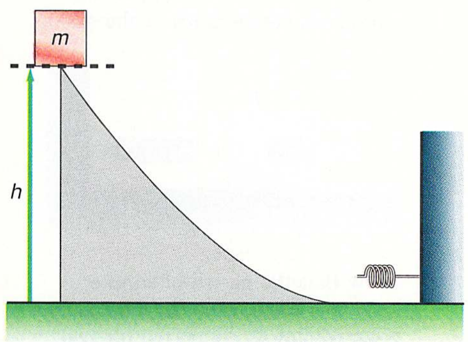
†38. In the simple pendulum shown in the diagram, find the tension in the string when the height of the pendulum is (a) h, (b) $h/2$, and (c) $h = 0$. The mass $m = 500$ g, the initial height $h = 15.0$ cm, and the length of the pendulum $l = 1.00$ m.



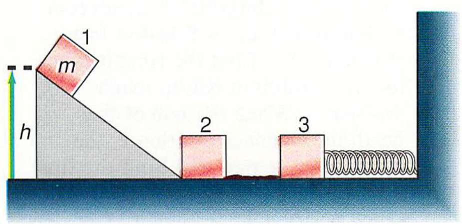
†39. A mass is attached to a horizontal spring. The mass is given an initial amplitude of 10.0 cm on a rough surface and is then released to oscillate in simple harmonic motion. If 10.0% of the energy is lost per cycle due to the friction of the mass moving over the rough surface, find the maximum displacement of the mass after 1, 2, 4, 6, and 8 complete oscillations.

†40. Find the maximum amplitude of vibration after 2 periods for a 85.0-g mass executing simple harmonic motion on a rough horizontal surface of $\mu_k = 0.350$. The spring constant is 24.0 N/m and the initial amplitude is 20.0 cm.

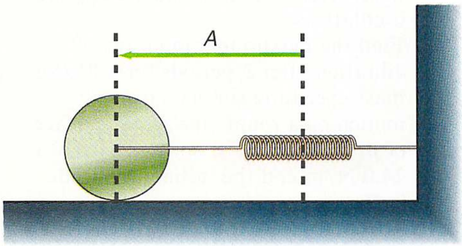
41. A 240-g mass slides down a circular chute without friction and collides with a horizontal spring, as shown in the diagram. If the original position of the mass is 25.0 cm above the table top and the spring has a spring constant of 18 N/m, find the maximum distance that the spring will be compressed.



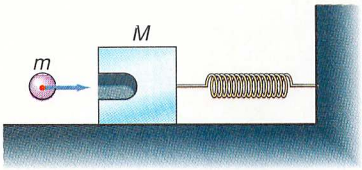
†42. A 235-g block slides down a frictionless inclined plane, 1.25 m long, that makes an angle of 34.0° with the horizontal. At the bottom of the plane the block slides along a rough horizontal surface 1.50 m long. The coefficient of kinetic friction between the block and the rough horizontal surface is 0.45. The block then collides with a horizontal spring of $k = 20.0$ N/m. Find the maximum compression of the spring.



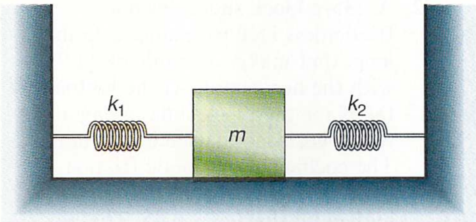
†43. A 335-g disk that is free to rotate about its axis is connected to a spring that is stretched 35.0 cm. The spring constant is 10.0 N/m. When the disk is released it rolls without slipping as it moves toward the equilibrium position. Find the speed of the disk when the displacement of the spring is equal to −17.5 cm.



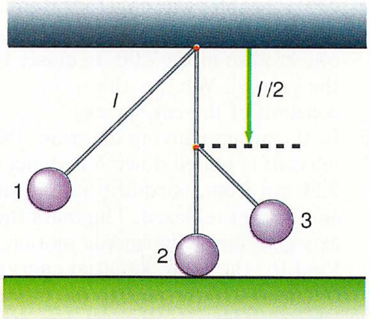
†44. A 25.0-g ball moving at a velocity of 200 cm/s to the right makes an inelastic collision with a 200-g block that is initially at rest on a frictionless surface. There is a hole in the large block for the small ball to fit into. If $k = 10$ N/m, find the maximum compression of the spring.



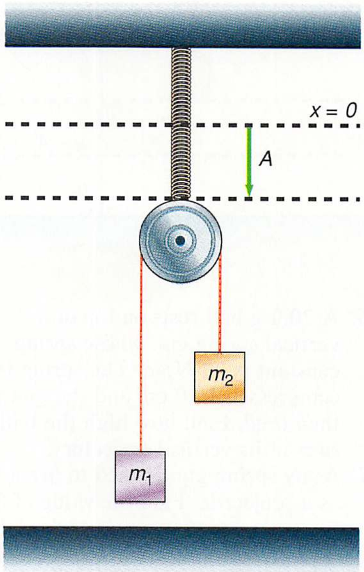
†45. Show that the period of simple harmonic motion for the mass shown is equivalent to the period for two springs in parallel.



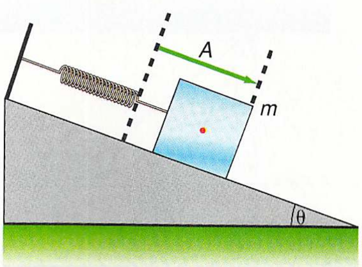
†46. A nail is placed in the wall at a distance of $l/2$ from the top, as shown in the diagram. A pendulum of length 85.0 cm is released from position 1. (a) Find the time it takes for the pendulum bob to reach position 2. When the bob of the pendulum reaches position 2, the string hits the nail. (b) Find the time it takes for the pendulum bob to reach position 3.



†47. A spring is attached to the top of an Atwood's machine as shown. The spring is stretched to $A = 10$ cm before being released. Find the velocity of m_2 when $x = -A/2$. Assume $m_1 = 28.0$ g, $m_2 = 43.0$ g, and $k = 10.0$ N/m.



†48. A 280-g block is connected to a spring on a rough inclined plane that makes an angle of 35.5° with the horizontal. The block is pulled down the plane a distance $A = 20.0$ cm, and is then released. The spring constant is 40.0 N/m and the coefficient of kinetic friction is 0.100. Find the speed of the block when the displacement $x = -A/2$.



49. The rotational analog of simple harmonic motion, is angular simple harmonic motion, wherein a body rotates periodically clockwise and then counterclockwise. Hooke's law for rotational motion is given by

$$\tau = -C\theta$$

where τ is the torque acting on the body, θ is the angular displacement, and C is a constant, like the spring constant. Use Newton's second law for rotational motion to show

$$\alpha = \frac{C}{I}\theta$$

Use the analogy between the linear result, $a = -\omega^2 x$, to show that the frequency of vibration of an object executing angular simple harmonic motion is given by

$$f = \frac{1}{2\pi}\sqrt{\frac{C}{I}}$$

Interactive Tutorials

50. Calculate the period T of a simple pendulum located on a planet having a gravitational acceleration of $g = 9.80$ m/s², if its length $l = 1.00$ m is increased from 1 to 10 m in steps of 1.00 m. Plot the results as the period T versus the length l.

51. The displacement x of a body undergoing simple harmonic motion is given by the formula $x = A\cos\omega t$, where A is the amplitude of the vibration, ω is the angular frequency in rad/s, and t is the time in seconds. Plot the displacement x as a function of t for an amplitude $A = 0.150$ m and an angular frequency $\omega = 5.00$ rad/s as t increases from 0 to 2 s in 0.10 s increments.

52. A mass $m = 0.500$ kg is attached to a spring on a smooth horizontal table. An applied force $F_A = 4.00$ N is used to stretch the spring a distance $x_0 = 0.150$ m. (a) Find the spring constant k of the spring. The mass is returned to its equilibrium position and then stretched to a value $A = 0.15$ m and then released. The mass then executes simple harmonic motion. Find (b) the angular frequency ω, (c) the frequency f, (d) the period T, (e) the maximum velocity v_{max} of the vibrating mass, (f) the maximum acceleration a_{max} of the vibrating mass, (g) the maximum restoring force F_{Rmax}, and (h) the velocity of the mass at the displacement $x = 0.08$ m. (i) Plot the displacement x, velocity v, acceleration a, and the restoring force F_R at any time t.

53. A mass $m = 0.350$ kg is attached to a horizontal spring. The mass is then pulled a distance $x = A = 0.200$ m from its equilibrium position and when released the mass executes simple harmonic motion. Find (a) the total energy E_{tot} of the mass when it is at its maximum displacement A from its equilibrium position. When the mass is at the displacement $x = 0.120$ m find, (b) its potential energy PE, (c) its kinetic energy KE, and (d) its speed v. (e) Plot the total energy, potential energy, and kinetic energy of the mass as a function of the displacement x. The spring constant $k = 35.5$ N/m.

54. A mass $m = 0.350$ kg is attached to a vertical spring. The mass is at a height $h_0 = 1.50$ m from the floor. The mass is then pulled down a distance $A = 0.220$ m from its equilibrium position and when released executes simple harmonic motion. Find (a) the total energy of the mass when it is at its maximum displacement A below its equilibrium position, (b) the gravitational potential energy when it is at the displacement $x = 0.120$ m, (c) the elastic potential energy when it is at the same displacement x, (d) the kinetic energy at the displacement x, and (e) the speed of the mass when it is at the displacement x. The spring constant $k = 35.5$ N/m.