The Language of Physics

Magnetic flux
A quantitative measure of the number of lines of the magnetic field that passes normally through a surface. It is measured in webers (p. 661).

Faraday's law of electromagnetic induction
Whenever the magnetic flux through a coil changes with time, an emf is induced in the coil. The magnetic flux can be changed by changing the magnetic field, the area of the loop, or the direction between the magnetic field and the area vector (p. 665).

Lenz's law
The direction of an induced emf is such that any current it produces, always opposes, through the magnetic field of the induced current, the change inducing the emf (p. 668).

AC generator
A device in which a coil of wire is manually rotated in an external magnetic field. Because the magnetic flux through the coil changes with time, the rotating coil has an alternating, sinusoidally varying, emf and current induced in the coil. The AC generator is a source of alternating current (p. 673).

Mutual induction
Changing the magnetic flux in one coil induces an emf in an adjacent coil (p. 675).

Self-induction
Changing the magnetic flux in a coil induces an emf in that coil. The induced emf opposes the changing magnetic flux (p. 677).

Inductor
A circuit element in which a self-induced emf accompanies a changing current (p. 678).

Summary of Important Equations

Magnetic flux
$$\Phi_M = BA\cos\theta \tag{23.1}$$

Induced electric field
$$E = \frac{F}{q} = vB\sin\theta \tag{23.3}$$

Magnitude of induced electric field
$$E = vB\sin\theta \tag{23.4}$$

Faraday's law of induction
$$\mathscr{E} = -\frac{\Delta\Phi_M}{\Delta t} \tag{23.13}$$
$$\mathscr{E} = -B\frac{\Delta A}{\Delta t}\cos\theta - A\frac{\Delta B}{\Delta t}\cos\theta \tag{23.14}$$

Faraday's law for N loops
$$\mathscr{E} = -N\frac{\Delta\Phi_M}{\Delta t} \tag{23.15}$$

Induced emf in a rotating coil
$$\mathscr{E} = \omega AB\sin\omega t \tag{23.22}$$

An alternating emf
$$\mathscr{E} = \mathscr{E}_{max}\sin\omega t \tag{23.24}$$

An alternating current
$$i = i_{max}\sin\omega t \tag{23.26}$$

emf induced in coil 2 by changing current in coil 1
$$\mathscr{E}_2 = -M\frac{\Delta i_1}{\Delta t} \tag{23.31}$$

Mutual inductance
$$M = \frac{-\mathscr{E}_2}{\Delta i_1/\Delta t} \tag{23.32}$$

Inductance of a solenoid
$$L = \mu_0 A l n^2 \tag{23.39}$$

Self-induced emf of a coil
$$\mathscr{E} = -L\frac{\Delta i}{\Delta t} \tag{23.40}$$

Inductance
$$L = \frac{-\mathscr{E}}{\Delta i/\Delta t} \tag{23.41}$$

Energy stored in magnetic field of an inductor
$$U_M = \tfrac{1}{2}LI^2 \tag{23.47}$$

Energy stored in magnetic field of a solenoid
$$U_M = \frac{1}{2}\frac{B^2 A l}{\mu_0} \tag{23.49}$$

Magnetic energy density
$$u_M = \frac{1}{2}\frac{B^2}{\mu_0} \tag{23.50}$$

Questions for Chapter 23

1. Describe the concept of magnetic flux. Could you also define an electric flux for an electric field?
†2. If changing the magnetic flux with time induces an electric field, does changing the electric flux with time induce a magnetic field?
3. If the metal wire MN in figure 23.2 were replaced with a wooden stick, how would this affect the experiment?
4. Show that if the area vector in figure 23.3 were defined in the opposite direction, the analysis would not be consistent with Lenz's law. Therefore, show that the choice of direction for the area vector **A** cannot be arbitrary.
5. Is it possible to change both the area of a loop and the magnetic field passing through the loop and still not have an induced emf in the loop?
6. Discuss Lenz's law.
7. Can an electric motor be used to drive an AC generator, with the output from the generator being used to operate the motor?
8. Discuss how energy is stored in the magnetic field of a coil. Compare this to the way energy is stored in the electric field of a capacitor.
†9. If changing an electric field with time produces a magnetic field, and changing a magnetic field with time produces an electric field, is it possible for these changing fields to couple together to propagate through space?

Problems for Chapter 23

23.2 Magnetic Flux

1. A rectangular coil 6.00 cm by 8.00 cm is located in a uniform magnetic field of 0.250 T. Find the flux through the coil when the plane of the coil is (a) perpendicular to **B**, (b) parallel to **B**, and (c) makes an angle of 60.0° with **B**.

2. A circular coil, 6.00 cm in diameter, is placed in a uniform magnetic field of 0.300 T. Find the flux through the coil when the coil makes an angle of 53.0° with **B**. What is the flux if the angle is increased to 90.0°?

23.3 Motional emf and Faraday's Law of Electromagnetic Induction

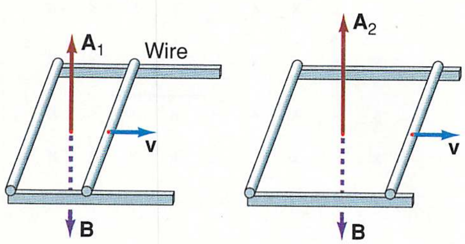
3. The magnetic flux through a coil of 10 turns, changes from 5.00×10^{-4} Wb to 5.00×10^{-3} Wb in 1.00×10^{-2} s. Find the induced emf in the coil.

4. A circular coil 6.00 cm in diameter is placed in a uniform magnetic field of 0.500 T. If B drops to 0 in 0.002 s find the maximum induced emf in the coil.

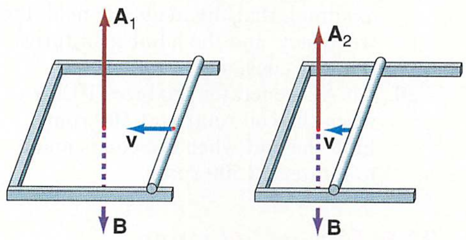
5. A 25-turn circular coil 6.00 cm in diameter is placed in, and perpendicular to, a magnetic field that is changing at 2.50×10^{-2} T/s. (a) Find the induced emf in the coil. (b) If the resistance of the coil is 25.0 Ω, find the induced current in the coil.

6. A circular coil 5.00 cm in diameter has a resistance of 2.00 Ω. If a current of 4.00 A is to flow in the coil, at what rate should the magnetic field change with time if (a) the coil is perpendicular to the magnetic field and (b) if the coil makes an angle of 30° with the field?

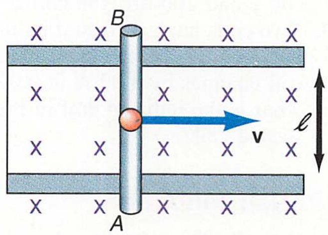
7. In the diagram, the magnetic field **B** is a constant and points downward. The wire moves to the right at the velocity **v**. Hence the area vector $\mathbf{A}_1$, which points upward, increases to $\mathbf{A}_2$, which also points upward. Find (a) the direction of Δ**A**, (b) the angle between Δ**A** and **B**, (c) the induced emf, and (d) the direction of the induced current.



8. In the diagram, the magnetic field **B** is a constant and points downward. The wire moves to the left at the velocity −**v**. Hence the area vector $\mathbf{A}_1$, which points upward, decreases to $\mathbf{A}_2$. Find (a) the direction of Δ**A**, (b) the angle between Δ**A** and **B**, (c) the induced emf, and (d) the direction of the induced current.



9. The wire AB in the diagram moves to the right with a velocity of 25.0 cm/s. If ℓ = 20.0 cm, B = 0.300 T, and the total resistance of the circuit is 50.0 Ω, find (a) the induced emf in wire AB, (b) the current flowing in the circuit, and (c) the direction of the current.



10. Repeat problem 9 if the wire AB is moving to the left with a velocity of 25.0 cm/s.

11. If wire AB of problem 9 moves with a velocity of 50.0 m/s to the right, at what rate should **B** change when the wire is at a distance of 5.00 cm from the left end of the loop, such that there will be no induced emf in the circuit?

12. Wire MN (l = 25.0 cm) of figure 23.2(a) is fixed 20.0 cm away from the galvanometer wire OL. The resistance of the circuit is 50.0 Ω. (a) If the magnetic field varies from 0 to 0.350 T in a time of 0.030 s, find the induced emf and current in the circuit during this time. (b) If the magnetic field remains constant at 0.350 T, find the induced emf and current in the coil. (c) If the magnetic field decays from 0.350 T to 0 T in 0.0200 s, find the induced emf, the current, and its direction in the wire.

13. An airplane is flying at 200 knots through an area where the vertical component of the earth's magnetic field is 3.50×10^{-5} T. If the wing span is 12.0 m, what is the difference in potential from wing tip to wing tip? Could this potential difference be used as a source of current to operate the aircraft's equipment?

14. A train is traveling at 40.0 km/hr in a location where the vertical component of the earth's magnetic field is 3.00×10^{-5} T. If the axle of the wheels is 1.50 m apart, what is the induced potential difference across the axle?

15. A coil of 10 turns and 35.0-cm² area is in a perpendicular magnetic field of 0.0500 T. The coil is then pulled completely out of the field in 0.100 s. Find the induced emf in the coil as it is pulled out of the field.

16. It is desired to determine the magnitude of the magnetic field between the poles of a large magnet. A rectangular coil of 10 turns, with dimensions of 5.00 cm by 8.00 cm, is placed perpendicular to the magnetic field. The coil is then pulled out of the field in 0.0500 s and an induced emf of 0.0250 V is observed in the coil. What is the value of the magnetic field between the poles?

17. It is desired to determine the magnetic field of a bar magnet. The bar magnet is pushed through a 10.0-cm diameter circular coil in 2.50×10^{-3} s and an emf of 0.750 V is obtained. Find the magnetic field of the bar magnet.

18. The flux through a 20-turn coil of 20.0 Ω resistance changes by 5.00 Wb/m² in a time of 0.0200 s. Find the induced current in the coil.

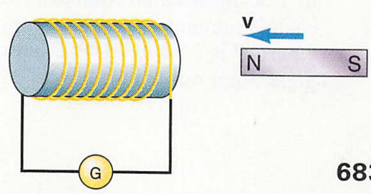
†19. Show that whenever the flux through a coil of resistance R changes by $\Delta\Phi_M$, there will always be an induced charge in the loop given by

$$\Delta Q = \frac{-N\Delta\Phi_M}{R}$$

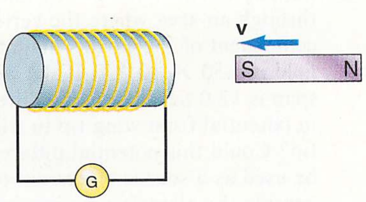
This induced charge becomes the induced current in the coil.

23.4 Lenz's Law

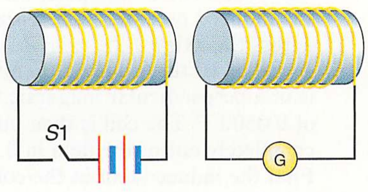
20. What is the direction of the induced current in the solenoid if a north magnetic pole is moved toward the solenoid in the diagram?



21. What is the direction of the induced current in the solenoid if a south magnetic pole is moved toward the solenoid in the diagram?



22. Find the direction of the current in the second solenoid when the switch $S1$ in the circuit of solenoid 1 is (a) closed and (b) opened.



23.5 The Induced emf in a Rotating Loop of Wire in a Magnetic Field—Alternating emf's and the AC Generator

23. A circular coil in a generator has 50 turns, a diameter of 10.0 cm, and rotates in a field of 0.500 T. What must be the angular velocity of the coil if it is to generate a maximum voltage of 50.0 V?

†24. A 50.0-Ω circular coil of wire of 20 turns, 5.00 cm in diameter, is rotated at an angular velocity of 377 rad/s in an external magnetic field of 2.00 T. Find (a) the maximum induced emf, (b) the frequency of the alternating emf, (c) the maximum current in the coil, and (d) the instantaneous current at 5.00 s.

25. An AC generator consists of a coil of 200 turns, 10.0 cm in diameter. If the coil rotates at 500 rpm in a magnetic field of 0.250 T, find the maximum induced emf.

†26. You are asked to design an AC generator that will give an output voltage of 156 V maximum at a frequency of 60.0 Hz. (a) Find the product of N, the number of turns in the coil, B, the magnetic field, and A, the area of the coil, that will give this value of voltage. (b) Pick a reasonable set of values for N, B, and A to satisfy the requirement.

27. At what angular speed should a coil of 1.00-m² area be rotated in the earth's magnetic field in a region where B is 3.50×10^{-5} T in order to generate a maximum emf of 1.50 V?

28. A circular coil of wire of 25.0-cm² area is placed in a magnetic field of 0.0500 T. What will the induced emf in the coil be if the coil is rotated at 20.0 rad/s (a) about an axis that is aligned with the magnetic field and (b) about an axis that is perpendicular to the magnetic field?

29. An AC generator is designed with a circular cross section of radius 1.25 cm to produce 120 V. What would the radius of the generator coil have to be if it is to produce 240 V, assuming that the magnetic field, the frequency, and the number of turns remains constant?

30. An AC generator produces 10.0 V when the coil rotates at 500 rpm. Find the emf when the coil is made to rotate at 1500 rpm.

23.6 Mutual Induction

31. An emf of 0.800 V is observed in a circuit when a nearby circuit has a current change at the rate of 200 A/s. What is the mutual inductance of the circuits?

32. Find the mutual inductance of two coaxial solenoids of 10.0-cm radius, 20.0 cm in length, with 120 turns in coil 1 and 200 turns in coil 2.

33. Two coils have a mutual inductance of 5.00 mH. If the current in the first coil changes by 3.00 A in 0.0200 s, what is the induced emf in the second coil?

23.7 Self-Induction

34. What is the self-induced emf in a coil of 5.00 H if the current through it is changing at the rate of 150 A/s?

35. A coil has an inductance of 5.00 mH. At what rate should current change in the coil to give an induced emf of 100 V?

36. The current through a 10.0-mH inductor changes at the rate of 250 A/s. Find the change of flux in the coil.

23.8 The Energy Stored in the Magnetic Field of an Inductor

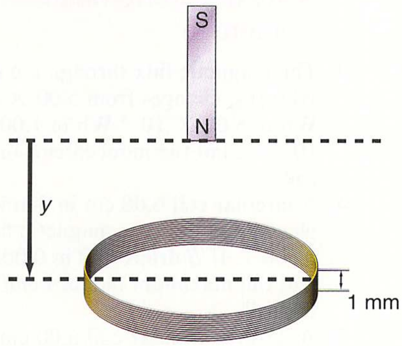
37. How much energy is stored in the magnetic field of a coil of 5.00-mH inductance, carrying a current of 5.00 A?

38. A solenoid has 2500 turns/m, a diameter of 10.0 cm, and a length of 20.0 cm. If the current varies from 0 to 10.0 A, how much energy is stored in the magnetic field of the solenoid?

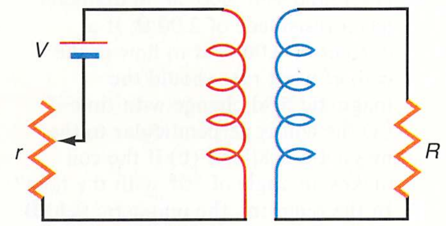
39. If 80.0 J of energy are stored in an inductor when the current changes from 5.00 A to 15.0 A, find the inductance of the inductor.

Additional Problems

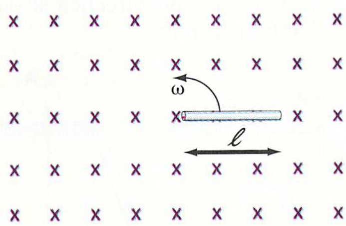
†40. A 10.0-cm bar magnet has a value of $B = 2.50 \times 10^{-3}$ T. It is 30.0 cm above a circular coil of radius 5.00 cm and 20 turns. The magnet is dropped from rest. Find (a) the velocity of the north pole of the bar magnet as it arrives at the coil, (b) the time it takes for the leading edge of the bar magnet to go through the 1.00-mm thickness of the coil, (c) the induced emf in the coil as the leading edge of the bar magnet enters the coil, (d) the emf in the coil while the entire bar magnet goes through the coil, and (e) the emf in the coil while the trailing edge of the bar magnet goes through the coil. State the assumptions you make in the solution of the problem.



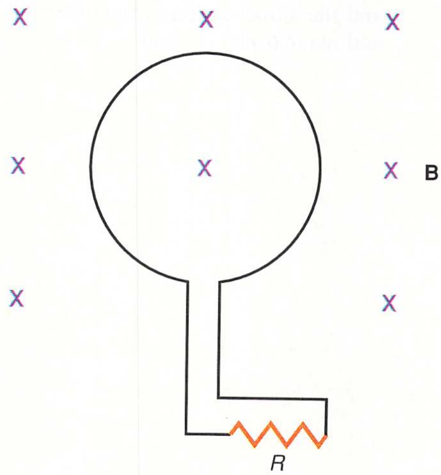
41. In the diagram, the two coils are wound in opposite senses about a common core. The resistor r is variable. Find the direction of the current in the resistor R when (a) r is increased and (b) r is decreased.



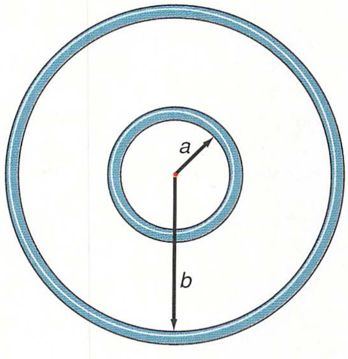
†42. A rod 25.0 cm long is rotated at an angular velocity of 20.0 rad/s in a uniform magnetic field of 4.5×10^{-3} T that points into the page. Find the induced emf from end to end of the rod.



43. A circular loop of radius 5.00 cm is placed in a uniform external magnetic field of 0.0400 T, directed into the page, as shown in the diagram. The circular loop is connected to a resistor, $R = 10.0\ \Omega$, by wires. The loop is now allowed to collapse until its area becomes effectively zero in 0.010 s. Find the magnitude and direction of the induced current in the resistor R.

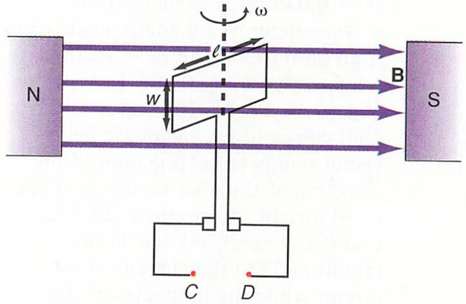


44. Find the induced emf in the inside loop when the current changes in the outside loop at the rate $\Delta i/\Delta t$.



45. A square loop of wire, 2.00 mm on each side, is placed at the center of a flat coil of 10 turns and 10.0 cm radius. The current in the coil increases from 0 to 1.00 A in 2.50 s. If the resistance of the loop is 0.15 Ω, find the emf induced in the square loop.

†46. A rotating coil of wire in a magnetic field is used to create a sparking device. The coil is connected by brushes to two wires C and D, the ends of which are separated by 1.00 mm. (a) Find the necessary emf between points C and D that will permit dielectric breakdown of the air gap between these points. (b) If N = 200 turns, the length of the coil is 10.0 cm, its width is 8.00 cm, and the magnetic field is 8.50×10^{-1} T, find the angular velocity ω of the coil to give the induced emf necessary for sparking.



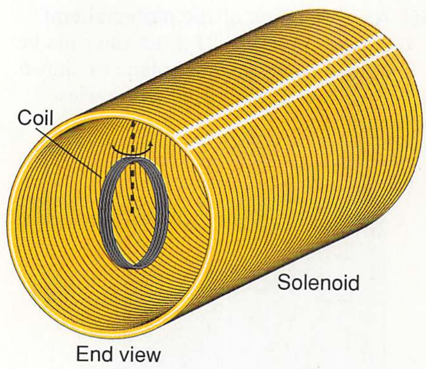
47. A 7.50-cm diameter circular loop of wire is initially oriented so that an external magnetic field of 4.00 T is normal to the plane of the loop. In 2.00 s, the loop rotates about an axis in its own plane so that it ultimately makes an angle of 45° with the field. During the *same* 2.00 s, the magnitude of the external field rises to 5.66 T. Determine the emf induced in the loop.

48. The current through a coil changes from 300 mA to 150 mA in 5.00×10^{-3} s. An induced emf of 2.00×10^{-2} V is obtained. Find (a) the inductance of the coil, (b) the initial energy in the field, and (c) the final energy in the field.

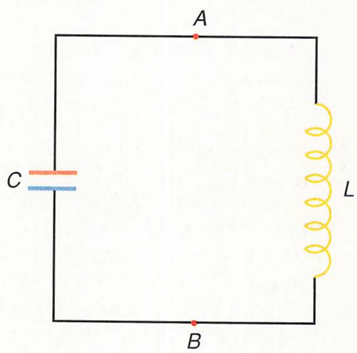
49. An air solenoid has an inductance of 5.00 mH. Find its inductance if a piece of iron ($\mu = 800\ \mu_0$) is placed within the solenoid.

50. A 5.00-mH inductor and a 50.0-Ω resistor are connected in series to a 24.0-V battery. Find (a) the final current in the circuit and (b) the energy stored in the magnetic field of the inductor.

†51. A coil 5.00 cm in diameter is placed inside a solenoid of 1000 turns/m. (a) If the current in the solenoid rises to 10.0 A in 2.00×10^{-2} s, find the induced emf in the inner coil. (b) When the current in the solenoid is constant at 10.0 A, what is the magnetic flux through the inner coil? (c) If the inner coil is now rotated at 20.0 rad/s, what is the induced emf in the coil? (d) If the inner coil is connected to a circuit with a resistance of 30.0 Ω, find the maximum current that will flow in the circuit.



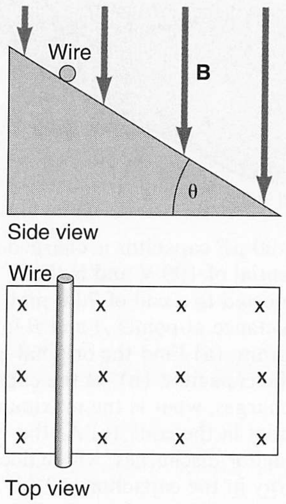
†52. A 5.00-μF capacitor is charged to a potential of 100 V and is then connected to a coil of 7.00-mH inductance at points A and B in the diagram. (a) Find the original energy in the capacitor. (b) As the capacitor discharges, what is the maximum current in the coil? (c) As the capacitor discharges, where does the energy in the capacitor go? (d) What will happen after the capacitor completely discharges?



†53. In the derivation of equation 23.18, the induced emf in wire cd was given by $\mathscr{E}_{cd} = vB\ell \sin\theta$. Using figure 23.5, find the angle between **v** and **B** for wire cd. Show that the sine of that angle reduces to the sine of the angle θ.

†54. It was shown in figure 23.2 that when the wire MN is moved to the right a current is induced in the wire from M to N. Show that this wire is a current-carrying wire in an external magnetic field, and as such experiences a force. Show that this force opposes the original motion and tends to slow down the motion of the wire and is a manifestation of Lenz's law.

†55. As a variation of the motional emf studied in figure 23.2, let the rails be placed on an inclined plane as shown. Find how the induced emf varies with time.



Interactive Tutorials

56. Magnetic flux. A uniform magnetic field, $B = 3.55 \times 10^{-2}$ T, passes through a plane surface of area $A = 9.35 \times 10^{-2}$ m² at an angle $\theta = 53.5°$ to the normal of the surface. Find the magnetic flux Φ_M passing through the plane surface.

57. Faraday's law. A coil of area $A = 0.035$ m² is connected to a galvanometer. A magnetic field varies from an initial value $B_i = 0.200$ T to a final value $B_f = 0.500$ T in a time of 1.50×10^{-3} s. The initial and final values of the magnetic field vector points out of the area in the direction of the area vector **A**. If the resistance of the circuit is 20.5 Ω, find (a) the induced emf in the circuit and (b) the current in the circuit while the magnetic field is increasing with time.

58. An AC generator. A coil with $N = 200$ turns of wire, with a cross-sectional area $A = 0.015$ m², is rotated at an angular speed of $\omega = 377$ rad/s in an external magnetic field $B = 0.225$ T. Find (a) the maximum induced emf $\mathscr{E}_{max}$, (b) the frequency f of the alternating emf, and (c) if the resistance of the circuit is $R = 100$ Ω, find the maximum current i_{max}. (d) Write the equation for the induced alternating emf $\mathscr{E}$ and the induced alternating current i and make a plot of both.