

3 Motions in the Universe

KEY IDEA

The regular motions in the universe can be discovered by observation and described mathematically.



PHYSICS AROUND US . . . Calculated Moves

How many times have you crossed a street today? Once? A dozen times? More? It's such an ordinary thing to do that you probably don't even keep track. Yet a simple act like crossing a street contains, within itself, a very important lesson about the way the universe works.

Think about what happens when you see a car approaching. You watch it for a while, estimate its speed, make an unconscious calculation about how long it will take before the car gets to your corner, and only then do you make a decision about whether to start across the street. You couldn't carry out this ordinary process if you didn't have an understanding of how objects moved—an understanding born of long experience

with cars and their behavior. Early in your life you observed cars in motion, came to some conclusions about their properties, and have used (and tested) that knowledge ever since. We cannot tell whether the car will turn left or right at the intersection or how much gas is in the tank, but we don't need to know that for estimating how quickly the car is approaching. Our observation and experience enable us to determine what we need to know for deciding when to cross the street.

In the same way, physicists observe the world and summarize their conclusions, often using a series of mathematical laws to do so. This process forms the core of what we call science.

PREDICTABILITY



The stars in the night sky, showing constellations. Most of these stars are not visible in the glare of city lights.

Among the most predictable objects in the universe are the lights we see in the sky at night—the stars and planets. People who live in today’s large metropolitan areas no longer pay much attention to the richness of the night sky’s shifting patterns. But think about the last time you were out in the country on a clear moonless night, far from the lights of town. There, the stars seem very close, very real. Now try to imagine what it was like before the development of artificial lighting in the nineteenth century. Human beings often experienced jet-black skies that were filled with brilliant pinpoint stars.

If you observe the sky closely, you notice that it changes; it’s never quite the same from one night to the next. Our ancestors also observed regularities in the arrangement and movement of stars and planets, and they wove these patterns into their religion and mythology. They knew, based on their observations, that when the Sun rose in a certain place, it was time to plant crops because spring was on its way. They came to know that there were certain times of the month when a full Moon would illuminate the ground, allowing them to continue harvesting and hunting after sunset. To these people, knowing the behavior of the sky was not an intellectual game or an educational frill. It was an essential part of their lives. No wonder, then, that astronomy, the study of the heavens, was one of the first sciences to develop.

By relying on their observations and records of the regular motion of the stars and planets, ancient observers of the sky were perhaps the first humans to accept the most basic tenet of science:

The universe is predictable and quantifiable.

Without the predictability of physical events, as we saw in Chapter 1, and our ability to quantify what we observe, as discussed in Chapter 2, the scientific method could not proceed.

Stonehenge and the Cycle of Seasons

No better symbol exists of humankind’s discovery of the predictability of nature than Stonehenge, the great prehistoric stone monument on Salisbury Plain in southern England. The structure consists of a large circular bank of earth, surrounding a ring of single upright stones, which, in turn, encircle a horseshoe-shaped structure of five giant stone archways. Each arch is constructed from three massive blocks—two vertical supports several meters tall capped by a great stone lintel. The open end of the horseshoe aligns with an avenue that leads northeast to another large stone, called the “heel stone” (Figure 3-1).

Stonehenge was built in spurts over a long period of time, starting about 2800 B.C. Despite various legends assigning it to the Druids, Julius Caesar, or the magician Merlin (who was supposed to have levitated the stones from Ireland), archaeologists have shown that it was built by several groups of people, none of whom had a written language and some of whom even lacked metal tools.

Stonehenge, like many similar structures scattered around the world, was built to mark the passing of time—a calendar based on the movement of objects in the sky. At Stonehenge, the seasons were marked by the alignment of the



Stonehenge is testimony to the predictability of the Earth’s seasons over the centuries.

stones with astronomical events. On midsummer's morning, for example, someone standing in the center of the monument sees the Sun rising directly over the heel stone.

Building a structure such as Stonehenge required the accumulation of a great deal of knowledge about the sky—knowledge that could only have been gained through many years of observation. Without a written language, people needed to pass complex information about the movements of the Sun, the Moon, and the planets from one generation to the next. How else could they have aligned their stones so perfectly that modern-day Druids in England can still greet the midsummer sunrise over the heel stone?

But as impressive as Stonehenge the monument might be, Stonehenge the symbol of universal regularity and predictability is even more impressive. If the universe were not regular and predictable—if repeated observation could not show us patterns that occur over and over again—the very concept of a monument such as Stonehenge would be impossible. And yet, it continues to stand after 4000 years, a testament to human ingenuity and to the possibility of predicting the behavior of the universe in which we live.

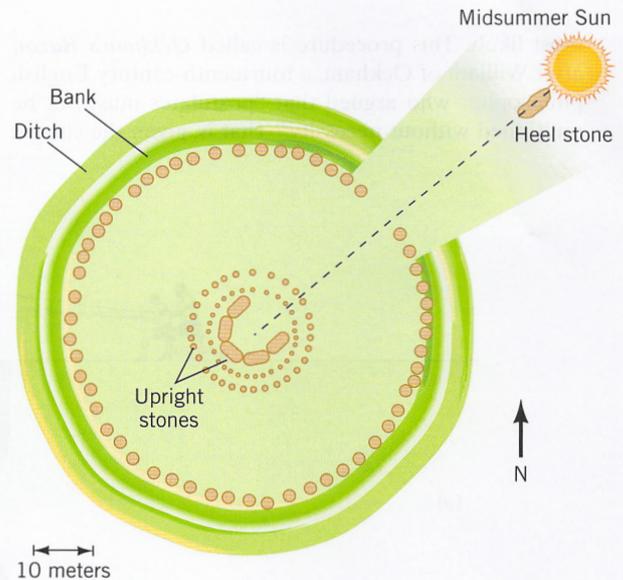


FIGURE 3-1. Stonehenge is built so that someone standing at the center will see the Sun rise over the heel stone on midsummer's morning.

LOOKING DEEPER

Stonehenge and Ancient Astronauts

Who built Stonehenge? Confronted by such an awesome stone monument, with its precise orientation and epic proportions, some writers evoke outside intervention by extraterrestrial visitors. Many ancient monuments, including the pyramids of Egypt, the Mayan temples of Central America, and the giant statues of Easter Island, have been ascribed to these mysterious aliens. One point often made is that the stones in the monument are simply too large to have been moved by people who didn't even have the wheel.

The largest stone at Stonehenge, about 10 meters (more than 30 feet) in length, weighs about 50 metric tons (50,000 kilograms or about 100,000 pounds) and had to be moved overland some 30 kilometers (20 miles) from quarries to the north. Could primitive people equipped only with wood and ropes have moved this massive block?

While Stonehenge was being built, the climate was cooler than it is now and it snowed frequently in southern England, so the stones could have been hauled on sleds. A single person can easily haul 100 kilograms on a sled (think of pulling a couple of your friends). How many people would it take to haul a 50,000-kilogram stone? To estimate the answer, we divide the total weight of the largest stone by the weight an individual can move:

$$\frac{50,000 \text{ kg}}{100 \text{ kg pulled by each person}} = 500 \text{ people}$$

Organizing 500 people for the job would have been a major social achievement in ancient times, but it was certainly physically possible (Figure 3-2).

Scientists cannot absolutely disprove the possibility that Stonehenge was constructed by some strange, forgotten technology. But why invoke such alien intervention when the concerted actions of a dedicated, hard-working human society would have sufficed? All of us are fascinated and awed by the mysterious and unknown, and an ancient structure such as Stonehenge, standing stark and bold on the Salisbury plain, certainly evokes these feelings.

When confronted with phenomena in a physical world, we should accept the simplest explanation as the

most likely. This procedure is called *Ockham's Razor*, after William of Ockham, a fourteenth-century English philosopher who argued that “postulates must not be multiplied without necessity.” That is, given the choice,

the simplest solution to a problem is most likely to be right. Scientists thus reject the notion of ancient astronauts building Stonehenge, and they relegate such speculation to the realm of pseudoscience.

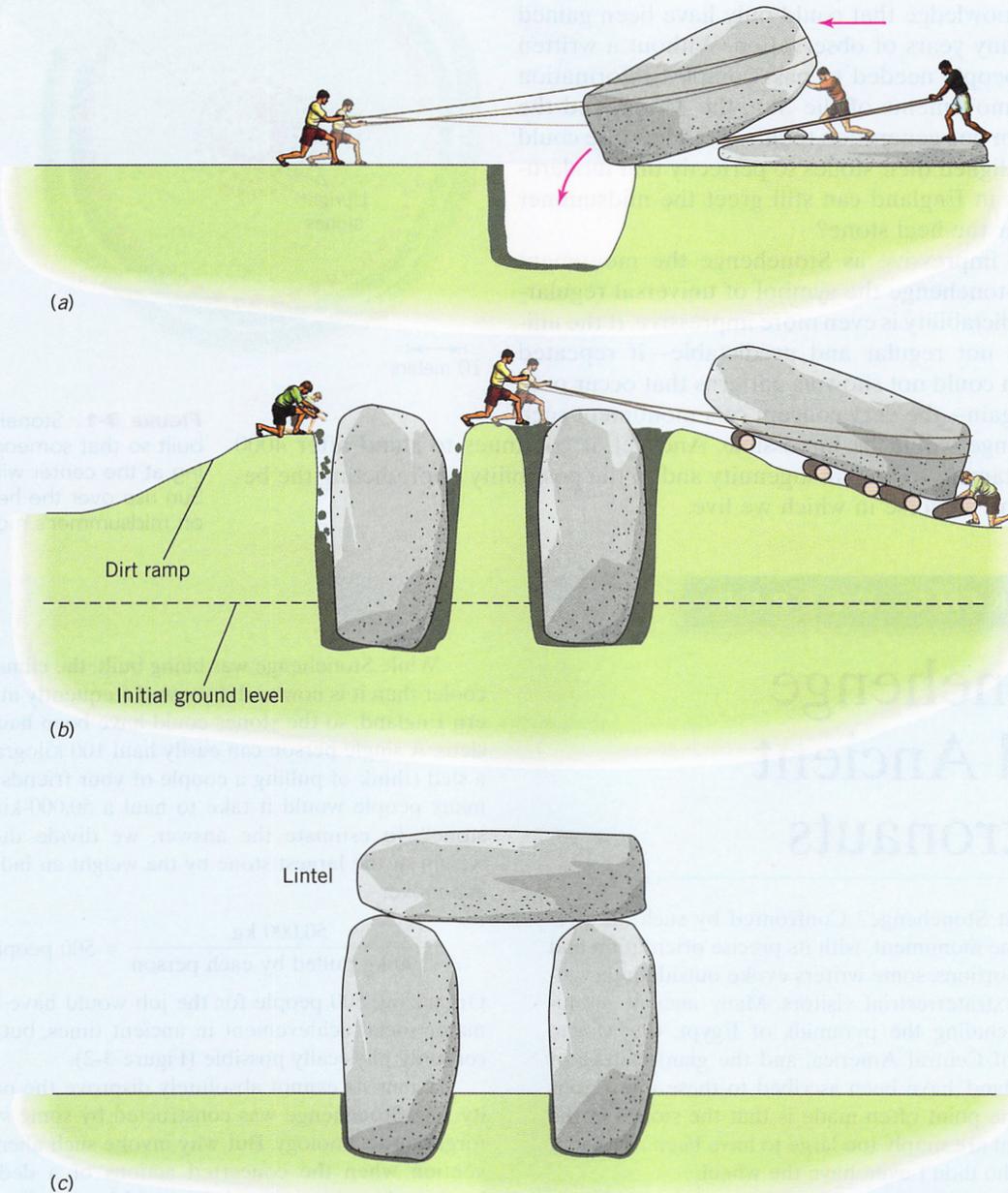


FIGURE 3-2. Perhaps the most puzzling aspect of the construction of Stonehenge is the raising of the giant lintel stones. As shown in this reconstruction, three steps in the process were probably (a) dig a pit for each of the upright stones; (b) pile dirt into a long sloping ramp up to the level of the two uprights so that the lintel stones could be rolled into place; and (c) cart away the dirt, thus leaving the stone archway.

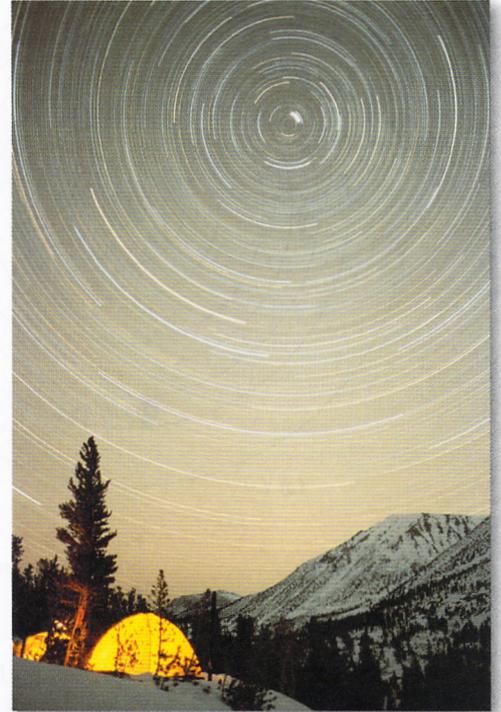
THE BIRTH OF MODERN ASTRONOMY

When you look up at the night sky, you see a dazzling array of objects. Thousands of visible stars fill the heavens and appear to move each night in stately, circular arcs centered on Polaris, the North Star. The relative positions of these stars never seem to change, and closely spaced groups of stars, called *constellations*, have been given names such as the Big Dipper and Leo the Lion. Moving across this fixed starry background are the Earth's Moon, with its regular succession of phases, and half a dozen planets that wander through the sky. You might also see swiftly streaking meteors or long-tailed comets, transient objects that grace the night sky from time to time.

The motion of planets can be especially complex. From night to night most planets, most of the time, appear to move gradually from east to west against the backdrop of the stars. But occasionally a planet seems to reverse its course, seemingly traveling backward with respect to the stars in retrograde motion for a few weeks (Figure 3-3). How can that be?

The Historical Background— Ptolemy and Copernicus

Claudius Ptolemy, an Egyptian-born Greek astronomer and geographer who lived in Alexandria in the second century AD, proposed the first plausible explanation for such complex celestial motions. Working with the accumulated observations of earlier Babylonian and Greek astronomers, he put together a singularly successful theory about how the



Arcs of stars around Polaris, the North Star.

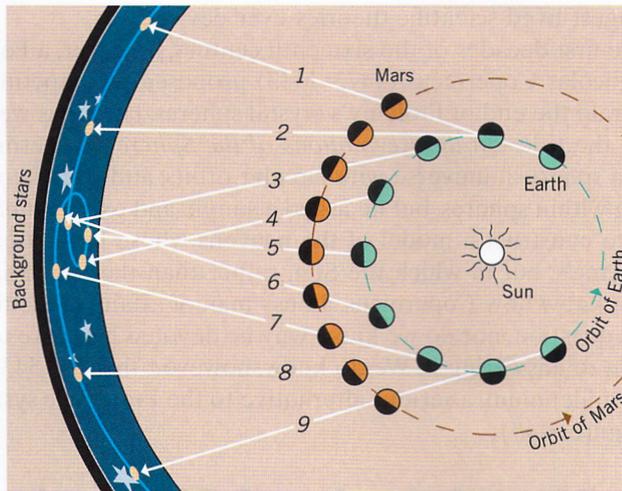


FIGURE 3-3. Today we know that the retrograde motion of Mars can be explained by the fact that as the Earth passes Mars in orbit, the position of Mars against the background of stars seems to reverse itself temporarily. This explanation can only work if the Earth moves, so Greek astronomers could not have evoked it.

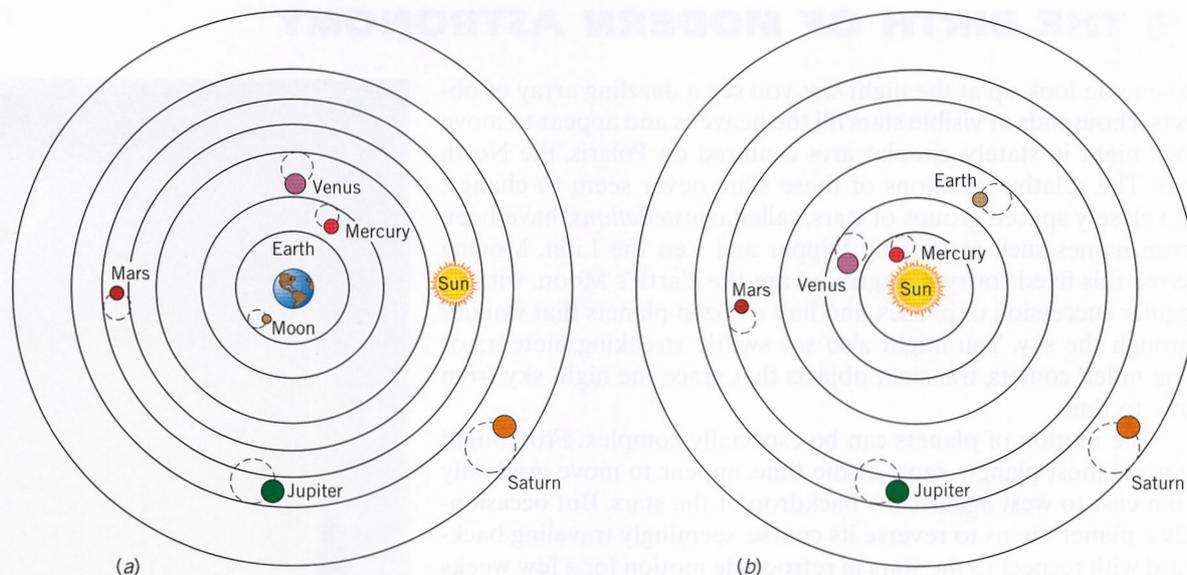
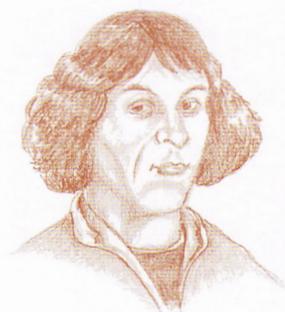


FIGURE 3-4. The Ptolemaic (a) and Copernican (b) systems. Both systems used circular orbits. The fundamental difference is that Copernicus placed the Sun at the center.

heavens had to be arranged to produce the display we see every night (Figure 3-4a). Earth sits unmoving at the center of Ptolemy's universe. Around it, on a series of concentric rotating spheres, move the stars and planets. The model was carefully crafted to take account of observations. The planets, for example, were attached to small spheres rolling inside of the larger spheres so that their uneven retrograde motion across the sky could be understood. This system remained the best explanation of the universe for almost 1500 years. It successfully predicted planetary motions, eclipses, and a host of other heavenly phenomena and was one of the longest-lived scientific theories ever devised.

During the first decades of the sixteenth century, however, a Polish cleric by the name of Nicolas Copernicus (1473–1543) proposed a competing hypothesis that was to herald the end of Ptolemy's crystal spheres. His ideas were published in 1543 under the title *On the Revolutions of the Spheres*. Copernicus retained the notion of a spherical universe with circular orbits, and even kept the idea of spheres rolling within spheres, but he asked a simple and extraordinary question. Is it possible to construct a model of the heavens whose predictions are as accurate as Ptolemy's, but in which the Sun, rather than the Earth, is at the center? We do not know how Copernicus, a busy man of affairs in medieval Poland, conceived this question, nor do we know why he devoted his spare time for most of his adult life to answering it. We do know, however, that in 1543, for the first time in over a millennium, a serious alternative to the Ptolemaic system was presented (see Figure 3-4b).



A portrait of Nicolas Copernicus.

Tycho Brahe and the Art of Observation



With the publication of the Copernican theory, two competing models of the universe confronted astronomers. The Ptolemaic and Copernican systems differed in a fundamental way that had far-reaching implications about the place of

humanity in the universe. They both described possible universes, but in one the Earth, and by implication humankind, was no longer at the center. The astronomers' task was to decide which model best describes our universe.

To resolve this question, astronomers had to compare the predictions of the two competing hypotheses. When astronomers tried to make comparisons, however, a fundamental problem became apparent. Although the models of Ptolemy and Copernicus made different predictions about the position of a planet at midnight or the time of moonrise, the differences were too small to be measured with equipment that was available at the time. The telescope had not yet been invented and astronomers had to record planetary positions by depending entirely on naked-eye measurements with awkward instruments. Until the accuracy of measurement was improved, the question of whether or not the Earth was at the center of the universe could not be decided.

Some scientists thrive on experimental challenges and they revel in devising new tricks for making measurements better than anyone else before. The Danish nobleman Tycho Brahe (1546–1601) was such a scientist. Abducted in infancy by his uncle, Tycho was raised in comfort and given the best possible education. His scientific reputation was firmly established at the age of 25, when he observed and described a new star in the sky (in fact, a type of exploding star called a supernova). By the age of 30, Brahe received from the Danish king the island of Hveen off the coast of Denmark and funds to build an observatory there.

Brahe built his career on designing and using vastly improved observational instruments. He determined each star or planet position with a *quadrant*, a large sloping device something like a gunsight. With this sort of instrument, you can record the position of a star or planet by measuring two angles—for example, the angle up from the horizon and the angle around from due north. Brahe constructed his sighting device of carefully selected materials, and he learned to correct his measurements for the inevitable contraction of brass and iron components that occurred during the cold Danish nights. Over a period of 25 years, he accumulated precise data on the positions of the planets with these instruments, compiling the most accurate record of planetary positions of his day.

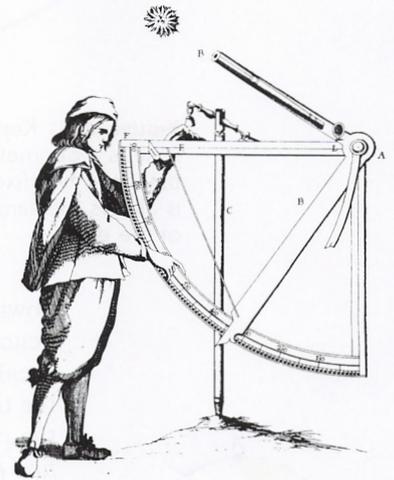
Kepler's Laws

After Tycho Brahe died in 1601, his data passed into the hands of his assistant, Johannes Kepler (1571–1630), a German mathematician who had joined Tycho 2 years before. Kepler was skilled in mathematics, and he was able to analyze Tycho Brahe's decades of planetary data in new ways. In the end, Kepler found that the data could be summarized in three basic mathematical statements about the solar system, known as **Kepler's laws of planetary motion**. The most important of these (shown in Figure 3-5) states that all planets, including the Earth, orbit the Sun in elliptical paths, not in perfect circles as had been previously assumed. In this picture, the spheres within spheres are gone. Not only do Kepler's laws give a more accurate description of what is observed in the sky, but they present a simpler picture of the solar system as well. And as we have seen in our discussion of Stonehenge, simple explanations are often a sign of deeper understanding of how nature works.

An *ellipse* is defined as a curve drawn so that the sum of the distances from any point on the curve to two fixed points is always the same. Imagine tacking



A portrait of Tycho Brahe in his observatory on the island of Hveen.



This instrument, called a quadrant, measures the angular position of stars and planets to the Earth.



Johannes Kepler (1571–1630).

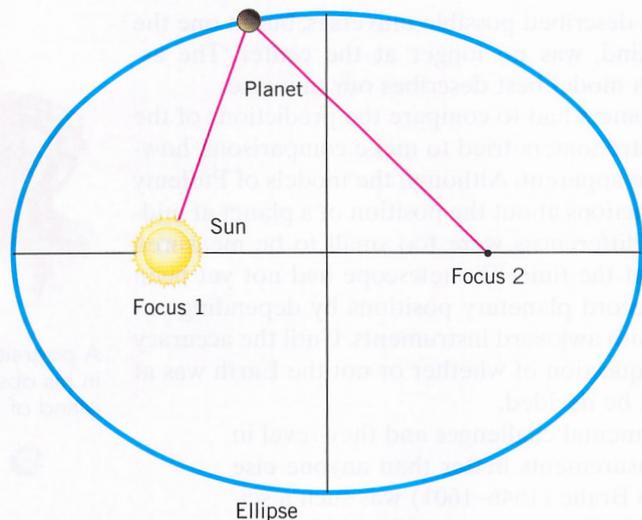


FIGURE 3-5. Kepler's first law, shown schematically. An ellipse is a geometrical figure in which the sum of the distances to two fixed points (each of which is called a focus) is always the same. For the planets, the Sun is at one focus of the ellipse.

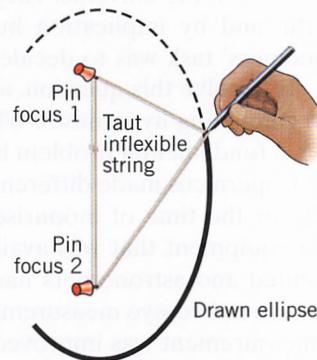


FIGURE 3-6. You can draw an ellipse by bracing a pencil against a string that is tacked down at each end by a pin, and drawing a curve while keeping the string taut. The two pins are at the foci of the ellipse. Note that as the two pins are brought closer together, the ellipse more nearly approximates a circle.

down a length of loose string at two points, then drawing a curve by bracing a pencil in the now-taut string, as shown in Figure 3-6. Each of the two fixed points is called a *focus*. What Kepler found was that the orbits of all planets known at the time, from Mercury to Saturn, have one focus at the Sun. The statement that the planets have elliptical orbits with one focus at the Sun is known as *Kepler's first law of planetary motion*.

Kepler's second law describes the speed at which the planets move in their elliptical orbits. If you remember the playground game in which you ran toward a post, then grabbed it on the fly, and swung around, you have a pretty good notion of the way the planets move. They speed up as they get closer to the Sun and then slow down in the farther parts of their orbits. Kepler's second law is usually stated in terms of equal areas, a concept illustrated in Figure 3-7. Imagine that a line drawn from the Sun to an orbiting planet sweeps out an area in a fixed period of time. *Kepler's second law* says that for a given time interval, this swept-out area is the same, no matter where the planet is in its orbit. A glance at Figure 3-7 should convince you that this means that planets move fastest when they are nearest the Sun and slowest when farther away.

Finally, Kepler turned his attention to the *period* of a planet's revolution—the time it takes for one complete orbit, or its “year.” Planets farther from the Sun have a longer year than those that are closer in for two reasons: (1) they have farther to go as they make their circuit, and (2) the outer planets travel more slowly than the inner ones. The net effect is a systematic relationship between the planet's period and its orbital distance from the Sun. *Kepler's third law* expressed this relationship between a planet's distance from the Sun and its period as a simple equation that allows us to predict the behavior of orbiting objects.

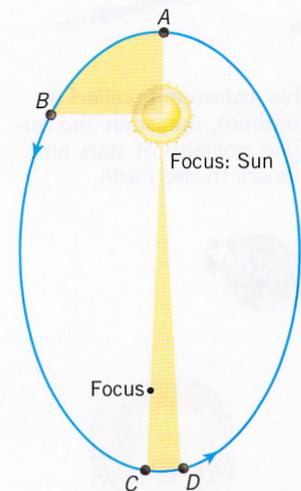


FIGURE 3-7. Kepler's second law of equal areas states that planets move fastest when they are closest to the Sun and slowest when at the farthest point in their orbits. Thus, a planet in an elliptical orbit sweeps out equal areas in equal times.

1. In words:

The farther a planet is from the Sun, the longer its year.

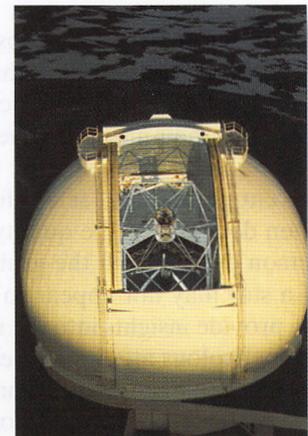
two steps of the scientific method—observation and pattern identification—that we describe in Chapter 1. But the next step, leading to a fundamental theoretical understanding of the sky, had yet to be taken. Kepler’s laws, as important as they are, give no insight into the basic mechanisms that make the solar system operate. The answer to that question was to come from an unexpected source.

THE BIRTH OF MECHANICS AND EXPERIMENTAL SCIENCE: GALILEO GALILEI

Mechanics is an old word for the branch of physics that deals with motions of material objects. A rock rolling down a hill, a ball thrown into the air, and a sailboat skimming over the waves are all fit subjects for mechanics. For sixteenth-century military leaders concerned with the behavior of cannonballs and other projectiles, mechanics was a science of practical interest. Since ancient times, philosophers had speculated on why objects move the way they do, but it wasn’t until about 1600 that our modern understanding of the subject began to emerge.

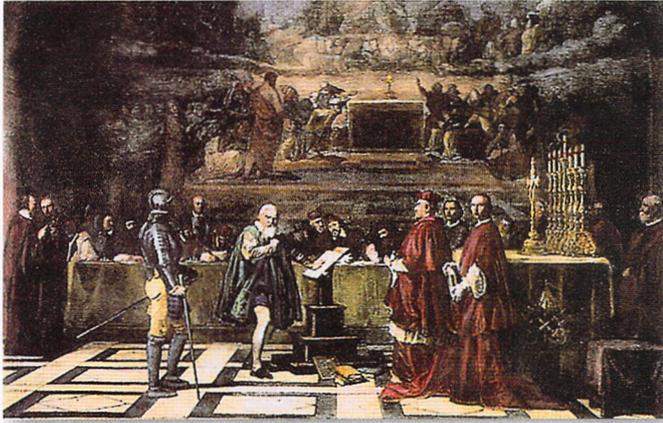
The Italian physicist and philosopher Galileo Galilei (1564–1642) was in many ways a forerunner of the twentieth-century scientist. A professor of mathematics at the University of Padua, he quickly became an advisor to the powerful court of the Medici at Florence as well as a consultant at the Arsenal of Venice, the most advanced naval construction center in the world at that time. He invented many practical devices, such as the first thermometer, the pendulum clock, and the proportional compass that draftsmen still use today. Galileo gained fame as the first person to observe the heavens with a telescope, which he built after hearing of the instrument from others. He was the first to see many astronomical phenomena, including the moons of Jupiter (now called the “Galilean moons”), craters and other surface features of Earth’s Moon, and sunspots.

From the scientist’s point of view, Galileo’s greatest achievement was his work on experimental technique. You can see why by considering his research on the behavior of objects thrown or dropped on the surface of the Earth—work that we discuss in more detail next. Greek philosophers had taught the reasonable idea that heavier objects must fall faster than light ones, because the



What Galileo saw through his telescopes and what we can see today. (a) **Telescopes:** Left, telescopes built and used by Galileo; right, the Keck Observatory Mauna Kea, Hawaii, has twin 394-inch telescopes.

(a)



A painting of the trial of Galileo.

In any case, the situation remained in this unsettled state until 1632, when Galileo published a book called *A Dialogue Concerning Two World Systems*, which was a long defense of the Copernican system. This publication led to the famous trial, in which Galileo excused himself of charges of heresy by denying that he held the views in his book. He was already an old man by this time, and he spent his last few years under virtual house arrest in his villa near Florence.

The legend of the trial of Galileo, in which a rigid hierarchy crushes an earnest seeker after truth (as in Bertoldt Brecht's play *Galileo*), bears little resemblance to the historical events. The Catholic Church had not banned Copernican

ideas; indeed, seminars on the Copernican system were given at the Vatican in the years before Galileo. Furthermore, Galileo's arguments in favor of the system were not very convincing. For example, much of the *Dialogues* is taken up by a completely incorrect discussion of the tides and Galileo had no convincing answers for why things fall if the Earth is not the center of the universe or why we don't notice any effects of the Earth's motion if it moves. His confrontational tactic of putting the Pope's favorite arguments into the mouth of a foolish character in the book brought a predictable reaction that earlier, more reasonable approaches had not. As often happens, under close inspection the simple myth associated with an historical event dissolves into something much more complex.

A footnote: In 1992, the Vatican reopened the case and, in effect, issued a retroactive not guilty verdict in the case of Galileo. The grounds for the reversal were that the original judges had not separated questions of faith from questions of scientific fact. ●

DESCRIBING MOTION

To lay the groundwork for understanding Galileo's study of moving objects, we have to begin with precise definitions of three familiar terms: speed, velocity, and acceleration. These terms are of basic importance throughout all areas of physics and we will use them often. We also note that these and many other terms common in everyday language are used in physics with precise definitions. Usually the definitions are not very different from what you would expect from their common use, but sometimes there are important differences. You need to be aware of these differences when you talk about the concepts of physics and try to explain how they apply in the world around us.

Speed

Speed is one of the many everyday words that has a precise definition in physics.

1. In words:

Speed is the distance an object travels divided by the time that it takes to travel that distance.

2. In an equation with words:

$$\text{Speed} = \frac{\text{Distance traveled}}{\text{Time of travel}}$$

3. In an equation with symbols:

$$s = \frac{d}{t}$$

where s is speed, d is distance traveled, and t is time of travel. This *direct relationship* between speed and distance is illustrated in graphical form in Figure 3-8. If you know the distance traveled and the time elapsed during the travel, you can calculate the speed.

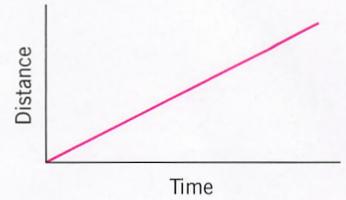


FIGURE 3-8. Graph of the distance traveled by an object moving at a constant speed.

From time to time we will need to use two variations of this equation. First, if you know the average speed, s , of an object and the time of travel, t , you can calculate how far the object has traveled, d :

$$\text{Distance traveled} = \text{Average speed} \times \text{Time of travel}$$

or
$$d = s \times t$$

Second, if you know the total distance traveled and the average speed of travel, you can calculate how long the journey takes:

$$\text{Time of travel} = \frac{\text{Distance traveled}}{\text{Average speed}}$$

or
$$t = \frac{d}{s}$$

Driving Your Car

If the speedometer on your car reads a constant 50 kilometers (31 miles) per hour, how far will you go in 15 minutes?

REASONING AND SOLUTION: We are given a speed and a time and we want to find a distance. We can apply the equation that relates time, speed, and distance in the form:

$$\text{Distance traveled} = \text{Average speed} \times \text{Time of travel}$$

However, this question also involves changing units. First, we must know the travel time in hours:

$$\frac{15 \text{ minutes}}{60 \text{ minutes/hour}} = \frac{1}{4} \text{ hour}$$

Then, using the relationship between distance and time given, we find:

$$\begin{aligned} \text{Distance} &= 50 \text{ kilometers/hour} \times \frac{1}{4} \text{ hour} \\ &= 12.5 \text{ kilometers (7.7 miles)} \end{aligned}$$

Your car will travel 12.5 kilometers (or 7.7 miles) in 15 minutes. ●

A word about units: You may have noticed that in Example 3-2 we put $\frac{1}{4}$ hour into the equation for the time instead of 15 minutes. The reason we did





this was that we needed to be consistent with the units in which an automobile speedometer measures speed. Since the speedometer dial reads in kilometers (or miles) per hour, we also put the time in hours to make the equation balance. A useful way to deal with situations such as this is to imagine the units are quantities that can be canceled in fractions, just like numbers. In this case, we have:

$$\begin{aligned}\text{Distance} &= \text{kilometers/hour} \times \text{hour} \\ &= \text{kilometers}\end{aligned}$$

If, however, we put the time in minutes, we'd have:

$$\text{Distance} = \text{kilometers/hour} \times \text{minutes}$$

and there would be no cancellation.

Whenever you do a problem like this, it's a good idea to check to make sure the units come out correctly. This important process is known as *dimensional analysis*.

Velocity

Velocity has the same numerical value as speed, but it is a vector quantity that also includes information about the direction of travel (see Chapter 2). The speed of a car might be 40 kilometers per hour, for example, while the velocity is 40 kilometers per hour due west. Velocity and speed are measured in units of distance per time, such as meters per second, feet per second, or miles per hour.

Acceleration

Acceleration measures the rate of change of velocity.

1. In words:

Acceleration is the change in velocity divided by the time it takes for that change to occur.

2. In an equation with words:

$$\text{Acceleration} = \frac{\text{Final velocity} - \text{Initial velocity}}{\text{Time}}$$

3. In an equation with symbols:

$$a = \frac{\Delta v}{t}$$

where Δv indicates the change in velocity. Like velocity, acceleration requires information about the direction and is therefore a vector.

When velocity changes, it may be by a certain number of feet per second or meters per second in each second. Consequently, the units of acceleration are meters per second per second, usually described as meters per second squared (and abbreviated m/s^2), where the first meters per second refers to the velocity, and the last per second refers to the time it takes for the velocity to change.

To understand the difference between acceleration and velocity, think about the last time you were behind the wheel of a car, driving along a straight highway and glancing at your speedometer. If the needle is unmoving (at 50 kilometers per hour, for example), you are moving at a constant speed. Suppose, however, that the needle isn't stationary on the speedometer scale (perhaps because you have your foot on the gas pedal or on the brake). Your speed is changing and, by our definition, you are accelerating. The higher the acceleration, the faster the needle moves. If the needle doesn't move, however, this doesn't mean you and the car aren't moving. As we have seen, an unmoving needle simply means that you are traveling at a constant speed without acceleration. Motion at a constant speed in a single direction is called **uniform motion**.

Deceleration

If you're driving a car and step on the brakes, the car slows down, or *decelerates*. This process involves a change in velocity, so it is actually acceleration. We use *deceleration* in everyday speech because we normally associate the term *acceleration* with speeding up. If you look at the definition of acceleration, however, you will notice that if the final velocity is less than the initial velocity (which is what happens when you step on the brakes), the acceleration is a negative number. So deceleration is simply a negative acceleration.

Average and Instantaneous Velocity

While it is fairly straightforward to measure speeds and velocities for objects that aren't accelerating, measuring becomes more complicated when objects accelerate. If your speedometer needle climbs steadily from 30 to 40 kilometers per hour, the speed of your car is constantly changing. At no point during this time interval is speed (or velocity) a constant. So how do we deal with objects whose speed is changing? How do we answer the question "How fast is it moving right now?"

To tackle this problem we need to make a distinction between average velocity and instantaneous velocity. The *average velocity* is simply the total distance traveled divided by the total time it takes to travel that distance. If the distance is a meter and it takes a second to travel that distance, then the average velocity is one meter per second.

Instantaneous velocity, on the other hand, is the velocity at a specific time. We can determine the instantaneous velocity of an accelerating object by thinking of the process this way: Suppose you had marked out a short distance on the pavement—a few feet, for example, or even a fraction of an inch. As the accelerating object goes by, you can time how long it takes to cross that small distance. If the time interval is short enough, the velocity won't change very much as the object crosses the small distance. You can think of the instantaneous velocity as the average velocity measured over a very small time interval. (This concept is explored numerically in Example 3-3 on page 73.)

For the record, what is actually measured in your car's speedometer is the time it takes a particular gear in the transmission to make one revolution. When the car moves a known distance forward, this gear turns once. The number of turns it makes is eventually translated into the number displayed on your speedometer.



Physics in the Making

Measuring Time Without a Watch

In our age of stop watches, digital timers, and atomic clocks, with Olympic races routinely measured to a hundredth or even a thousandth of a second, it's hard to imagine measuring time without an accurate instrument. But when Galileo set out to study accelerated motion in the 1600s, measuring time was a formidable technological challenge. Think about how you might determine small time intervals if all you had was a clock that ticked off seconds but could record no shorter times.



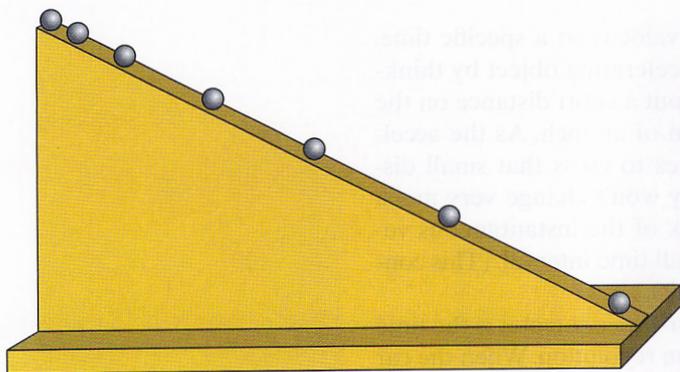
Galileo wanted to document as accurately as possible the way falling objects accelerate, but these measurements required knowledge of both distance and time. Since objects that fall straight down moved much too fast for him to measure, he devised an experiment in which balls rolled down a gently inclined plane. However, even these balls moved too fast to time with available clocks (see Figure 3-9).

Galileo tried a variety of different methods to measure these small increments of time. He experimented with his own heartbeat, but his pulse proved too irregular. He tested rapidly swinging pendulums, but they were difficult to start and stop precisely. He had more success measuring the weight of water that accumulated when a steady flow was started and stopped to coincide with the period of an object's fall, but irregularities in the flow and uncertainties in starting and stopping the water limited the accuracy of that method.

Galileo's most ingenious solution for measuring time intervals relied on his musical training. Galileo stretched lute strings across the rolling ball's path, so that there was a discernible twang when the ball passed over the string. He then adjusted the distance between the strings until he heard the notes coming at precisely equal intervals. A musician with a good ear can tell if notes in a series are off by as little as $\frac{1}{64}$ second. Thus, even though he did not have clocks capable of measuring time intervals better than a second or so, with this scheme he could be certain that several very short time intervals were the same.

When Galileo got the time intervals between twangs just right, he measured the distances between lute strings. He found that if a ball had traveled 1 inch in the first twang, then it traveled 4 inches by the end of two twangs, 9 inches by

Galileo's apparatus: inclined plane



Time	Distance
0	0
1	1
2	4
3	9
4	16
5	25
6	36

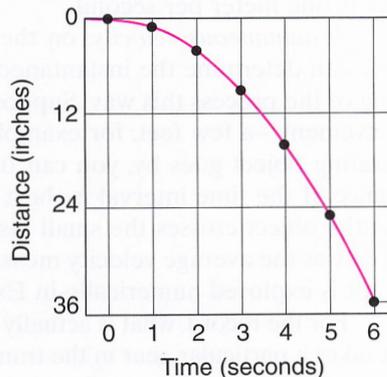


FIGURE 3-9. Galileo's falling-ball apparatus, with a table of measurements and a graph of distance versus time.

the end of three twangs, 16 inches by the end of four twangs, and so on. In other words, he found that the distance traveled by an accelerating object depends on the *square* of the time, not just on the time itself.

Modern versions of the rolling ball experiment, relying on laser beams and electronic timers, have greatly improved the accuracy of Galileo's experiment, but they produce exactly the same result. ●

RELATIONSHIPS AMONG DISTANCE, VELOCITY, AND ACCELERATION

The Velocity of an Accelerating Object

Galileo's experiments revealed a simple relationship between an object's acceleration and the distance it travels.

1. In words:

When an object is accelerated in a uniform way from a standing start, the distance it covers in a given time depends on the square of the time.

2. In an equation with words:

$$\text{Distance traveled} = \frac{1}{2} \times \text{Acceleration} \times \text{Time}^2$$

3. In an equation with symbols:

$$d = \frac{1}{2} at^2$$

This relationship between the distance traveled and time can also be represented in a graph, as shown in Figure 3-10. This is the same kind of graph that we show in Figure 2-8 for a squared relationship.

Suppose that at the instant the object starts accelerating (time equals zero), the velocity is also zero, but the object begins to accelerate at a uniform rate. In this case, the instantaneous velocity of the falling body is given by:

1. In words:

The instantaneous velocity of a uniformly accelerating object that started at rest equals the acceleration multiplied by the total time of acceleration.

2. In an equation with words:

$$\text{Velocity} = \text{Acceleration} \times \text{Time}$$

3. In an equation with symbols:

$$v = a \times t$$

The *direct relationship* between the velocity and time of travel for a given distance is illustrated in Figure 3-11.

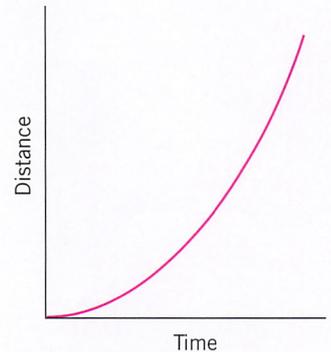


FIGURE 3-10. Graph of the distance traveled by a uniformly accelerating object.

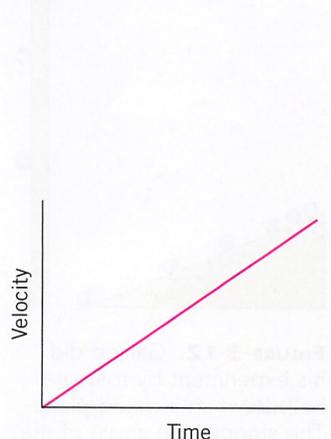


FIGURE 3-11. Graph of the velocity of a uniformly accelerating object.

The relationships among distance, time, velocity, and acceleration enable you to calculate a lot of information about an object's motion from some simple measurements you can make with a ruler and a stop watch. The thing to remember is that during motion with uniform velocity, the distance covered is the average velocity multiplied by the elapsed time; however, during accelerated motion, the distance covered is the acceleration multiplied by one-half the square of the elapsed time. Example 3-3 at the end of the chapter shows how to use these equations to analyze the motion of a sprinter.



Connection

The Evolution of Speed

A hallmark of advancing technology is an increase in speed. Year by year, cars and planes are faster, computers are faster, medical procedures are faster—even food service is faster (most of the time).

Thousands of years ago humans learned to increase their natural speed by domesticating and riding horses. The top human speed in modern times is about 22 miles per hour in a sprint or 16 miles per hour in a 1-mile run. But racehorses can run over 35 miles per hour in a $1\frac{1}{2}$ -mile run. People on horseback could hunt animals such as buffalo for food and escape other predators such as wolves. Greater speed meant improvements in life.

By the nineteenth century, machines surpassed horses in speed. Around 1830, Peter Cooper's steam-engine locomotive, the *Tom Thumb*, pulled ahead of a horse-drawn carriage pulling the same load in a race, but broke down before the end and the horse won. Journalists of the time speculated about the dangers to the human body of such high speeds as 60 miles per hour, but by 1860, railroads had expanded across the country. Their ability to carry huge amounts of freight changed the world's economy. Today, of course, freight trains are still a commonplace, but land speed belongs to the passenger trains. High-speed bullet trains in France and Japan routinely cruise at 175 miles per hour between cities.

Automobiles powered by steam engines were developed as early as the eighteenth century, but were not practical. True land vehicles that did not run on rails had to wait until the development of the internal combustion engine, patented in 1879. Within a few decades, some commercial automobiles could exceed 100 miles per hour on the open road.

However, the true kings of speed are airplanes. The Wright brothers' first airplane flight in 1903 lasted 12 seconds and covered 120 feet. Five years later they were flying 10 miles at a stretch at average speeds of 40 miles per hour. Today, planes can fly more than three times faster than the speed of sound. The SR-71 Blackbird, the fastest vehicle ever built other than spacecraft, has achieved speeds of almost 2200 miles per hour and is one of the great achievements of modern technology. (See Looking at Speed on page 65.) ●

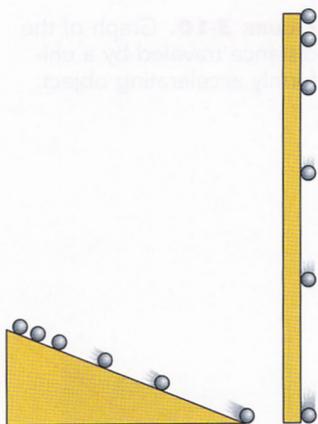


FIGURE 3-12. Galileo did his experiment by rolling a ball down an inclined plane. The steeper the angle of the plane, the faster the fall. A plane at a right angle to the ground corresponds to a freely falling object.

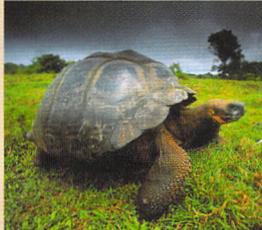
Acceleration due to Gravity

In Galileo's experiments, he produced greater accelerations by increasing the angle of the incline down which the balls rolled (Figure 3-12). Eventually, at very steep angles, the motion of the balls became too fast to measure the time intervals. In our everyday lives, most falling objects fall straight down, equivalent to a plane tilted at an angle of 90 degrees.

Looking at Speed

You can probably walk much faster than a tortoise, but you wouldn't want to race against a cheetah. Not unless you drove in a car, which could easily pass any animal, or in a jet plane, which is faster than anything other than spacecraft. And speaking of spacecraft, some probes have been launched to take close-up looks at comets, which can move far faster than anything on Earth.

10^{-1} m/s



Giant tortoise, about 0.11 meter/second (0.25 miles/h)

10^1 m/s



Cheetah, about 31 meters/second (70 miles/h)

10^5 m/s



Comet West, about 45,000 meters/second (100,000 miles/h)

10^2 m/s



Race car, about 90 meters/second (200 miles/h)

10^3 m/s



SR-71 Blackbird jet, about 1100 meters/second (2500 miles/h)

You can verify that there is an acceleration involved by dropping an object and watching it fall. Notice that at the instant you release the object it barely moves. Can you see that it's moving faster at the end of its fall than at the beginning?

The acceleration of a freely falling body is so important that physicists give it a special name, **acceleration due to gravity**, and assign it a special letter, ***g***. Galileo's experiments led him to the hypothesis that any object dropped near the Earth's surface, no matter how heavy or light, falls with exactly the same constant acceleration. In the absence of complications such as air resistance or wind, which may slow down or alter the direction of a falling object, it makes no difference how massive an object is—all objects experience exactly the same acceleration. (In Chapter 4 you will learn why this is so.)

The numerical value of *g* can be determined by measuring the actual motion of objects. The modern value of *g* is given by:

$$g = 32 \text{ feet/s}^2 = 9.8 \text{ m/s}^2$$

The velocity of a falling object is given by:

$$\text{Velocity of a falling object} = g \times \text{Time}$$

These equations tell us that in the first second, a falling object accelerates from a stationary position to a velocity of 9.8 m/s (about 22 miles per hour) straight down. After two seconds, the velocity doubles to 19.6 m/s; after three seconds, it triples to 29.4 m/s, and so on.

Ironically, Galileo probably never performed the one experiment for which he is most famous—dropping two different weights from the leaning Tower of Pisa to see which would land first. Had he done so, in fact, the effects of friction between the air and the falling bodies probably would have slowed the lighter one slightly more than the heavier object, so that the two would have been perceived to fall at slightly different rates.

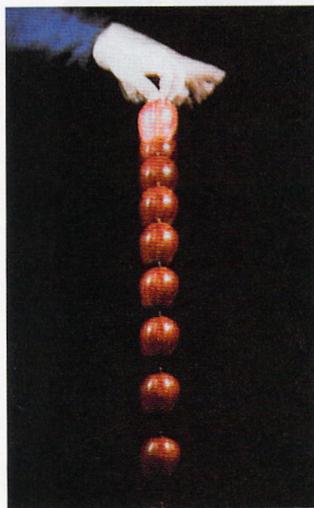


FIGURE 3-13. A multiple-exposure photograph captures the accelerated motion of a falling apple. In each successive time interval, the apple falls farther.



Develop Your Intuition: Freely Falling Objects

According to Galileo's results, every object at the surface of the Earth, once released, should accelerate downward at the same rate (Figure 3-13). But we know that if a leaf and an acorn fall from a branch at the same time, the acorn reaches the ground well before the leaf, even if they have equal mass. Was Galileo wrong?

To understand this problem, think for a minute about *how* the leaf and the acorn fall. The acorn plummets straight down to the ground, accelerating as it goes. The leaf, on the other hand, flutters down slowly. It appears that the air has a much greater effect on the leaf than on the acorn. The acorn, being compact, has a much smaller resistance relative to its weight exerted on it by the air than does the leaf. In fact, if we repeated this experiment on the Moon (where there is no air) or in a tube from which air has been removed, the leaf would not flutter and the two objects would fall at exactly the same rate. (This experiment has actually been done, both on the Moon and in a vacuum tube, with a feather and a rock. Both reached the ground at the same time.)

MOVEMENT IN TWO DIMENSIONS: THE LAW OF COMPOUND MOTION

Until now, we have talked only about motion along a straight line—called motion in one dimension. However, if you throw a baseball or ride in a car on a highway, you experience motion in more than one dimension. The baseball, for example, follows an arching path as it travels forward. The car goes around turns as well as moving ahead. In Galileo’s time, analyzing the motion of projectiles such as baseballs was particularly important because the cannon had just been introduced into warfare, and cannonballs, like baseballs, move in two-dimensional arcs.

The central question in analyzing two-dimensional motion is this: how does the motion in one direction affect motion in the other dimension? For example, when a baseball is thrown, does its speed in the horizontal direction affect how high it goes? Galileo proposed what we now call **the law of compound motion**. It states that:

Motion in one dimension has no effect on motion in another dimension.

Projectile Motion



Let’s look at an example of projectile motion to see how this law works. Suppose you stand on a high cliff and throw a rock outward, as shown in the graph of Figure 3-14. The law of compound motion tells us that we should think of the rock’s path as being made up of two separate parts or components. In the horizontal direction, the rock moves at a constant velocity imparted to it by your hand. In the vertical direction, the rock accelerates downward like any other falling object.

Suppose, for example, that the rock is moving 12 meters per second in the horizontal direction. At the end of 1 second, the rock is 12 meters from the cliff. At the same time, however, the rock is falling with an acceleration of 9.8 meters per second per second, so at the end of 1 second it is 4.9 meters down (remember, $\text{distance} = \frac{1}{2}at^2$, where in this case $a = g = 9.8 \text{ m/s}^2$). The rock’s position, then, is 12 meters out and 4.9 meters down, as shown in Figure 3-14. A second later, it is 24 meters out and 19.6 meters down, and so on. The path, illustrated in Figure 3-15, is called a *parabola*.

The law of compound motion also applies to projectiles thrown up from the ground, such as a baseball hit by a bat or a football thrown or kicked by a player. In this case, the vertical motion of the ball is an ever-slowing motion upward, followed by a fall like that in Figure 3-14 (if the effects of wind are negligible). The horizontal motion is, as in Figure 3-14, movement at a constant velocity. The combination is an arc, also in the shape of a parabola, that begins at the ground, goes up to the peak in a path that is the mirror image of that in Figure 3-14, and then goes back to the ground exactly as in Figure 3-14.

Physics in the Making The Range of a Cannonball

The distance a projectile travels before it comes back to the ground is called its range. In the fifteenth century, when cannons were first being introduced into the military affairs of Europe, finding the range of a cannonball was of more than

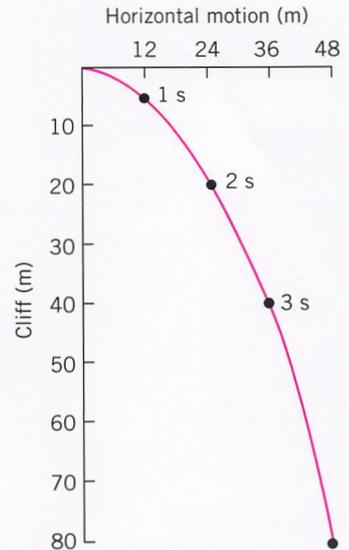


FIGURE 3-14. The trajectory of a rock thrown off a cliff. The motion in the horizontal direction is independent of the motion in the vertical direction.

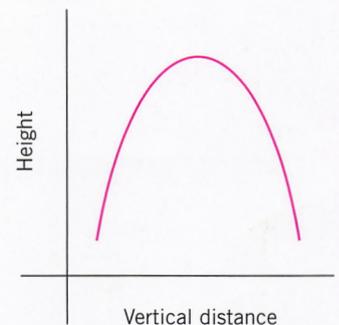


FIGURE 3-15. An object thrown upward at the Earth’s surface follows an arching path, called a parabola.



academic interest. The Duke of Milan, having acquired some cannons, wanted to know how to get the maximum range from his new purchases. He called in his chief engineer, a man named Tartaglia (The Stutterer), and told him to find out how to do it. What followed was one of the earliest episodes of the new experimental method in science.

The prevailing thought about projectiles at the time, following the teachings of Aristotle, was that motion could be divided into two classes: natural motion, in which an object followed its natural inclination to fall toward the center of the Earth, and violent motion, which was imposed by an outside agency such as gunpowder. Scholars believed that a cannonball would move off in a straight line until the violent motion was expended and then fall straight down. They, like Aristotle, arrived at this conclusion by simply thinking about the situation.

Tartaglia, on the other hand, took a different approach. He went out to a field outside Milan and shot off cannonballs, noting the distance the ball went for different elevations of the barrel and different gunpowder charges. He was the first person to realize that the maximum range occurs when the projectile leaves the ground at an angle of 45 degrees. (This is true in the absence of air resistance. The maximum range with air resistance requires a slightly smaller launch angle above the ground; for example, 43 degrees for a baseball, 38 degrees for a golf ball.) More important, however, Tartaglia showed that observation and experiment were an essential piece of the scientific method. ●

Motion in a Circle

The motion of an object moving in an arc or circle presents a more difficult challenge to analyze than a batted baseball or a rock thrown from a cliff. The basic problem is that when we look at the regular motion of an object moving in a circle at constant speed, our first impression may be that it is not accelerating. *But it is!*

If you have difficulty envisioning this acceleration, you are in good company. Many famous scientists, including Galileo, tried to work out the properties of uniform circular motion and failed. But recall from the definition of velocity that velocity involves *both speed and direction*. The ordinary kind of accelerated motion we discuss earlier, such as stepping on your car's gas pedal or brake, involves a change of speed without a change of direction. We have no difficulty recognizing this motion as acceleration. Uniform circular motion, on the other hand, involves a change of direction without a change of speed. This, too, changes the velocity, and therefore requires a compensating acceleration.

Consider a simple example illustrating this point. Imagine holding one end of a string that has a ball attached to the other end and whirling the ball around your head (Figure 3-16). You have to pull on the string all the time—you can feel the tension in your hand. If you let go or if the string breaks, the ball doesn't keep moving in a circle; it flies off in a straight line tangent to the circle. Just as your car won't pick up speed unless you keep your foot on the gas pedal, the ball won't move in a circle unless you keep pulling the string. In both cases, your actions produce an acceleration.

In fact, the acceleration of an object moving in a circle of radius r with a velocity whose magnitude (speed) is v turns out to be:

1. In words:

The acceleration of an object moving in a circle is equal to the square of its speed divided by the radius of the circle.

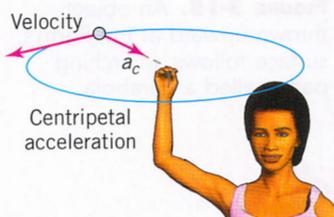
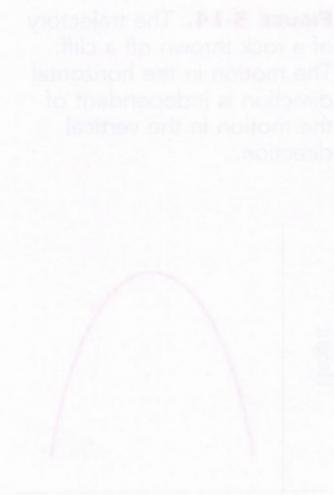


FIGURE 3-16. The acceleration of an object moving in a circle. The object moves at the same speed, and only the direction changes.

2. In an equation with words:

$$\text{Acceleration (centripetal)} = \frac{\text{Velocity}^2}{\text{Radius}}$$

3. In an equation with symbols:

$$a_c = \frac{v^2}{r}$$

Another interesting fact about this acceleration is that it is directed inward toward the center of the circle, perpendicular to the velocity. (To grasp this point, remember that when you whirl the ball around your head you are actually pulling inward on the string.) For this reason, it is called the *centripetal acceleration* (which means center-seeking acceleration). We encounter centripetal acceleration again in Chapter 5 when we discuss the orbits of satellites.

With Galileo's work, then, physicists began to isolate and observe the motion of material objects in nature and to summarize their results in mathematical relationships. However, why bodies behave this way remained undiscovered. The man who made this discovery was the great English mathematician and physicist Isaac Newton, who was born in 1642, the year of Galileo's death. We discuss Newton's extraordinary contributions in the next two chapters.

THINKING MORE ABOUT

Our Place in the Ordered Universe

Galileo's work and writings had great significance in human society beyond their central importance in science. To understand their importance, we need to recognize the role of Ptolemy's Earth-centered solar system and Aristotle's principles of physics within the context of the general culture and religious beliefs of the time.

The Catholic Church held the idea that humans were the supreme achievement of God's creation. In the eyes of religious leaders, each person's spiritual salvation was the primary concern of our life on Earth, far outweighing issues such as the best model of the solar system. Ptolemy's model placed our planet at the center of the universe, which fit with the Church's sense of the importance of humans in God's overall plan. The Copernican model made Earth just one of several planets orbiting the Sun, which undermined the idea of humans' being central to the rest of the universe. Church leaders feared that such a model

might confuse people and lead to doubt about the importance of individual salvation. This fear was not confined to the Catholic Church; both Martin Luther and John Calvin, who spearheaded the Protestant Reformation, condemned the Copernican model.

Church views about how the world worked were based on the explanations of Aristotle, which by the time of Galileo had been unchallenged for some 2000 years. Thus, when Galileo's experiments led to conclusions that differed from those of Aristotle, they also seemed to challenge the authority of the Church. Galileo reported that moons orbited the planet Jupiter, but the Church believed that everything in the heavens revolved around the Earth. Galileo noted dark spots on the Sun but, according to Church doctrine, God created the Sun as a perfect source of light, without blemish. Galileo showed that objects of different mass fell at the same rate, but religious philosophers, following Aristotle's teaching, said that heavier objects fell faster so as to reach the center of the Earth, and thus the center of the universe, more quickly.

Galileo did recant his discoveries, thus avoiding the fate of Giordano Bruno, who did not renounce his belief in the Copernican model and was burned at the stake in 1600. But the Church was fighting a losing battle. By the nineteenth century, not only was the Copernican model of the solar system considered as established fact, but it was beginning to be recognized that the Sun is

simply a star, not essentially different from other stars in the sky except for its proximity.

Galileo's troubles arose because his research challenged the prevailing mores of his day. Under what circumstances should society at large restrict the research activities of scientists? What research topics are considered immoral or illegal today? Who should decide these limits?

Summary

Since before recorded history, astronomers have observed regularities in the heavens and have built monuments such as Stonehenge to help establish order in their lives. Models, such as the Earth-centered system of Ptolemy and the Sun-centered system of Copernicus, attempted to explain these regular motions of stars and planets. Astronomers such as Tycho Brahe made ever more precise measurements of star and planet positions. These data led mathematician Johannes Kepler to propose his **laws of planetary motion**, which, among other things, state that planets orbit the Sun in elliptical orbits, not circular orbits as had been previously assumed.

Meanwhile, Galileo Galilei and other scientists investi-

gated the science of **mechanics**, which is the study of how objects move near the Earth's surface. These workers recognized two fundamentally different kinds of motion: **uniform motion**, which means constant **speed** and direction (**velocity**), and **acceleration**, which entails a change in either speed or direction of travel. Galileo devised experiments to study falling objects, and he discovered that all things fall with the constant rate of acceleration of 9.8 meters per second per second, which is called the **acceleration due to gravity (g)**. He also discovered the **law of compound motion**, which states that the motion in one dimension has no effect on motion in another dimension.

Key Terms

acceleration The change in velocity divided by the time it takes for that change to occur. Acceleration can involve changes of speed, changes in direction, or both. (p. 60)

acceleration due to gravity (g) The velocity change of a freely falling body at the Earth's surface. (p. 66)

Kepler's laws of planetary motion Three basic mathematical statements about the solar system: *Kepler's first law of planetary motion* states that the planets have elliptical orbits with one focus at the Sun; *Kepler's second law* says that for a given time interval, the swept-out area is the same, no matter where the planet is in its orbit; *Kepler's third law* expresses the relationship between a planet's distance from the Sun and its period as a simple equation that allows scientists to predict the behavior of orbiting objects. (p. 53)

law of compound motion Galileo's proposition that motion in one dimension has no effect on motion in another dimension. (p. 67)

mechanics The branch of physics that deals with motions of material objects. (p. 56)

speed The distance an object travels divided by the time that it takes to travel that distance. (p. 58)

uniform motion Motion at a constant speed in a single direction. (p. 61)

velocity A vector quantity that has the same numerical value as speed but also includes information about the direction of travel. (p. 60)

Key Equations

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\text{Acceleration} = \frac{\text{Final velocity} - \text{Initial velocity}}{\text{Time}}$$

For an object starting at rest and experiencing constant acceleration:

$$\text{Distance} = \frac{1}{2} \times \text{Acceleration} \times \text{Time}^2$$

$$\text{Velocity of falling object} = g \times \text{Time}$$

$$\text{Acceleration due to gravity} = g = 9.8 \text{ meters/second}^2$$

For an object in circular motion:

$$\text{Acceleration} = \frac{\text{Velocity}^2}{\text{Radius}}$$



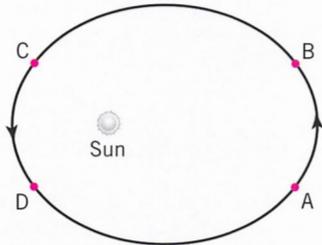
Review

- How did Stonehenge allow ancient people to make predictions?
- Why do scientists argue that ancient astronauts did not build Stonehenge?
- What are the characteristic movements of some of the objects you see in the night sky?
- Describe the main features of the Ptolemaic and Copernican systems of the universe. In what ways are they similar?
- What did Tycho Brahe try to do to resolve the question of the structure of the universe?
- What was Kepler's role in interpreting Tycho Brahe's data?
- What is Kepler's first law of planetary motion? What assumption of the Copernican system did this law refute?
- What is Kepler's second law of planetary motion? According to this law, at what point in its orbit does a planet move fastest?
- What is Kepler's third law of planetary motion? Given this law, what are the relative lengths of the year for Earth, Venus (next closest planet to the Sun), and Mars (next farthest away planet from the Sun)?
- How are the works of Tycho Brahe and Kepler an example of the scientific method?
- What is mechanics? Provide an example of an event that might be studied by this discipline.
- What new observations did Galileo make with his telescope? Why were these discoveries controversial?
- Define speed. How can you calculate speed from a known distance traveled and time of travel?
- What is the difference between speed and velocity?
- Define acceleration. How can you tell if your car is accelerating by looking at the speedometer?
- What is instantaneous velocity?
- What two quantities did Galileo have to measure in his rolling ball experiment? How might you improve on his experiment using modern technology?
- How did Galileo slow down the rate at which objects fall in the laboratory?
- How did Galileo measure time?
- Why is the measurement of time important in mechanics?
- How are Galileo's experiments in mechanics examples of the scientific method?
- What is g ?

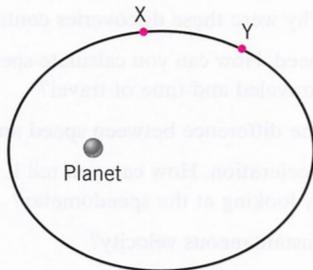
Questions

- The asteroid belt is a large collection of rocks and boulders that lies about three times as far from the Sun as the Earth does. How long is the orbital period, or year, for one of these rocks?
- Which of the following objects are in uniform motion and which are in accelerated motion? Explain each response.
 - A car heading north at 35 mph
 - A car going around a curve at 50 mph
 - A dolphin leaping out of the water
 - An airplane cruising at 30,000 feet at 500 mph
 - A book resting on your desk
 - The Moon
- People who put oversized wheels on their cars often find that their actual speed is significantly greater than the speedometer indicates. Can you explain why this is so?

4. A planet orbits the Sun in an elliptical orbit (see figure). The distance along the orbit from A to B is the same as the distance from C to D. Compare the time it takes the planet to move from A to B to the time it takes the planet to move from C to D. Explain your reasoning.

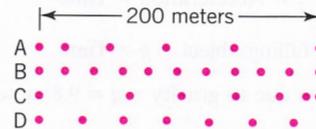


5. Suppose two different moons, X and Y, follow the same elliptical orbit around a planet. Which moon is moving faster according to the figure? Will the faster moon ever catch up to the slower one? Explain.

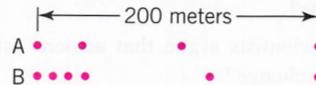


6. By extending the logic used to define instantaneous velocity, define instantaneous acceleration.
7. Consider a comet with a very elongated, elliptical orbit around the Sun. Using Kepler's three laws of motion, describe the speed of the comet as it orbits the Sun.
8. Astronomers investigating other solar systems have found many systems in which the planets have very elongated, elliptical orbits, rather than almost circular orbits as in our own system.
- Does the existence of highly elliptical orbits for planets violate Kepler's laws? Why or why not?
 - What effect do you think a highly elliptical orbit might have on the chances of the planet developing life? (*Hint:* What would happen to Earth's oceans if the planet spent time far away from the Sun?)
9. As you drive north on the highway at 65 miles per hour, the cars in the opposing lane are traveling south at 65 miles per hour. Do the cars in the opposing lane have the same speed as you do? Do they have the same velocity? Explain.
10. Unfortunately, your car has developed an oil leak. One drop of oil falls from your engine every 3 seconds, leaving a trail of oil drops on the road. In the figure are four patterns of oil drops you've left over the same 200-meter stretch of road. For which one(s) is your car accelerating?

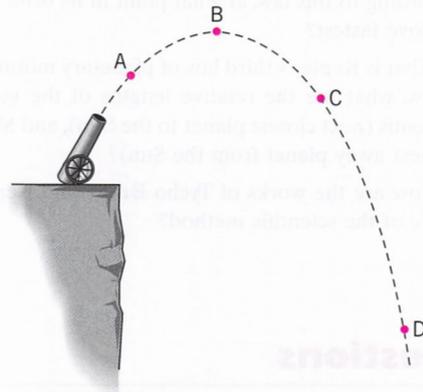
For which one(s) is your car moving at a constant speed? For which one is your average speed the greatest?



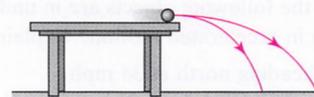
11. Unfortunately, your car has developed an oil leak. One drop of oil falls from your engine every 3 seconds, leaving a trail of oil drops on the road. In the figure are two patterns of oil drops you've left over the same 200-meter stretch of road. In which case do you achieve the highest instantaneous speed? In which case do you have the highest average speed? In which case do you achieve the greatest instantaneous acceleration (acceleration at a specific point)?



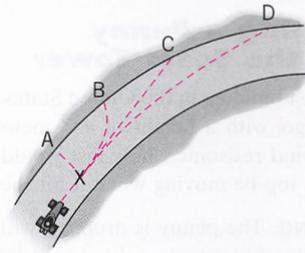
12. Is it possible for your speed to be zero when your acceleration is not zero? Explain.
13. A bowling ball and a volleyball are dropped at the same time from the top of a tall building. Neglecting air drag, which one will hit the ground first? Would your answer change if we did not neglect air resistance? How?
14. A cannon fires a shot from a high cliff as shown in the figure. Where in the cannonball's trajectory is its acceleration the greatest, A, B, C, or D? Where is its speed the greatest? Where is its speed 0? Is its acceleration ever 0? If so, where?



15. You roll a ball off a horizontal tabletop. In which case will the ball take longer to hit the floor: if it is moving fast or if it is moving slowly? Explain.



16. Two balls roll off a horizontal tabletop. One is moving fast and one is moving slowly. Which one hits the ground with a higher speed? Explain.
17. A race car driver is driving on a circular track. If he doubles his speed, how much greater will his centripetal acceleration be? What if he triples his speed?
18. A car is rounding a corner on an cold winter day. If the road suddenly turns to ice (at the X in the figure), where will the car run off the road?



Problem-Solving Examples

EXAMPLE
3-3

Out of the Blocks

A sprinter accelerates from the starting blocks to a speed of 11 meters per second in 1.5 seconds. Answer the following questions about the sprinter's speed, acceleration, time, and distance run. In each case, answer the question by substituting into the appropriate motion equation.

1. What is his acceleration?

SOLUTION: Acceleration is defined as the change in velocity divided by the time interval of that change.

$$\text{Acceleration} = \frac{\text{Final velocity} - \text{Initial velocity}}{\text{Time}}$$

In this case, the sprinter starts from rest at the beginning of the race, so his initial velocity is 0.

$$\begin{aligned} \text{Acceleration} &= \frac{11 \text{ meters/second}}{1.5 \text{ seconds}} \\ &= 7.3 \text{ m/s}^2 \end{aligned}$$

2. How far does the sprinter travel during this acceleration?

SOLUTION: We saw from Galileo's experiments that distance traveled is proportional to the square of the time interval.

$$\begin{aligned} \text{Distance} &= \frac{1}{2} \times \text{Acceleration} \times \text{Time}^2 \\ &= \frac{1}{2} \times (7.3 \text{ m/s}^2) \times (1.5 \text{ s})^2 \\ &= 8.2 \text{ meters} \end{aligned}$$

3. How fast is the sprinter going when he's halfway through the period of acceleration?

SOLUTION: This question asks for the runner's instantaneous velocity. We can find it from the values of the acceleration (7.3 m/s^2) and the time interval (0.75 s).

$$\text{Instantaneous velocity} = \text{Acceleration} \times \text{Time}$$

At $\frac{3}{4}$ second, his instantaneous velocity is

$$\begin{aligned} \text{Velocity} &= 7.3 \text{ m/s}^2 \times 0.75 \text{ s} \\ &= 5.5 \text{ m/s} \end{aligned}$$

4. How far has the sprinter traveled in the first 0.75 second?



A sprinter gets a fast start by pushing off from angled blocks.

SOLUTION: This question is the same as part 2 but over a different time interval.

$$\begin{aligned} \text{Distance} &= \frac{1}{2} \times \text{Acceleration} \times \text{Time}^2 \\ &= \frac{1}{2} \times (7.3 \text{ m/s}^2) \times (0.75 \text{ s})^2 \\ &= \frac{1}{2} \times (7.3 \text{ m/s}^2) \times 0.56 \text{ s}^2 \\ &= 2.05 \text{ m} \end{aligned}$$

Notice that halfway through the 1.5-second period of acceleration the sprinter has not covered half of the 8.2 meters (see part 2). This feature is common to accelerated motion. The sprinter moves much faster and farther during the second half of the period of acceleration and therefore covers more ground.

5. Assuming the sprinter covers the remaining 91.8 meters at a constant speed of 11 m/s, what will be his time for the event?

SOLUTION: We have already calculated that the time to cover the first 8.2 meters is 1.5 seconds. The time required to cover the remaining 91.8 meters at a constant velocity of 11 meters per second is

$$\begin{aligned} \text{Time} &= \frac{\text{Distance}}{\text{Velocity}} \\ &= \frac{91.8 \text{ m}}{11 \text{ m/s}} \\ &= 8.35 \text{ s} \end{aligned}$$

Thus, Total time = $1.5 + 8.35 = 9.85$ seconds

For reference, the world record for the 100-meter dash, set by Tim Montgomery of the United States in 2002, is 9.78 seconds. ●

EXAMPLE
3-4**Dropping a Penny from the Sears Tower**

The tallest building in the United States is the Sears Tower in Chicago, with a height of 443 meters (1454 feet). Ignoring wind resistance, how fast would a penny dropped from the top be moving when it hit the ground?

REASONING: The penny is dropped with 0 initial velocity. We first need to calculate the time it takes to fall 443 meters. From this time we can calculate the velocity at impact.

SOLUTION:

Step 1. Time of fall. The distance traveled by an accelerating object is:

$$\begin{aligned}\text{Distance} &= \frac{1}{2} \times \text{Acceleration} \times \text{Time}^2 \\ &= \frac{1}{2} \times 9.8 \text{ m/s}^2 \times t^2 \\ &= 4.9 \text{ m/s}^2 \times t^2\end{aligned}$$

The given distance of the fall equals 443 meters, so rearranging gives:

$$\begin{aligned}t^2 &= \frac{443 \text{ m}}{4.9 \text{ m/s}^2} \\ &= 90.41 \text{ s}^2\end{aligned}$$

Taking the square root of both sides gives the time of fall:

$$t = 9.5 \text{ s}$$

Step 2. Velocity at impact. The velocity of an accelerating object is:

$$\begin{aligned}\text{Velocity} &= \text{Acceleration} \times \text{Time} \\ &= 9.8 \text{ m/s}^2 \times 9.5 \text{ s} \\ &= 93.1 \text{ m/s}\end{aligned}$$

This velocity is about 200 miles per hour—a high speed indeed. A penny traveling at such a velocity could easily kill a person, so *don't* try this experiment!

In fact, most objects dropped in air do not accelerate indefinitely. Because of air resistance, an object accelerates only until it reaches its *terminal velocity*; and it continues falling at a constant velocity after that point. The terminal velocity for a penny is somewhat less than 200 miles an hour, still fast enough to cause serious injury. We return to the topic of terminal velocity in Chapter 4, after we have studied more about force and motion. ●

EXAMPLE
3-5**Throwing a Ball Straight Up**

Suppose you throw a ball straight up into the air with an initial speed of 25 meters per second (about 55 miles per hour).

1. How high will it go?
2. How long will it take to return to the ground?

REASONING AND SOLUTION:

1. The motion of the ball is the result of two effects: the velocity, pointing up, and the acceleration, pointing down. When the ball is thrown straight up, it decelerates because of the effects of gravity. It moves more and more slowly as it climbs, until finally it stops and starts to fall back down. We can use the equation for the velocity of an accelerating body to tell us how long it takes for the velocity to be reduced to 0. Then we can use the equation for the distance traveled by a decelerating body to tell us how far it has traveled (and hence how high it will go).

The velocity of the ball as it moves up is:

Final velocity = Initial velocity + Acceleration \times Time,

At the top of the throw, $v = 0$, so

$$0 = (25 \text{ m/s}) - (9.8 \text{ m/s}^2) \times t$$

and the time it takes for the ball to stop moving up is

$$t = \frac{25 \text{ m/s}}{9.8 \text{ m/s}^2} = 2.55 \text{ s}$$

Then, to calculate the distance traveled by the ball in 2.55 seconds, apply the distance equation, making sure to include the effects of both the velocity pointing up and the acceleration pointing down:

$$\begin{aligned}d &= [v_0 \times t] - \left[\frac{1}{2} g \times t^2\right] \\ &= [(25 \text{ m/s}) \times 2.55 \text{ s}] - \left[\frac{1}{2} (9.8 \text{ m/s}^2) \times (2.55 \text{ s})^2\right] \\ &= 63.75 \text{ m} - 31.86 \text{ m} \\ &= 31.89 \text{ m}\end{aligned}$$

2. There are two ways to determine how long it takes for the ball to return to the ground. One simple way is to note that it takes the ball just as long to fall as it did to climb up, so that the total time of flight is 2×2.55 seconds, or 5.10 seconds.

The other way is to note that the problem of tracing the ball's path once it starts down is exactly the same as dropping a ball from rest from a height of 31.89 meters. If we work out how long it takes for the ball to fall and add it to the 2.55 seconds it took to get to the top of the throw, we'll have the total time the ball was in the air.

The time it takes a ball to fall 31.89 meters is

$$\begin{aligned}d &= \frac{1}{2} g \times t^2 \\ 31.89 \text{ m} &= \frac{1}{2} \times 9.8 \text{ m/s}^2 \times t^2 \\ t^2 &= \frac{31.89 \times 2}{9.8} = 6.51 \text{ s}^2 \\ t &= 2.55 \text{ s}\end{aligned}$$

Thus, the total time the ball is in the air is $2.55 + 2.55 = 5.10$ seconds.

Note in this example how we handled the calculation of a distance due to an upward velocity and a

downward acceleration. Both factors affect how high the ball goes; the distance traveled is the sum of the distances due to each factor separately. ●

EXAMPLE
3-6

The Human Cannonball

One of the attractions you may see at the circus is the human cannonball (Figure 3-17). This person is lowered into the barrel of a giant cannon, only to be shot out, travel through the air, and finally land in a safety net to the sound of applause from the audience. Suppose he emerges from the cannon's mouth with a vertical velocity of 20 m/s and a horizontal velocity of 8 m/s. How far away would you have to place the net to make sure he landed safely?

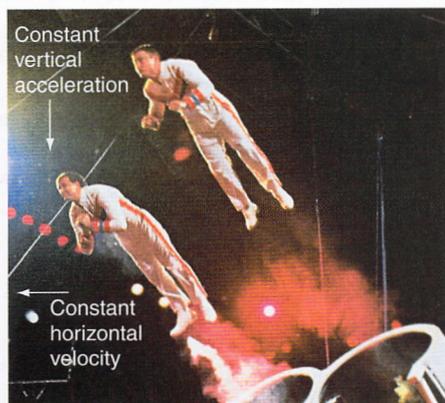


FIGURE 3-17. The motion of a human cannonball illustrates the law of compound motion. The accelerated up-and-down motion in the vertical direction is completely independent of the uniform motion in the horizontal direction.

REASONING: The way to approach this problem is to remember that the horizontal and vertical motions are independent of each other. In the vertical direction we have a problem just like Example 3-5, in which an object is thrown upward but there is a constant downward acceleration equal to g . In this direction, the object slows down as it moves up, then stops and falls back down. The time for the up-and-down trip is twice the time it takes to get from the ground to the top of the arc. Then we can use

the total time of up-and-down travel, multiplied by 8 m/s, to tell us the distance the object travels horizontally, and hence where to place the net.

SOLUTION:

Step 1. How long will it take to get to the top of the arc?

The velocity in the vertical direction is:

Final velocity = Initial velocity + Acceleration \times Time
where initial velocity is 20 m/s and final velocity at the top of the arc is 0. We get

$$0 = 20 \text{ m/s} + (-9.8 \text{ m/s}^2 \times t)$$

where all values are in SI units. Note that the acceleration due to gravity has a negative sign because the acceleration is downward. Rearranging this equation, we can solve for time, t :

$$\begin{aligned} 9.8 \text{ m/s}^2 \times t &= 20 \text{ m/s} \\ t &= \frac{20 \text{ m/s}}{9.8 \text{ m/s}^2} \\ &= 2.04 \text{ s} \end{aligned}$$

In other words, the human cannonball takes about 2 seconds to get to the top of the arc and two more seconds to come down, for a total flight time of about 4 seconds.

Step 2. Where should you place the net?

In the horizontal direction, the human cannonball is travelling at a steady 8 m/s. In four seconds the horizontal distance traveled is:

$$\begin{aligned} \text{Distance} &= \text{Velocity} \times \text{Time} \\ &= 8 \text{ m/s} \times 4.08 \text{ s} \\ &= 32.6 \text{ m} \end{aligned}$$

The net, then, should be placed 32.6 meters (about 106 feet) from the mouth of the cannon.

Note that in the absence of air resistance, the vertical motion is the same for the ball in Example 3-5 and the human in Example 3-6. This equivalence is part of the great strength and beauty of physics: the same principles apply to what may at first seem to be very different situations. ●

Problems

1. If a race car completes a 3-mile oval track in 58 seconds, what is its average speed? Did the car accelerate during the 58 seconds?
2. If your car goes from 0 to 60 miles per hour in 6 seconds, what is your average acceleration?
3. The hare and the tortoise are at the starting line together. When the gun goes off, the hare moves off at a constant speed of 10 meters per second. (Ignore the acceleration required to get the animal to this speed.) The tortoise starts more slowly, but accelerates at the rate of 2 meters per sec-

ond per second. Make a table showing the positions of the two racers after 1 second, 2 seconds, 3 seconds, and so forth. How long will it be before the tortoise passes the hare?

- Someone in a car going past you at the speed of 20 meters per second drops a small rock from a height of 2 meters. How far from the point of the drop will the rock hit the ground? (*Hint:* Find how long it will take the rock to fall and then apply the law of compound motion.)
- The Statue of Liberty weighs nearly 205 metric tons. If a person can pull an average of 100 kg, how many people would it take to move the Statue of Liberty?
- The weight of the space shuttle is about 4.5 million pounds. How many people would it take to move it? (See problem 5.)
- The eccentricity of an ellipse is a measure of how elongated, or oval, it is. It is defined for a planet's orbit as the distance between the two foci divided by twice the average distance to the sun, which resides at one of the foci (see Figure 3-18). A perfect circle has an eccentricity of zero since the two foci are in the same position.

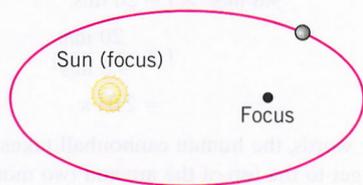


FIGURE 3-18. The eccentricity of a planetary orbit is defined as the ratio of the distance between the foci and twice the average distance to the Sun.

- Calculate the eccentricities for the following solar system objects. All data are in terms of the average distance of the Earth from the Sun, called the astronomical unit (AU).
- | Object | $f_1 f_2$ (AU) | Average distance |
|----------------|----------------|------------------|
| Earth | 0.017 | 1.0 |
| Mars | 0.14 | 1.52 |
| Pluto | 9.8 | 39.5 |
| Halley's comet | 17.4 | 17.9 |
- Which object has the most nearly circular orbit? Which object has the most elliptical orbit?
- For the planets and comet in the list in problem 7, calculate the orbital periods using Kepler's third law of planetary motion.
 - Consider the orbit of a typical comet around the sun given in Figure 3-19, which is marked at five different positions, A, B, C, D, and E. Using Kepler's second law of planetary

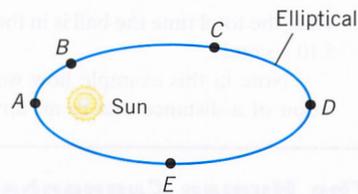


FIGURE 3-19. The elliptical orbit of a hypothetical comet around the Sun is shown with five positions along the orbit.

motion, rank those positions in order of their relative speeds, with the position for the fastest speed first.

- Imagine that a new asteroid is discovered in the solar system with a circular orbit and an orbital period of 8 years.
 - What is the average distance of this object from the Sun in Earth units?
 - Between which planets would this new asteroid be located?
- The four Galilean moons of Jupiter are Io, Europa, Ganymede, and Callisto. Their average distances from Jupiter and orbital periods are listed below in terms of Io's values.

Moon	Relative average distance	Relative orbital period
Io	1.00	1.00
Europa	1.59	2.00
Ganymede	2.54	4.05
Callisto	4.46	9.42

- Plot the square of the relative orbital period versus the cube of the relative average distance for each moon. In words, state the pattern you find in your graph.
 - From this information, do you agree or disagree that Kepler's third law (as applied to the moons of Jupiter) holds for Jupiter's four Galilean moons? Explain.
- An average person can walk 1 kilometer in 10 minutes.
 - What is the speed in miles per hour? In kilometers per hour?
 - How long would it take an average person to walk 3.5 miles? To walk 10 km?
 - How far can an average person walk in 45 minutes? In 1.5 hours?
 - The North American continental plate is moving away from the European continental plate at a constant speed of 4.2 cm per year.
 - If the average distance between the two plates is 7000 km and the two plates maintained their constant speeds, how long ago were the two continental plates together?
 - In 1 million years (10^6 years), how large will the separation be between the two plates?

14. A typical motorist in the United States travels 25,000 miles in his or her car every year. If you assume that the average speed of the car while traveling is 45 miles per hour (remember that the car is not always moving), calculate the total number of hours an average motorist spends in his or her car. How many hours per day is this? Do you think that the average speed is a reasonable estimate? Explain.
15. The typical airborne speed of an intercontinental B747 jet is 530 miles per hour, while the airborne speed of the supersonic Concorde is 1500 miles per hour. If each airliner were to circumnavigate the Earth (25,000 miles), what would be the difference in air time spent by the two aircraft?
16. It takes light (speed = 3.0×10^8 m/s) 8.33 minutes to travel from the Sun to the Earth and 1.3 seconds from the Moon to the Earth. What is the Sun's average distance from the Earth? The Moon's?
17. While traveling out in the country at 50 miles per hour, your car's engine (and brakes) stops working and you coast to a stop in 25 seconds. What was your average acceleration during the time after the motor shut off?
18. Starting from rest, a train reaches a final, constant speed in 35 seconds while accelerating at a constant rate of 3 km/hour/s.
- What is the final speed of the train?
 - What is the total distance traveled by the train during this period of constant acceleration? (Be careful here with your units.)
19. A rock falls to the bottom of a tall canyon, falling freely with no air resistance, for 4.5 seconds. Make a table of the distance traveled by the rock and its velocity after 1.0, 2.0, 3.0, 3.5, and 4.5 seconds.
20. In a safety test, a car traveling at 65 miles per hour crashes directly into a wall, coming to a complete stop. The time of contact for the crash was 0.25 s. What is the deceleration of the car in terms of the acceleration of gravity (i.e., the number of g's)?
21. A baseball is hit off the edge of a cliff horizontally at a speed of 30 m/s. It takes the ball 3 seconds to reach the ground, with no air resistance.
- How far from the cliff wall does the ball land?
 - How high is the cliff wall?
22. Sammy Sosa pops up a baseball directly over the batter's box. It takes the ball 5.0 seconds to reach the waiting glove of the catcher.
- What is the instantaneous speed of the ball at the top of the ball's path?
 - What is the instantaneous speed of the ball immediately after it was in contact with the bat?
 - How far above the ground did the ball travel? (Assume that the ball was caught at the same height that it was hit.)
23. Two balls are released simultaneously from the same height, 10 meters above the ground. The first ball is released at rest and the second ball is released with a horizontal velocity of 15 m/s. Which ball reaches the ground first? Why?
24. A girl grabs a bucket of water and swings it around her in a horizontal circle, at a constant speed of 2 m/s at an arm's length of 0.7 meters. What is the centripetal acceleration of the bucket of water?
25. The space shuttle orbits the Earth in a near-circular orbit at a constant speed approximately 100 miles above the Earth's surface. If we assume that the centripetal acceleration is equal to the acceleration due to gravity at sea level (9.8 m/s^2) and the orbital radius is equal to the radius of the Earth (6380 km):
- What is the average speed of the space shuttle?
 - How long does the space shuttle take to make one orbit around the Earth?
26. The Moon moves around the Earth in a near-circular orbit of radius 3.84×10^8 m in 27.3 days. What is the centripetal acceleration of the Moon in m/s^2 ?
27. Some people who study the history of life on Earth have suggested that every 26 million years a hitherto unknown companion star to the sun comes near the solar system, sending a storm of comets into the inner solar system. (In this scheme, the dinosaurs were wiped out when one of these comets hit the Earth 65 million years ago). From Kepler's laws, what would the axis of the elliptical orbit of this companion, named Nemesis, have to be for its period to be 26 million years? Compare that axis to the distance to the nearest star.
28. Scientists who study the Earth have found that Europe and North America are separating from each other at the rate of about 5 centimeters per year. Assuming this rate has been constant throughout history, estimate the age of the Atlantic Ocean. (*Hint:* How wide is the ocean now?) Compare this number to the age of the Earth.

Investigations

- Read the Bertold Brecht play *Galileo*, which dramatizes Galileo Galilei's heresy trial. Discuss the dilemma faced by scientists whose discoveries offend conventional ideas. What areas of scientific research does today's society find offensive or immoral? Why?

- What other kinds of models of the universe did old civilizations develop? Look up those of the Mayans, the Chinese, and the Indians of the American Southwest, and describe some of their models. What features do these models have in common?
- Find out how Galileo came to the idea of the pendulum clock. What did he actually observe that led him to this development?
- Drop a wadded-up sheet of paper and a flat one side by side. Which reaches the ground first? Why? What do you think would happen if this experiment were done in a vacuum?
- When you are in a car traveling at a constant speed, throw a ball up and describe its motion as you see it. Is there a difference when the car is being accelerated?
- Drop a helium-filled balloon. Does it fall with acceleration g ? Why? What do you think would happen if you dropped the balloon in a vacuum?
- Investigate different technologies that scientists use to measure time. What is the shortest time interval that can be measured and how is such a measurement accomplished? What sorts of experiments require this kind of measurement?



WWW Resources

See the *Physics Matters* home page at www.wiley.com/college/trefil for valuable web links.

- www.mcm.acu.edu/academic/galileo/ars/arshtml/mathofmotion1.html A website based on a video series, *The Art of Renaissance Science*, which includes a discussions of Galileo's contributions to the mathematics of motion, to science, and to art via the development of painting perspective.
- galileo.imss.firenze.it/museo/b/egalilg.html The Galileo room of the History of Science Museum in Florence, Italy. The museum contains originals and models of apparatus described in this chapter, and Galileo's preserved right middle finger.
- observe.arc.nasa.gov/nasa/education/reference/orbits/orbits.html A partially animated NASA tutorial on satellite motion and Kepler's laws.
- liftoff.msfc.nasa.gov/toc.asp?s=Satellites Contains tutorials on types of satellites (including a section on geosynchronous satellites) and an extensive section on tracking current Earth-orbiting spacecraft live via the web.
- www.fourmilab.ch/solar/solar.html An online solar system orrery showing positions of the solar planets and a few comets.
- jersey.uoregon.edu/vlab/Cannon/index.html The cannon Java applet simulates projectile motion, allowing control of launch conditions.
- www.mcm.acu.edu/academic/galileo/ars/arshtml/mathofmotion1.html A website based on a video series, *The Art of Renaissance Science*, which includes a discussion of Galileo's contributions to the mathematics of motion, to science and to art via the development of painting perspective.