The argument up to now may seem somewhat academic, in that we have described some of the characteristics of black holes, but do they really exist in nature? That is, is it possible for any objects in the universe to become black holes? The answer is yes. In the ordinary evolution of very massive stars, black holes can be formed. A star is essentially a gigantic nuclear reactor converting hydrogen to helium in a process called *nuclear fusion.* Think of the star as millions of hydrogen bombs going off at the same time, thereby producing enormous quantities of energy and enormous forces outward from the star. There is an equilibrium between the gravitational forces inward and the forces outward caused by the exploding gases. Eventually, when all the nuclear fuel is used, there is no longer an equilibrium condition. The gravitational force causes the gas to become very compact. If the star is large enough, it is compressed below its Schwarzschild radius and a black hole is formed. For an evolving star to condense into a black hole it must be approximately 25 times the mass of the sun. When the star condenses to a black hole it does not stop at the event horizon but continues to reduce in size until it becomes a singularity, a point mass. That is, the entire mass of the star has condensed to the size of a point.

There is experimental evidence that a black hole has been found as a companion of the star Cygnus X-1 and more are looked for every day.

Since time slows down in a gravitational field, the effect becomes much more pronounced in the vicinity of the black hole. If a person were to fall into the black hole he would eventually be crushed due to the enormous gravitational forces. Time would slow down for him as he approached the event horizon. At the event horizon, time would stand still for him.

The Schwarzschild black hole is an example of a nonrotating massive body. However, just as the sun and planets rotate about their axes, a more general solution of a black hole should also be concerned with the rotation of the massive body. The solution to the rotating black hole is called a *Kerr black hole,* after Roy Kerr, a New Zealand mathematician. The rotating black hole[4] (essentially an accelerating black hole) drags spacetime around with it, forming a second event horizon, thus leaving a space between the first event horizon and the second event horizon. It has been speculated that it may be possible to enter the first event horizon, but not the second, and exit somewhere else in either another universe or in this universe in another place and/or time.

It has also been speculated that there might also exist white holes in space. That is, mass is drawn into a black hole, but would be spewed out of a white hole. In fact some physicists have speculated that a black hole in one universe is a white hole in another universe.

4. See interactive tutorial problem 15.

The Language of Physics

Spacetime diagram
A graph of a particle's space and time coordinates. The time coordinate is usually expressed as τ, which is equal to the product of the speed of light and the time (p. 891).

World line
A line in a spacetime diagram that shows the motion of a particle through spacetime. A world line of a particle at rest or moving at a constant velocity is a straight line in spacetime. The world line of a light ray makes an angle of 45° with the τ-axis in spacetime. The world line of an accelerated particle is a curve in spacetime (p. 891).

Light cone
A cone that is drawn in spacetime showing the relation between the past and the future of a particle in spacetime. World lines within the cone are called timelike because they are accessible to us in time. Events outside the cone are called spacelike because they occur in another part of space that is not accessible to us and hence is called elsewhere (p. 894).

Invariant interval
A constant value in spacetime that all observers agree on, regardless of their state of motion. The equation of the invariant interval is in the form of a hyperbola in spacetime. Because of the hyperbolic form of the invariant interval, Euclidean geometry does not hold in spacetime. The reason for length contraction and time dilation is the fact that spacetime is non-Euclidean. The longest distance in spacetime is the straight line (p. 894).

Equivalence principle
On a local scale, the physical effects of a gravitational field are indistinguishable from the physical effects of an accelerated coordinate system. Hence, an accelerated frame of reference is equivalent to an inertial frame of reference in which gravity is present, and an inertial frame is equivalent to an accelerated frame in which gravity is absent (p. 906).

The general theory of relativity
The laws of physics are the same in all frames of reference (note that there is no statement about the constancy of the velocity of light as in the special theory of relativity) (p. 907).

Warped spacetime
Matter causes spacetime to be warped so that the world lines of particles in spacetime are curved. Hence, matter warps spacetime and spacetime tells matter how to move. Gravity is a consequence of the warping of spacetime by matter (p. 908).

Gravitational red shift
Time elapsed on a clock in a gravitational field is less than the time elapsed on a clock in a gravity-free space. This effect of the slowing down of a clock in a gravitational field is manifested by observing a spectral line from an excited atom in a gravitational field. The wavelength of the spectral line of that atom is shifted toward the red end of the electromagnetic spectrum (p. 913).

Summary of Important Equations

Tau in spacetime
$$\tau = ct \quad (30.1)$$

Velocity in a spacetime diagram
$$v = c \tan\theta \quad (30.2)$$

The square of the invariant interval
$$(\Delta s)^2 = c^2(\Delta t)^2 - (\Delta x)^2 \quad (30.8)$$
$$(\Delta s)^2 = c^2(\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2 \quad (30.9)$$
$$(\Delta s)^2 = (\Delta\tau)^2 - (\Delta x)^2 \quad (30.11)$$

Slowing down of a clock in a gravitational field
$$\Delta t_f = \Delta t_g\left(1 + \frac{gh}{c^2}\right) \quad (30.31)$$

Gravitational red shift of wavelength

$$\lambda_f = \lambda_g\left(1 + \frac{gh}{c^2}\right) \quad (30.35)$$

Gravitational red shift of frequency

$$\nu_f = \nu_g\left(1 - \frac{gh}{c^2}\right) \quad (30.37)$$

Change in frequency per unit frequency

$$\frac{\Delta\nu}{\nu_g} = \frac{gh}{c^2} \quad (30.38)$$

Questions for Chapter 30

1. Discuss the concept of spacetime. How is it like space and how is it different?
2. How many light cones are there in your classroom?
3. Why can't a person communicate with another person who is elsewhere?

†4. Discuss the twin paradox on the basis of figure 30.6(b).

5. Using figure 30.7, discuss why the scales in the S' system are not the same as the scales in the S system.

†6. Considering some of the characteristics of spacetime, that is, it can be warped, and so forth, could spacetime be the elusive ether?

7. What does it mean to say that spacetime is warped?
8. Describe length contraction by a spacetime diagram.
9. Describe time dilation by a spacetime diagram.
10. Discuss simultaneity with the aid of a spacetime diagram.

Problems for Chapter 30

30.1 Spacetime Diagrams

1. Draw the world line in spacetime for a particle moving in (a) an elliptical orbit, (b) a parabolic orbit, and (c) a hyperbolic orbit.

30.2 The Invariant Interval

2. Find the angle that the world line of a particle moving at a speed of $c/4$ makes with the τ-axis in spacetime.
3. The world line of a particle is a straight line making an angle of 30° below the τ-axis. Determine the speed of the particle.
4. The world line of a particle is a straight line of length 150 m. Find the value of Δx if $\Delta\tau = 200$ m.
5. (a) On a sheet of graph paper draw the hyperbolas representing the invariant interval of spacetime as shown in figure 30.7. (b) Draw the S'-axes on this diagram for a particle moving at a speed of $c/4$.
6. Using the graph of problem 5, draw a rod 1.50 units long at rest in the S frame of reference. (a) From the graph determine the length of the rod in the S' frame of reference. (b) Determine the length of the rod using the Lorentz contraction equation.
7. Using the graph of problem 5, draw a rod 1.50 units long at rest in the S' frame of reference. (a) From the graph determine the length of the rod in the S frame of reference. (b) Determine the length of the rod using the Lorentz contraction equation.

30.6 The Gravitational Red Shift

8. One twin lives on the ground floor of a very tall apartment building, whereas the second twin lives 200 ft above the ground floor. What is the difference in their age after 50 years?
9. The lifetime of a subatomic particle is 6.25×10^{-7} s on the earth's surface. Find its lifetime at a height of 500 km above the earth's surface.
10. An atom on the surface of Jupiter ($g = 23.1$ m/s^2) emits a ray of light of wavelength 528.0 nm. What wavelength would be observed at a height of 10,000 m above the surface of Jupiter?

Additional Problems

†11. Using the principle of equivalence, show that the difference in time between a clock at rest and an accelerated clock should be given by

$$\Delta t_R = \Delta t_A \left(1 + \frac{ax}{c^2}\right)$$

where Δt_R is the time elapsed on a clock at rest, Δt_A is the time elapsed on the accelerated clock, a is the acceleration of the clock, and x is the distance that the clock moves during the acceleration.

†12. A particle is moving in a circle of 1.00-m radius and undergoes a centripetal acceleration of 9.80 m/s^2. Using the results of problem 11, determine how many revolutions the particle must go through in order to show a 10% variation in time.

13. The pendulum of a grandfather clock has a period of 0.500 s on the surface of the earth. Find its period at an altitude of 200 km. *Hint:* Note that the change in the period is due to two effects. The acceleration due to gravity is smaller at this height even in classical physics, since

$$g = \frac{GM}{(R + h)^2}$$

To solve this problem, use the fact that the average acceleration is $g = \dfrac{GM}{R(R + h)}$, and assume that $\Delta t_f = \Delta t_g\left(1 + \dfrac{gh}{c^2}\right)$.

14. Compute the fractional change in frequency of a spectral line that occurs between atomic emission on the earth's surface and that at a height of 325 km.

Interactive Tutorials

15. A rotating black hole. Assume the sun were to collapse to a black hole as described in the "Have you ever wondered . . . ?" section. (a) Calculate the radius of the black hole, which is called the Schwarzschild radius R_S. Since the sun is also rotating, angular momentum must be conserved. Therefore as the sun collapses the angular velocity of the sun must increase, and hence the tangential velocity of a point on the surface of the sun must also increase. (b) Find the radius of the sun during the collapse such that the tangential velocity of a point on the equator is equal to the velocity of light c. Compare this radius to the Schwarzschild radius. Some characteristics of the sun are radius, $r_0 = 6.96 \times 10^8$ m, mass of sun $M = 1.99 \times 10^{30}$ kg, and the angular velocity of the sun $\omega_0 = 2.86 \times 10^{-6}$ rad/s.

16. Gravitational red shift. An atom on the surface of the earth emits a ray of light of wavelength $\lambda_g = 528.0$ nm, straight upward. (a) What wavelength λ_f would be observed at a height $y = 10{,}000$ m? (b) What frequency ν_f would be observed at this height? (c) What change in time would this correspond to?