c. To determine the speed of the electron, equation 29.96 is rearranged as

$$\text{KE} = mc^2 - m_0c^2$$

$$= \frac{m_0c^2}{\sqrt{1 - v^2/c^2}} - m_0c^2 = \left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1\right)m_0c^2$$

$$\frac{1}{\sqrt{1 - v^2/c^2}} - 1 = \frac{\text{KE}}{m_0c^2}$$

$$\frac{1}{\sqrt{1 - v^2/c^2}} = \frac{\text{KE}}{m_0c^2} + 1 = \frac{0.300 \text{ MeV}}{0.511 \text{ MeV}} + 1 = 1.587$$

$$\sqrt{1 - v^2/c^2} = \frac{1}{1.587} = 0.630$$

$$1 - \frac{v^2}{c^2} = (0.630)^2 = 0.397$$

$$\frac{v^2}{c^2} = 1 - 0.397 = 0.603$$

$$v = \sqrt{0.603c^2}$$

$$v = 0.776c$$

Hence, the speed of the electron is approximately seven-tenths the speed of light.

d. To determine the relativistic mass of the electron, we use equation 29.86:

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

$$= \frac{9.11 \times 10^{-31} \text{ kg}}{\sqrt{1 - (0.776c)^2/c^2}}$$

$$= 14.4 \times 10^{-31} \text{ kg}$$

The relativistic mass has increased by approximately 1.6 times the rest mass.

e. The momentum of the electron, found from equation 29.90, is

$$p = mv = \frac{m_0}{\sqrt{1 - v^2/c^2}}v$$

$$= (14.4 \times 10^{-31} \text{ kg})(0.776)(3.00 \times 10^8 \text{ m/s})$$

$$= 3.35 \times 10^{-22} \text{ kg m/s}$$

The Language of Physics

Relativity
The observation of the motion of a body by two different observers in relative motion to each other. At speeds approaching the speed of light, the length of a body contracts, its mass increases, and time slows down (p. 843).

Inertial coordinate system
A frame of reference that is either at rest or moving at a constant velocity (p. 845).

Galilean transformations
A set of classical equations that relate the motion of a body in one inertial coordinate system to that in a second inertial coordinate system. All the laws of classical mechanics are invariant under a Galilean transformation, but the laws of electromagnetism are not (p. 846).

Invariant quantity
A quantity that remains a constant whether it is observed from a system at rest or in motion (p. 850).

Ether
A medium that was assumed to pervade all space. This was the medium in which light was assumed to propagate (p. 853).

Michelson-Morley experiment
A crucial experiment that was performed to detect the presence of the ether. The results of the experiment indicated that if the ether exists it cannot be detected. The assumption is then made that if it cannot be detected, it does not exist. Hence, light does not need a medium to propagate through. The experiment also implied that the speed of light in free space is the same everywhere regardless of the motion of the source or the observer (p. 858).

Special or Restricted Theory of Relativity
Einstein stated his special theory of relativity in terms of two postulates.

Postulate 1: The laws of physics have the same form in all inertial frames of reference.

Postulate 2: The speed of light in free space has the same value for all observers, regardless of their state of motion.

In order for the speed of light to be the same for all observers, space and time itself must change. The special theory is restricted to inertial systems and does not apply to accelerated systems (p. 859).

Lorentz transformations
A new set of transformation equations to replace the Galilean transformations. These new equations are derived by the two postulates of special relativity. These equations show that space and time are intimately connected. The effects of relativity only manifests itself when objects are moving at speeds approaching the speed of light (p. 862).

Proper length
The length of an object that is measured in a frame where the object is at rest (p. 865).

Lorentz-Fitzgerald contraction
The length of a rod in motion as measured by an observer at rest is less than its proper length (p. 866).

Proper time
The time interval measured on a clock that is at rest relative to the observer (p. 869).

Time dilation
The time interval measured on a moving clock is less than the proper time. Hence, moving clocks slow down (p. 870).

Proper mass or rest mass
The mass of a body that is at rest in a frame of reference (p. 877).

Relativistic mass
The mass of a body that is in motion. The relativistic mass is always greater than the rest mass of the object (p. 877).

Relativistic linear momentum
The product of the relativistic mass of a body and its velocity (p. 878).

Relativistic energy
The product of the relativistic mass of a body and the square of the speed of light. This total energy is equal to the sum of the kinetic energy of the body and its rest mass energy (p. 880).

Rest mass energy
The product of the rest mass and the square of the speed of light. Hence, mass can manifest itself as energy, and energy can manifest itself as mass (p. 880).

The law of conservation of mass-energy
Mass can be created or destroyed as long as an equal amount of energy vanishes or appears, respectively (p. 882).

Summary of Important Equations

Galilean transformation of coordinates

$$x = x' + vt \quad (29.1)$$
$$y = y' \quad (29.2)$$
$$z = z' \quad (29.3)$$
$$t = t' \quad (29.4)$$

Galilean transformation of velocities

$$v_x = v'_x + v \quad (29.11)$$
$$v'_x = v_x - v \quad (29.13)$$
$$v'_y = v_y \quad (29.14)$$
$$v'_z = v_z \quad (29.15)$$

Lorentz transformation equations of coordinates

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}} \quad (29.49)$$
$$y' = y$$
$$z' = z$$
$$t' = \frac{t - xv/c^2}{\sqrt{1 - v^2/c^2}} \quad (29.50)$$

Inverse Lorentz transformation equations of coordinates

$$x = \frac{x' + vt'}{\sqrt{1 - v^2/c^2}} \quad (29.51)$$
$$y = y' \quad (29.52)$$
$$z = z' \quad (29.53)$$
$$t = \frac{t' + x'v/c^2}{\sqrt{1 - v^2/c^2}} \quad (29.54)$$

Length contraction

$$L = L_0\sqrt{1 - v^2/c^2} \quad (29.60)$$

Time dilation

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}} \quad (29.64)$$

Lorentz transformation of velocities

$$V'_x = \frac{V_x - v}{1 - (v/c^2)V_x} \quad (29.75)$$
$$V'_y = \frac{V_y\sqrt{1 - v^2/c^2}}{1 - (v/c^2)V_x} \quad (29.76)$$
$$V'_z = \frac{V_z\sqrt{1 - v^2/c^2}}{1 - (v/c^2)V_x} \quad (29.77)$$

Relativistic mass

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}} \quad (29.86)$$

Linear momentum

$$\mathbf{p} = m\mathbf{v} = \frac{m_0\,\mathbf{v}}{\sqrt{1 - v^2/c^2}} \quad (29.90)$$

Newton's second law

$$F = \frac{\Delta p}{\Delta t} = \frac{\Delta(mv)}{\Delta t}$$
$$F = \frac{\Delta}{\Delta t}\left(\frac{(m_0 v)}{\sqrt{1 - v^2/c^2}}\right) \quad (29.92)$$

Relativistic kinetic energy

$$\text{KE} = mc^2 - m_0c^2 \quad (29.96)$$
$$\text{KE} = (\Delta m)c^2 \quad (29.98)$$

Total relativistic energy

$$E = mc^2 \quad (29.99)$$

Rest mass energy

$$E_0 = m_0c^2 \quad (29.101)$$

Law of conservation of relativistic energy

$$E = \text{KE} + E_0 \quad (29.102)$$

Electron volt

$$1\ \text{eV} = 1.60 \times 10^{-19}\ \text{J} \quad (29.103)$$
$$\text{u} = 1.66 \times 10^{-27}\ \text{kg} \quad (29.104)$$

Unified mass unit

$$\text{u} = 931.493\ \text{MeV} \quad (29.105)$$

Questions for Chapter 29

1. If you are in an enclosed truck and cannot see outside, how can you tell if you are at rest, in motion at a constant velocity, speeding up, slowing down, turning to the right, or turning to the left?
†2. Does a length contract perpendicular to its direction of motion?
†3. Lorentz explained the negative result of the Michelson-Morley experiment by saying that the ether caused the length of the telescope in the direction of motion to be contracted by an amount given by $L = L_0\sqrt{1 - v^2/c^2}$. Would this give a satisfactory explanation of the Michelson-Morley experiment?
4. If the speed of light in our world was only 100 km/hr, describe some of the characteristics of this world.
†5. Does time dilation affect the physiological aspects of the human body, such as aging? How does the body know what time is?
6. Are length contraction and time dilation real or apparent?
7. An elementary particle called a neutrino moves at the speed of light. Must it have an infinite mass? Explain.
†8. It has been suggested that particles might exist that are moving at speeds greater than c. These particles, which have never been found, are called tachyons. Describe how such particles might exist and what their characteristics would have to be.
9. In the equation for the total relativistic energy of a body, could there be another term for the potential energy of a body? Does a compressed spring, which has potential energy, have more mass than a spring that is not compressed?
†10. When helium is formed, the difference in the mass of helium and the mass of its constituents is given off as energy. When the deuteron is formed, the difference in mass is also given off as energy. Could the formation of deuterium be used as a source of commercial energy?
11. If the speed of light were infinite, what would the Lorentz transformation equations reduce to?
†12. Can you apply the Lorentz transformations to a reference frame that is moving in a circle?

Problems for Chapter 29

29.1 Introduction to Relative Motion

1. A projectile is thrown straight upward at an initial velocity of 25.0 m/s from an open truck at the same instant that the truck starts to accelerate forward at 5.00 m/s². If the truck is 4.00 m long, how far behind the truck will the projectile land?
2. A projectile is thrown straight up at an initial velocity of 25.0 m/s from an open truck that is moving at a constant speed of 10.0 m/s. Where does the projectile land when (a) viewed from the ground (S frame) and (b) when viewed from the truck (S' frame)?
3. A truck moving east at a constant speed of 50.0 km/hr passes a traffic light where a car just starts to accelerate from rest at 2.00 m/s². At the end of 10.0 s, what is the velocity of the car with respect to (a) the traffic light and (b) with respect to the truck?
4. A woman is sitting on a bus 5.00 m from the end of the bus. If the bus is moving forward at a velocity of 7.00 m/s, how far away from the bus station is the woman after 10.0 s?

29.2 The Galilean Transformations of Classical Physics

5. The woman on the bus in problem 4 gets up and (a) walks toward the front of the bus at a velocity of 0.500 m/s. What is her velocity relative to the bus station? (b) The woman now walks toward the rear of the bus at a velocity of 0.500 m/s. What is her velocity relative to the bus station?

29.3 The Invariance of the Mechanical Laws of Physics under a Galilean Transformation

†6. Filling in the steps omitted in the derivation associated with figure 29.8, show that the law of conservation of momentum is invariant under a Galilean transformation.
†7. Show that the law of conservation of energy for a perfectly elastic collision is invariant under a Galilean transformation.

29.5 The Michelson-Morley Experiment

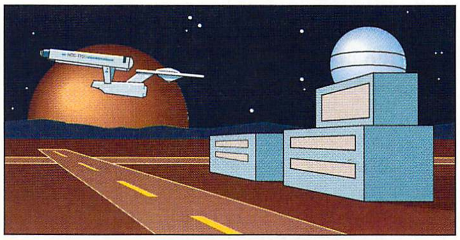
8. A boat travels at a speed V of 5.00 km/hr with respect to the water, as shown in figure 29.10. If it takes 90.0 s to cross the river and return and 95.0 s for the boat to go the same distance downstream and return, what is the speed of the river current?

29.7 The Lorentz Transformation

9. A woman on the earth observes a firecracker explode 10.0 m in front of her when her clock reads 5.00 s. An astronaut in a rocket ship who passes the woman on earth at $t = 0$, at a speed of $0.400c$ finds what coordinates for this event?
10. A clock in the moving coordinate system reads $t' = 0$ when the stationary clock reads $t = 0$. If the moving frame moves at a speed of $0.800c$, what time will the moving clock read when the stationary observer reads 15.0 hr on her clock?
†11. Use the Lorentz transformation to show that the equation for a light wave, equation 29.25, has the same form in a coordinate system moving at a constant velocity.

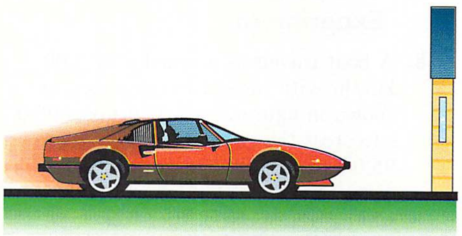
29.8 The Lorentz-Fitzgerald Contraction

12. The USS *Enterprise* approaches the planet Seti Alpha 5 at a speed of $0.800c$. Captain Kirk observes an airplane runway on the planet to be 2.00 km long. The air controller on the planet says that the runway on the planet is how long?



13. The starship *Regulus* was measured to be 100 m long when in space dock. If it approaches a planet at a speed of $0.400c$, how long does it appear to an observer on the planet?

14. How fast must a 15.0-ft car move in order to fit into a 1.00-ft garage? Could you park the car in this garage?



15. A comet is observed to be 130 km long as it moves past an observer at a speed of $0.700c$. How long does the comet seem when it travels at a speed of $0.900c$ with respect to the observer?

16. A meterstick at rest makes an angle of 30.0° with the x-axis. Find the length of the meterstick and the angle it makes with the x'-axis for an observer moving parallel to the x-axis at a speed of $0.650c$.

29.9 Time Dilation

17. A particle is observed to have a lifetime of 1.50×10^{-6} s when it is at rest in the laboratory. (a) What is its lifetime when it is moving at $0.800c$? (b) How far will the particle move with respect to the moving frame of reference before it decays? (c) How far will the particle move with respect to the laboratory frame before it decays?

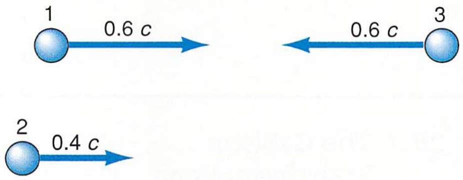
18. A stroboscope is flashing light signals at the rate of 2100 flashes/min. An observer in a rocket ship traveling toward the strobe light at $0.500c$ would see what flash rate?

19. A particle has a lifetime of 0.100 s when observed while it moves at a speed of $0.650c$ with respect to the laboratory. What is its lifetime in its rest frame?

29.10 Transformation of Velocities

20. A spaceship traveling at a speed of $0.600c$ relative to a planet launches a rocket backward at a speed of $0.500c$. What is the velocity of the rocket as observed from the planet?

21. The three electrons are moving at the velocities shown in the diagram. Find the relative velocities between (a) electrons 1 and 2, (b) electrons 2 and 3, and (c) electrons 1 and 3.



29.11 The Law of Conservation of Momentum and Relativistic Mass

22. What is the mass of the following particles when traveling at a speed of $0.86c$: (a) electron, (b) proton, and (c) neutron?

23. Find the speed of a particle at which the mass m is equal to (a) $0.100\ m_0$, (b) $1.00\ m_0$, (c) $10.0\ m_0$, (d) $100\ m_0$, and (e) $1000\ m_0$.

24. Determine the linear momentum of an electron moving at a speed of $0.990c$.

25. How fast must a proton move so that its linear momentum is 8.08×10^{-19} kg m/s?

26. Compute the speed of a neutron whose total energy is 1.88×10^{-10} J.

29.12 The Law of Conservation of Mass-Energy

27. An isolated neutron is capable of decaying into a proton and an electron. How much energy is liberated in this process?

28. Since it takes 540 kcal to convert 1.00 kg of water to 1.00 kg of steam at 100 °C, what is the increase in mass of the steam?

29. What is the kinetic energy of a proton traveling at $0.800c$?

30. Through what potential difference must an electron be accelerated if it is to attain a speed of $0.800c$?

31. What is the total energy of a proton traveling at a speed of 2.50×10^8 m/s?

32. Calculate the speed of an electron whose kinetic energy is twice as large as its rest mass energy.

Additional Problems

33. If an ion-engine in a spacecraft can produce a continuous acceleration of $0.200\ \text{m/s}^2$, how long must the engine continue to accelerate if it is to reach the speed of $0.500c$?

†34. The volume of a cube is V_0 in a frame of reference where it is at rest. Show that the volume observed in a moving frame of reference is given by

$$V = V_0\sqrt{1 - v^2/c^2}$$

35. The distance to Alpha Centari, the closest star, is about 4.00 light years as measured from earth. What would this distance be as observed from a spaceship leaving earth at a speed of $0.500c$? How long would it take to get there according to a clock on the spaceship and a clock on earth?

36. A muon is an elementary particle that is observed to have a lifetime of 2.00×10^{-6} s before decaying. It has a typical speed of 2.994×10^8 m/s. (a) How far can the muon travel before it decays? (b) These particles are observed high in our atmosphere, but with such a short lifetime how do they manage to get to the surface of the earth?

†37. Show that the formula for the density of a cube of material moving at a speed v is given by

$$\rho = \frac{\rho_0}{1 - v^2/c^2}$$

†38. A proton is accelerated to a speed of $0.500c$. Find its (a) kinetic energy, (b) total energy, (c) relativistic mass, and (d) momentum.

†39. Show that the speed of a particle can be given by

$$v = c\sqrt{1 - (E_0/E)^2}$$

where E_0 is the rest mass energy of the particle and E is its total energy.

†40. An electron is accelerated from rest through a potential difference of 4.00×10^6 V. Find (a) the kinetic energy of the electron, (b) the total energy of the electron, (c) the velocity of the electron, (d) the relativistic mass, and (e) the momentum of the electron.

†41. From the solar constant, determine the total energy transmitted by the sun per second. How much mass is this equivalent to? If the mass of the sun is 1.99×10^{30} kg, approximately how long can the sun continue to radiate energy?

†42. A reference frame is accelerating away from a rest frame. Show that Newton's second law in the form $F = ma$ does not hold in the accelerated frame.

Interactive Tutorials

43. Length contraction. The length of a rod at rest is found to be $L_0 = 2.55$ m. Find the length L of the rod when observed by an observer in motion at a speed $v = 0.250c$.

44. Time dilation. A clock in a moving rocket ship reads a time duration $\Delta t_0 = 15.5$ hr. What time elapses, Δt, on earth if the rocket ship is moving at a speed $v = 0.355c$?

45. Relative velocities. Two spaceships are approaching a space station, as in figure 29.15. Spaceship 1 has a velocity of $0.55c$ to the left and spaceship 2 has a velocity of $0.75c$ to the right. Find the velocity of rocket ship 1 as observed by rocket ship 2.

46. Relativistic mass. A mass at rest has a value $m_0 = 2.55$ kg. Find the relativistic mass m when the object is moving at a speed $v = 0.355c$.

47. The length of a rod at rest is $L_0 = 1.00$ m and its mass is $m_0 = 1.00$ kg. Find the length L and mass m of the rod as its speed v in the axial direction increases from $0.00c$ to $0.90c$, where c is the speed of light ($c = 3.00 \times 10^8$ m/s). Plot the results.

48. An accelerated charged particle. An electron is accelerated from rest through a potential difference $V = 4.55 \times 10^5$ V. Find (a) the kinetic energy of the electron, (b) the rest mass energy of the electron, (c) the total relativistic energy of the electron, (d) the speed of the electron, (e) the relativistic mass of the electron, and (f) the momentum of the electron.