5

The Law of Universal Gravitation

KEY IDEA

Gravity is a universal attractive force that acts between any two masses.

PHYSICS AROUND US . . . In an English Garden

One of life's most familiar phenomena is gravity. If you drop a book, it falls. If you slip on the ice, you fall. Acorns fall from trees and rain falls from the sky. But have you ever looked up at the full moon on a clear night and wondered why it didn't fall? Almost 350 years ago the young Isaac Newton did just that.

According to Isaac Newton, it happened this way: he was walking in an apple orchard one fall day when he noticed an apple falling from a tree. From the first law of motion he knew that the change in the state of motion of the apple (from stationary to falling) had to have occurred because of the action of a force—a force known as gravity.

At the same time as he saw the apple fall, he noticed the Moon in the sky behind the tree. He knew that the Moon went around the Earth in a roughly circular orbit and if the laws of motion applied to the Moon, then the action of a force—some kind of force—was required to keep the Moon from flying off into space in a straight line. In a sense, Newton saw that the Moon is falling, just like the apple, but is also moving forward at the same time.

At this point, Newton asked a simple but very profound question: was it possible that the same force that caused the apple to fall also kept the Moon in its orbit? From this question came our modern understanding of the working of the solar system.

THE UNIVERSAL FORCE OF GRAVITY

Newton's three laws of motion don't say anything about the nature of the forces that act in the universe. Discovering the nature of those forces is a separate problem from understanding their effects. Much of the progress of science during and after Newton's time has been associated with the discovery and description of the forces that act in the world around us. Newton's second great contribution to science was to elucidate the nature of one of these forces.

Gravity is an attractive force that acts between any two objects in the universe. It is the most familiar (and insistent) force in our daily lives. It holds you down in your chair and keeps you from floating off into space. It guarantees that when we drop a ball or a book or a glass, they fall. The ancients knew the effects of what we call gravity, and Galileo and many of his contemporaries studied its quantitative properties. We discuss some of that work in connection with falling bodies in Chapter 3. It was Isaac Newton, however, who revealed the true universal nature of gravity.

Everyone knew that gravity pulled objects toward the Earth, but until Newton, most people assumed that gravity was local and operated only near the planet's surface. People believed that farther out, in the realm of the stars and planets, different rules applied to the turning of the celestial spheres. They would say that terrestrial gravity operated on the Earth and celestial gravity operated in the heavens, but that the two forces had little to do with each other. In their minds, there was no connection between a planet in orbit around the Sun and an apple falling toward the ground. Isaac Newton discovered that these two seemingly different kinds of gravity were, in fact, one and the same. In modern language, in a remarkable union of seemingly disparate elements, he unified earthly and heavenly gravity. This was another important step in simplifying our understanding of the universe; we can now explain a wide range of observations by applying one law instead of two.

Let's look at this problem in more detail (Figure 5-1). As we point out in Physics Around Us, the fact that the Moon moves more or less in a circle implies that some sort of force must be acting on it to keep it in orbit. The question that Newton asked had to do with the exact nature of that force. He knew that there was one force acting in the orchard—the force that caused the apple to accelerate as it fell to Earth. Newton's insight was that the force holding the Moon in its orbit could be the same as the force that made the apple fall—the familiar force of gravity. This force not only pulls the apple down, but it also extends out to the orbit of the Moon and keeps it from flying off in a straight line.

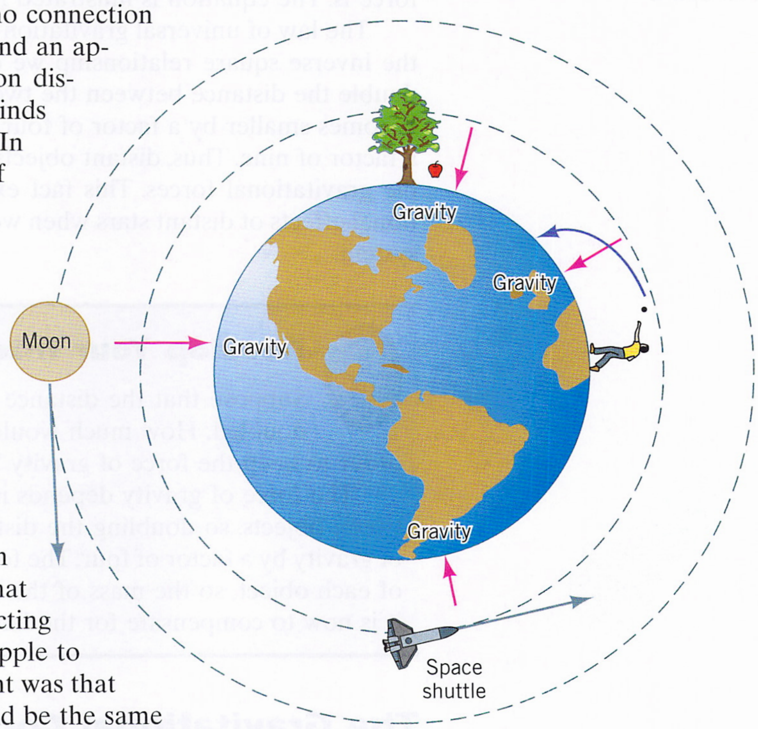


FIGURE 5-1. An apple falling, a ball being thrown, the space shuttle orbiting the Earth, and the orbiting Moon all display the influence of the force of gravity.

Eventually, Newton realized that the orbits of all the planets could be understood if gravity is not restricted to the surface of the Earth but is a force found throughout the universe. He formulated this insight (an insight that has been overwhelmingly confirmed by observations) in what is called **Newton's law of universal gravitation.**

1. In words:

Between any two objects in the universe there is an attractive force (gravity) that is proportional to the masses of the objects and inversely proportional to the square of the distance between them.

2. In an equation with words:

$$\text{Force of gravity} = \text{Constant} \times \frac{\text{First mass} \times \text{Second mass}}{\text{The square of the Distance between the masses}}$$

3. In an equation with symbols:

$$F = G \times \frac{m_1 \times m_2}{d^2}$$

where G is a number known as the **gravitational constant** (see next section).

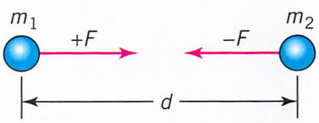


FIGURE 5-2. Two masses separated by a distance. The force of gravity falls off rapidly as the objects get farther apart.

This law tells us that the more massive two objects are, the greater the gravitational force between them. If one of the masses doubles, then the force of gravity doubles. However, the greater the distance between them, the smaller the force is. The equation is illustrated in Figure 5-2.

The law of universal gravitation is the first example we have encountered of the inverse square relationship we discuss in Chapter 2. This means that if we double the distance between the two objects, the force of gravity between them becomes smaller by a factor of four. Triple that distance and the force drops by a factor of nine. Thus, distant objects have to be very massive to exert appreciable gravitational forces. This fact explains why we never consider the gravitational effects of distant stars when we talk about the orbits of planets in our solar system.

Develop Your Intuition: Moving the Sun

Suppose that the distance between the Earth and the Sun suddenly doubled. How much would the mass of the Sun have to increase in order to keep the force of gravity between the two the same?

The force of gravity depends inversely on the square of the distance between objects, so doubling the distance would cause a decrease in the force of gravity by a factor of four. The force of gravity depends directly on the mass of each object, so the mass of the Sun would have to be four times as big as it is now to compensate for the larger distance.

The Gravitational Constant

The equation for gravitational force incorporates the gravitational constant—the number, G, which is a constant of proportionality (see Chapter 2). In the equation for gravity, G expresses the exact numerical relation between the masses of two objects and the distance between them, on the one hand, and the gravitational force between them on the other. Henry Cavendish, a physicist at Oxford

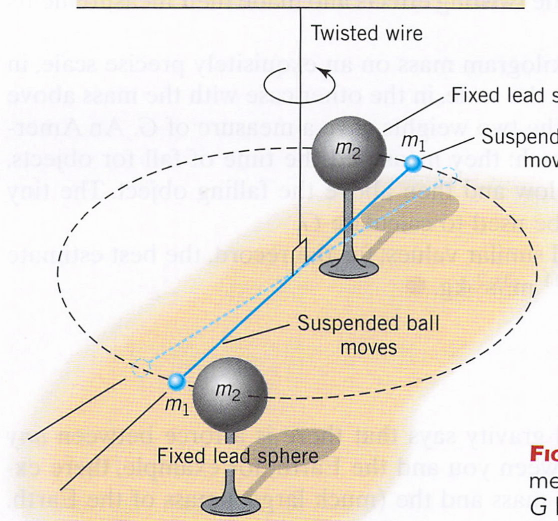


FIGURE 5-3. The Cavendish balance measures the gravitational constant G by balancing the gravitational force against the force exerted by a twisted wire.

University in England, first measured G in 1798 by using the apparatus shown in Figure 5-3. Cavendish suspended from a wire a dumbbell made of two small balls. Then he brought two large lead spheres near the balls. The resulting gravitational attraction between the small balls and the lead spheres turned the dumbbell until the twisted wire exerted a counterbalancing force strong enough to stop the rotation. By measuring the amount of twisting force (also known as a torque) on the wire, Cavendish measured the force on the dumbbells. This measured force, together with the known masses of the dumbbells, the masses of the heavy spheres, and knowledge of the distance between them, gave him the numerical value of everything in Newton's law of universal gravitation except G, which he then calculated using simple arithmetic. In metric units, the value of G is 6.674×10^{-11} m^3/s^2-kg. Most physicists think that this constant is universal, having the same value everywhere and at all times in our universe.

Ongoing Process of Science

The Value of *G*

It might surprise you to learn that the measurement of G is still of great interest to scientists around the world. It turns out that most fundamental physical constants, such as the mass of an electron or the electrical force between two charged particles, are known with great precision and accuracy, to many decimal places. But G is still only known, at best, to the fourth decimal place.

In November 1998 a group of 45 physicists met in London to celebrate the 200th anniversary of Cavendish's original experiment and to compare notes on several new determinations of the constant. All of these workers performed meticulous experiments: they enclosed their apparatus in a vacuum, they eliminated all stray magnetic and electrical fields, they checked and rechecked every aspect of their work.

Rival groups in France, Russia, and the United States used modern versions of the original Cavendish experiment, with weights suspended on a long wire or metal strips. They looked for subtle twisting effects and made their measurements over and over again.

A Zurich group weighed a kilogram mass on an exquisitely precise scale, in one case with a large mass below the scale, in the other case with the mass above the scale. The tiny difference in the two weights gave a measure of *G*. An American group tried a similar approach: they measured the time of fall for objects, with heavy weights held first below and then above the falling object. The tiny difference in falling time could be used to calculate *G*.

All of these techniques yield similar values; for the record, the best estimate for *G* is now about 6.674×10^{-11} m^3/s^2-kg. ●

Weight and Mass

Recall that the law of universal gravity says that there is a force between *any* two objects in the universe. Between you and the Earth, for example, there exists a force proportional to your mass and the (much larger) mass of the Earth. The distance between you and the center of the Earth is the radius of the Earth. This distance, which we denote by R_E, is about 6400 km (4000 miles); this is the number you would put in for the distance if you were calculating the force with which you are attracted to the Earth.

The gravitational attraction between you and the Earth would accelerate you downward if you weren't standing on the ground. As it is, the ground exerts an equal and opposite force to cancel gravity, a force you can feel in the soles of your feet.

An ordinary bathroom scale makes use of this interplay of forces. Inside the scale is a spring or some other mechanism that, when compressed, exerts an upward force. This upward force exerted by the scale keeps you from falling. The size of this counterbalancing force registers on a display, allowing you to measure your weight.

Weight, in fact, is just the force of gravity on an object. Your weight depends on where you are: on the surface of the Earth you have one weight, on the surface of the Moon another, and in the depths of interstellar space you would weigh next to nothing. Your weight is related to your mass, which is the amount of matter in your body (see Chapter 4). However, weight is different from mass. In interstellar space your weight would be zero, but your mass would not change.

Develop Your Intuition: Weight on a Mountaintop

Do you weigh the same at sea level as you do on a mountaintop?

The law of universal gravitation says that the farther apart two objects are, the smaller is the gravitational attraction between them. On the mountaintop, you are farther from the center of the Earth than you are at sea level, so you would actually weigh slightly less at a higher altitude. (In Problem 5 you get a chance to work out exactly what your weight reduction would be.)

The Acceleration due to Gravity: *g*

If we denote the mass and radius of the Earth as M_E and R_E, and your own mass as m, then the force that the Earth is exerting on you right now is given by the equation of gravity:

$$\text{Force} = G \times \frac{\text{First mass} \times \text{Second mass}}{\text{Distance}^2}$$

$$\text{Your weight} = G \times \frac{\text{Earth's mass} \times \text{Your mass}}{\text{Earth's radius}^2}$$

$$= G \times \frac{M_E \times m}{R_E^{\,2}}$$

or, rearranging the terms:

$$\text{Your weight} = m \times \left(\frac{G \times M_E}{R_E^{\,2}} \right)$$

Note that this equation is in the form: Force = Mass × [something in square brackets]. If we compare this to Newton's second law, Force = Mass × Acceleration, we see that what's in square brackets must be the acceleration you would feel if gravity were the only force acting on you. This missing number is identical to the quantity we call g in Chapter 3. In other words, g is the acceleration due to gravity at the Earth's surface:

$$\text{Force} = \text{Mass} \times g = \text{Weight}$$

and

$$g = \frac{G \times M_E}{R_E^{\,2}}$$

This result is extremely important. For Galileo, g was a number to be measured, but whose value he could not predict. For Newton, on the other hand, g was a number that could be calculated purely from the size and mass of the Earth.

One way of keeping weight and mass distinct in your mind is to remember that weight can change from one place to another; for instance, apples would weigh less on the Moon than on Earth, as shown in Figure 5-4. The mass of the

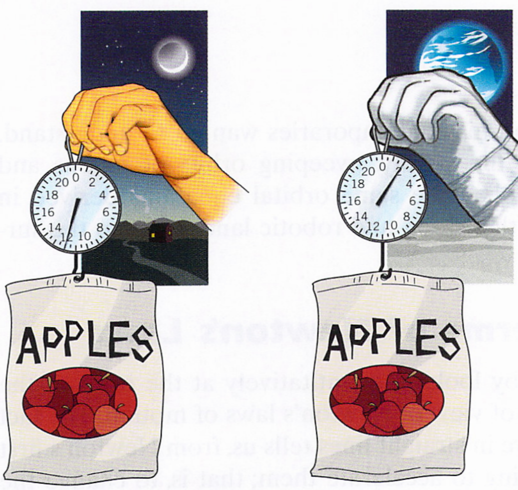


FIGURE 5-4. The weight of an object is different on Earth and on the Moon, but its mass is the same.

apples however, which measures the amount of material in the apples, is the same everywhere in the universe.

EXAMPLE 5-1

The Earth's Gravity

Given that the mass of the Earth is 6×10^{24} kilograms, what is the acceleration due to gravity at the Earth's surface?

REASONING AND SOLUTION: To answer this question, we just have to put the Earth's mass and radius into the expression for g given previously.

$$g = G \times \frac{M_E}{R_E{}^2}$$

$$g = 6.674 \times 10^{-11}\ \text{m}^3/\text{s}^2\text{-kg} \times \frac{6 \times 10^{24}\ \text{kg}}{(6.4 \times 10^6\ \text{m}^2)^2}$$

$$= 9.8\ \text{meters/second}^2$$

This number is the same constant that Galileo and others measured. Notice that because we now understand where g comes from, we can predict the appropriate value of gravitational acceleration not only for the Earth, but also for any object in the universe, provided we know its mass and radius. ●

EXAMPLE 5-2

Weighty Matters

A cantaloupe has a mass of 0.5 kilograms. What does it weigh?

REASONING AND SOLUTION: We are given a mass and want to find its weight. To answer this question, we have to calculate the force of gravity exerted on the cantaloupe at the Earth's surface. The relation between mass and weight is:

$$\begin{aligned}\text{Weight} &= \text{Mass} \times g\\ &= 0.5\ \text{kg} \times 9.8\ \text{m/s}^2\\ &= 4.9\ \text{kg-m/s}^2 = 4.9\ \text{newtons}\end{aligned}$$

This value is the weight of the cantaloupe. Note that the kilogram is not a unit of weight, despite its popular use. ●

GRAVITY AND ORBITS

Of all the motions that Newton and his contemporaries wanted to understand, none were more fascinating than the stately, sweeping orbits of moons and planets. Today's scientists have applied the same orbital equations derived in Newton's time to send humans to the Moon and robotic landrovers to the surface of the red planet Mars.

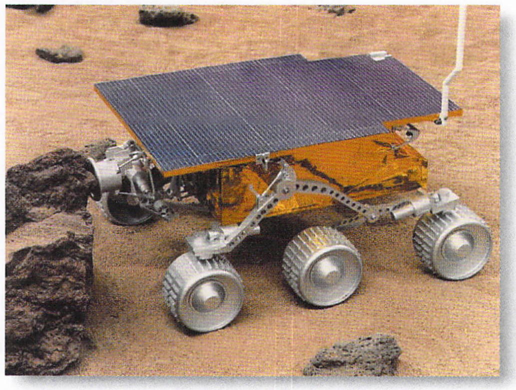
Circular Motion in Terms of Newton's Laws

Let's begin our analysis of orbits by looking quantitatively at the orbit of the Moon (or a planet) from the point of view of Newton's laws of motion. The fact that moons and planets do not move in straight lines tells us, from Newton's first law, that there must be a force acting to accelerate them; that is, to change the direction of their motion. In Chapter 3, we saw that the acceleration, a_c, required to keep an object such as the Moon moving around in a circular path of radius r with a constant velocity v is

$$a_c = \frac{v^2}{r}$$

This quantity is the centripetal acceleration. Newton's second law tells us that the force needed to produce this acceleration equals the product of the mass of the object times its acceleration. So the force needed to keep the object moving in a circle is

$$F_c = ma_c = \frac{mv^2}{r}$$



The robot rover *Sojourner* explored the surface of Mars for several days in 1997 before breaking down from the harsh conditions.

An object stays in a circular path only as long as this force acts. If the force disappears, Newton's first law of motion tells us that the object moves off in a straight line. Think of swinging a weight on a string in a circle around your head. The force that keeps the weight circling is the tension in the string; you can feel that force in your fingers. However, if you let go of the string, the weight doesn't keep circling around, but it flies off in a straight line in whatever direction it happened to be going at the moment of release.

Discus throwers use exactly this kind of action (Figure 5-5). First they spin around so that the discus, held at arm's length, is moving in a circular path (becase the thrower's hand is exerting a force, pulling the discus in). Then they release the discus so that it flies out over the field in a straight line.

The force in circular motion is directed toward the center of the circle, and hence is sometimes called the **centripetal force** (which means center-seeking force). The action of the centripetal force is illustrated for a weight on a string in Figure 5-6. The natural tendency of the weight is to move off in a straight line, as shown at point A. To keep it from flying off, you have to exert a force to pull it back into the circle. It then wants to fly off again, so that you have to pull it back again, and so on. The tug you feel in your hand as the weight goes around

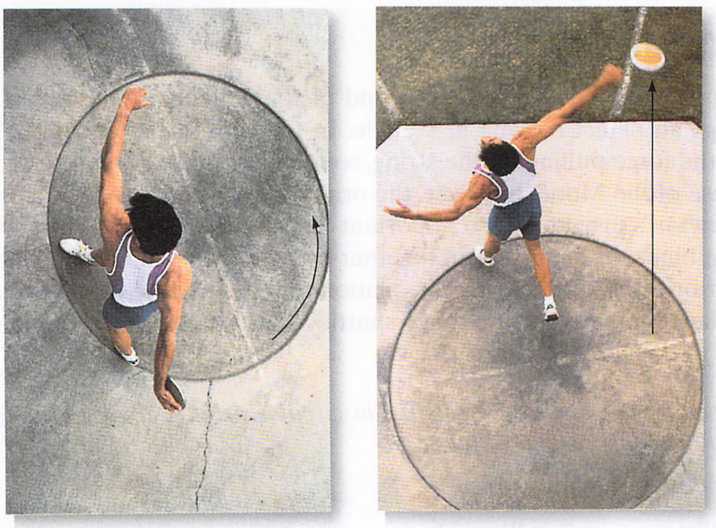


FIGURE 5-5. A discus moves in a circular path as long as the athlete holds on to it, but it moves in a straight line once it is released.

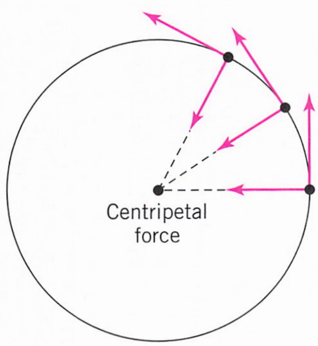


FIGURE 5-6. Centripetal force acts to keep the weight moving in a circle at the end of the string.

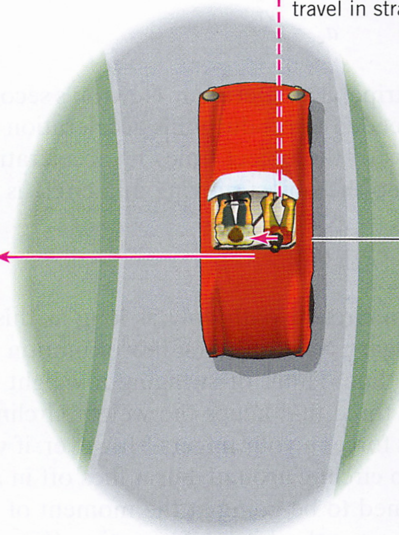


FIGURE 5-7. When you're driving in a car moving around a curve, you feel the car seat and door of the car pushing you in toward the center of the circle.

is the reaction to your constantly pulling on the string, constantly tugging the weight back.

Another familiar example of centripetal force is a person sitting in a car as it goes around a curve (Figure 5-7). As the car starts into the turn, the passenger tends to move ahead in a straight line, following Newton's first law. The car seat or car door must exert a contact or frictional force to keep the passenger moving in a curve along with the car. This contact force is the centripetal force: the force that makes the passenger move in a circle. The passenger feels as if she is being thrown toward the outside of the car, but that is due to her inertia resisting the force pushing her toward the inside.

The Orbit Equation

Centripetal force is not a different kind of force; it simply describes any force that keeps an object moving in a circle. In the case of the circling weight, you can feel the force pulling on the string, so identifying centripetal force is simple. In the case of the Moon, however, the origin of the centripetal force isn't so obvious. Newton's insight was so important because he realized that gravity is the force that binds the solar system together by pulling the planets and moons back into their orbits. In fact, the basic equation that defines the orbit of a satellite—be it a planet, a moon, or the space shuttle—follows from this statement.

1. In words:

The centripetal force on a satellite in circular orbit is equal to the force of gravity exerted on that object.

2. In an equation with words:

$$\frac{\text{Satellite mass} \times \text{Satellite velocity squared}}{\text{Orbital distance}} = \frac{G \times \text{Satellite mass} \times \text{Central mass}}{\text{Distance squared}}$$

3. In an equation with symbols:

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

where m is the mass of the satellite, M is the mass of the central body, r is the distance from the satellite to the center of the central body, and v is the speed of the satellite in its orbit.

This equation is simply an application of Newton's second law: force (the gravitational force, GMm/r^2) equals mass (m) times acceleration (v^2/r).

If we multiply both sides of this equation by r and divide both sides by m, we find that it reduces to

$$v^2 = \frac{GM}{r}$$

This orbit equation is a good one to examine because it tells us that for a given distance, r, between a satellite and its central body, there is one and only one speed, v, at which the satellite can move and remain in orbit (Figure 5-8). It also tells us that the speed at which a satellite has to move doesn't depend in any way on its mass. For example, a grapefruit orbiting the Earth at the same distance as the Moon would circle the Earth every 29 days, just as the Moon does. All modern satellites obey the orbit equation; for some examples of satellite applications, see Connection on page 106.

The force of gravity is not much different at the distance of a typical satellite orbit, such as the orbit of the space shuttle (6400 km + 200 km above Earth's surface), than it is where you are sitting (r = 6400 km). Why, then, does the shuttle stay in orbit? The reason is that enormous amounts of energy have been expended to get the shuttle moving very fast, so that its tendency to fly off in a straight line is just balanced by gravity. In that sense, the only reason you aren't in orbit at this moment is that you're not moving fast enough! How fast does the shuttle have to move? See Example 5-5 at the end of this chapter.

Apparent Weightlessness

We often see pictures of astronauts floating around in the space shuttle or the Space Station and we speak of them as being weightless. In fact, the force of the Earth's gravity at the orbit of the shuttle is pretty much the same as it is at

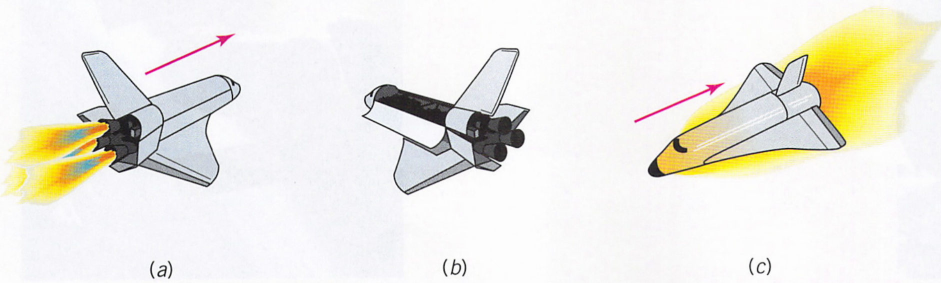


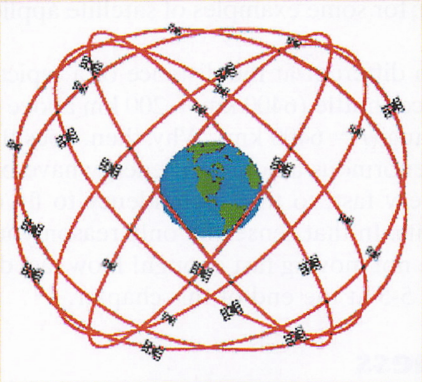
FIGURE 5-8. (*a*) The space shuttle reaches orbit by using its rocket boosters to achieve high acceleration. (*b*) Once in stable orbit, the space shuttle's speed is determined by the orbit equation; it does not use its engines at all. (*c*) To return to Earth, the shuttle fires its small thruster rockets to slow down, enabling Earth's gravity to pull it back down to the surface.

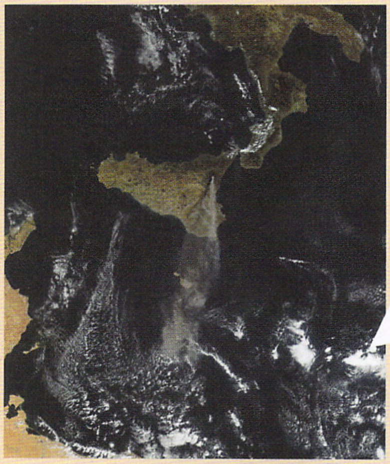
Physics and Modern Technology—Satellites

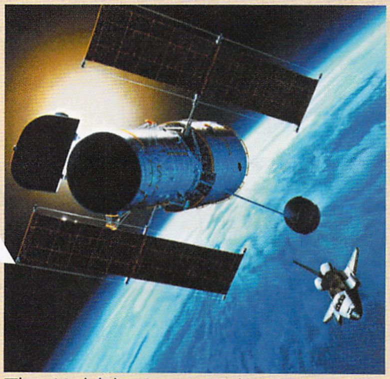
The physics of orbiting objects was first worked out by Kepler and Newton in the seventeenth century. Not until 1957 did advances in rocket technology and electronic instrumentation lead to a successful satellite launch. Today hundreds of satellites circle the Earth, from simple observation satellites with a camera to the International Space Station. However, the physics involved in all these orbits is the same.

With a navigation system in your car, you can locate your position on a road map of your neighborhood.


Long-distance cell phones receive messages relayed by a network of communications satellites.


The Global Positioning System (GPS) uses 24 satellites to pinpoint the position of an object to within 15 meters or less, anywhere on the globe.


A weather satellite took this picture of a volcanic smoke plume rising from Mt. Etna in Sicily.


The Hubble Space Telescope can see astronomical objects from above Earth's atmosphere, greatly increasing its effectiveness.

the surface, so the astronauts aren't weightless. The force of gravity has not suddenly gotten smaller. Astronauts float in the shuttle because of the spaceship's acceleration as it moves around its orbit.

As seen from outside the spaceship, the only force acting is the Earth's gravity. Consequently, the spaceship is continuously pulled toward the Earth—otherwise it would escape into space. You can think of the spaceship (and its contents) as falling toward the Earth, even as it speeds along in its orbit. The point is that everything in the ship is falling at the same rate. So if the astronaut steps on a scale, that scale is falling at the same rate he is, and his weight registers as zero. We achieve apparent weightlessness in the presence of gravity.

Develop Your Intuition: Weight in an Elevator

When you're in an elevator, you usually feel heavier when the elevator starts up and lighter when it starts down. Why?

These shifts in apparent weight, which would actually register as changes in your weight on a scale, come about even though the force of the Earth's gravity is essentially constant throughout the elevator trip (Figure 5-9). When you start upward, the elevator floor is accelerated into your feet, exerting a force that accelerates you upward. By Newton's third law of motion, your feet exert an equal and opposite force on the floor—or on a scale, if you're standing on one. This extra force causes the spring in the scale to compress and the reading to increase. The reverse process causes a feeling of partial weightlessness when the elevator starts down.

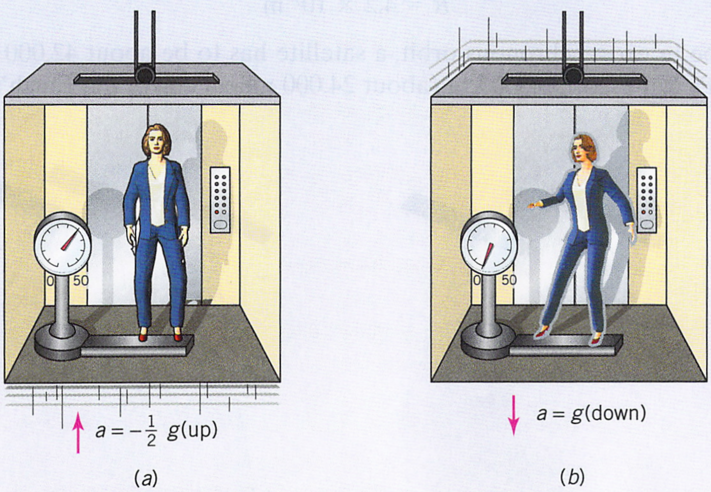


FIGURE 5-9. A person on a scale in an elevator feels (*a*) heavier while accelerating up, against gravity, and (*b*) lighter while accelerating down, with gravity.

Connection

Geosynchronous Orbits

Much of our modern communications system depends on relaying signals through satellites in orbit above the Earth. A particularly useful orbit is one in which the satellite moves just fast enough so that it appears to remain stationary above a point on the Earth.

Consider, for example, a satellite in orbit above the Earth's equator. If the satellite completes one revolution in 24 hours, that satellite appears to hover over the same spot on the Earth's surface, since that spot also completes a revolution in 24 hours. Such a satellite is said to be in *geosynchronous orbit.* What would the radius of the orbit have to be for the orbit to be geosynchronous?

If the satellite's orbit is located a distance R from the center of the Earth, then the satellite must travel a distance $2\pi R$ in 1 day. A day consists of 24 hours, each of which has 60 minutes of 60 seconds each. Consequently, the number of seconds in a day is

$$t = (24 \text{ hours/day}) \times (60 \text{ minutes/hour}) \times (60 \text{ seconds/minute}) = 86{,}400 \text{ s}$$

Now, the speed of the satellite must be

$$v = \frac{2\pi R}{t}$$

If we put this value of the velocity into the orbit equation, we find that

$$\frac{GM}{R} = \frac{4\pi^2 R^2}{t^2}$$

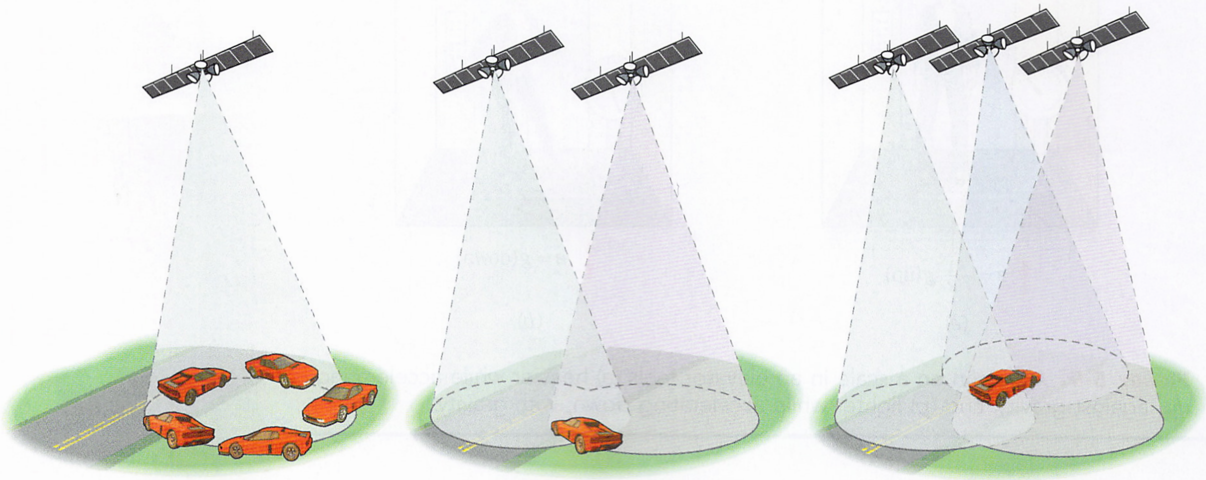
so that the radius of the orbit is

$$R^3 = \frac{GMt^2}{4\pi^2}$$

If we put in the mass of the Earth (6×10^{24} kg) and the value of t, we find that

$$R = 4.2 \times 10^7 \text{ m}$$

Thus, to be in geosynchronous orbit, a satellite has to be about 42,000 km from the Earth's center, or 36,000 km (about 24,000 miles) above the Earth's surface.



(*a*) (*b*) (*c*)

FIGURE 5-10. In the Global Positioning System (GPS), (*a*) a satellite identifies the receiver in a car as being somewhere on a circle. (*b*) When a second satellite identifies the same receiver as being on another circle, the car must be located at one of the two points where the circles intersect. (*c*) The third satellite determines exactly where the car is located.

This is a very high orbit. For reference, the space shuttle normally orbits a little over 100 miles above the surface.

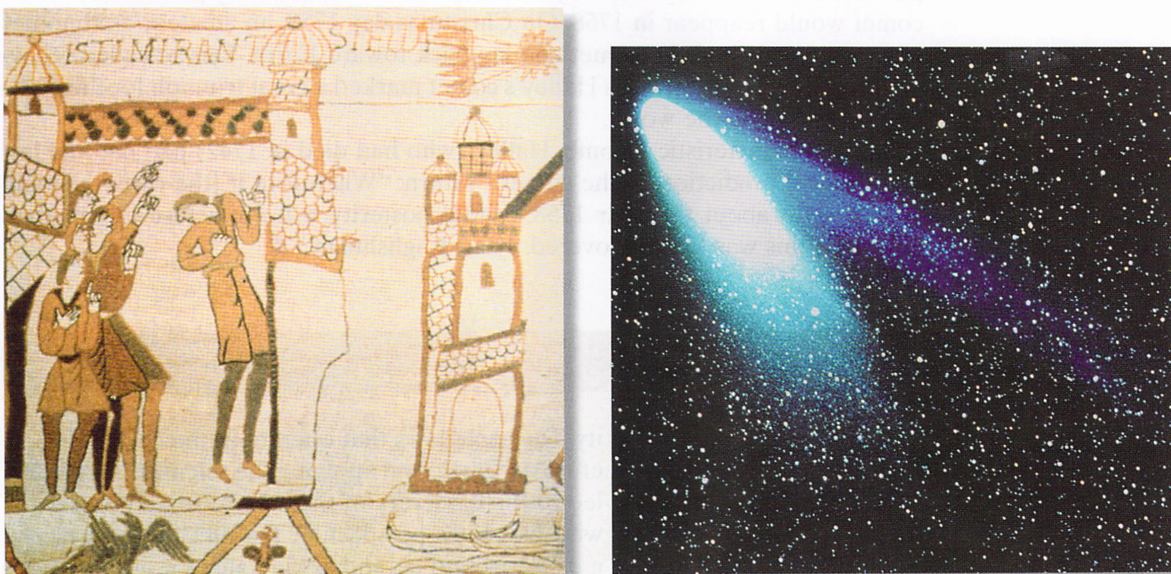
The Global Positioning System (GPS), first developed by the United States Air Force, consists of 24 satellites placed in six orbits. These orbits are about 11,000 miles above the Earth's surface; they are not geosynchronous, so they move relative to the Earth's surface. Portable computers on your car, your plane, or even in your hand can pick up signals from three or more of these satellites and use them to determine your location on the Earth to within a few yards (Figure 5-10). ●

Physics in the Making

The Recovery of Halley's Comet

Of all celestial phenomena, none seemed more portentous and magical to the ancients than comets. These glorious lights in the sky, with their luminous sweeping tails, are not like the planets. They appear sporadically, and even in Newton's day appeared unpredictably. Yet even comets are subject to Newton's laws.

We associate the discovery of the orbital nature of comets with the British astronomer Edmond Halley (1656–1742). Halley led an adventurous (even swashbuckling) life before he settled down as Britain's Astronomer Royal. At various points in his life, he ran a diving company, captained a Royal Navy survey ship (facing down a mutiny in the process) and, if we are to believe the legend, used his growing reputation as an astronomer to travel around European capitals as a secret agent for his country. He was the first European astronomer to produce modern maps of the skies of the Southern Hemisphere, and his navigational maps of the Earth's magnetic field were used well into the nineteenth century.



(*a*) (*b*)

Halley's comet: (*a*) on the Bayeux tapestry, showing the comet in 1066 A.D., and (*b*) in a telescope photo from 1986.

Halley was so wedded to the adventurous life that when he was proposed for a professorship at Oxford, one colleague wrote, "Mr. Halley expects [the professorship], who now talks, swears, and drinks brandy like a sea captain; so much that I fear that his ill behavior will deprive him of the advantage of this vacancy."

Nevertheless, he got the post at Oxford (the house he lived in is still there). About the time of Halley's appointment, astronomers were starting to think about explaining comets. Even though they knew about universal gravitation, they didn't have the mathematical tools to solve the problem of comets' elongated orbits. The reason for this difficulty is that, except for orbits that are nearly circular like those of the planets, the distance between a satellite and its central body varies considerably from one point to the next. Hence the gravitational force is not the same at each point around the orbit. The problem of deducing the shape of an orbit under these circumstances is a difficult one, but one that could be dealt with using the new mathematics of calculus, which was invented independently by Isaac Newton and the German mathematician Gottfried Leibniz.

In 1684, Halley visited Newton at Cambridge. Newton told him over dinner that according to his calculations, all bodies subject to a gravitational force would move in orbits shaped like ellipses. Bolstered by this information, Halley analyzed the historical records of some 24 comets. Knowing that the orbits had to be elliptical, he was able to use the observations to determine exactly the elliptical path along which each comet moved.

He found that three recorded comets—those that had appeared in 1531, 1607, and 1682—seemed to be following the same orbit. He realized that the sightings represented not three separate comets, but one comet that was appearing over and over again at intervals of about 75 or 76 years.

Predicting the next appearance of the comet wasn't as simple as you might think, because the gravitational effects of Jupiter and Saturn could change the period of the comet by several years. After some work, Halley predicted that the comet would reappear in 1758. On Christmas day 1758, an amateur astronomer in Germany sighted the comet coming back toward Earth. This so-called recovery of what is now known as Halley's comet marked a great triumph for the Newtonian picture of the world.

With characteristic aplomb, Halley (who had died in 1742) had had the last word on his prediction of the comet's return: "Wherefore if [the comet] should return again about the year 1758, candid posterity will not refuse to acknowledge that this was first discovered by an Englishman." ●

THINKING MORE ABOUT

The Clockwork Universe: Predictability

Newton bequeathed to posterity a picture of the universe that is beautiful and ordered. The planets orbit the Sun in stately paths, forever trying to move off in straight lines, forever prevented from doing so by the inward tug of gravity. The same laws that operate in the cosmos operate on Earth, and applying the scientific method led to the discovery of these laws. To an observer with Newton's perspective, the universe was like a clock, wound up and ticking along according to definite laws.

The Newtonian universe seemed regular and predictable. If you knew the present state of a system and the forces acting on it, the laws of

Whitewater is an example of a chaotic system, in which a small change in the initial position can produce a large difference in the outcome.

motion would allow you to predict its entire future. This notion was taken to the extreme by the French mathematician Pierre Simon Laplace (1749–1827), who proposed the notion of the *Divine Calculator.* His argument (in modern language) was this: if we knew the position and velocity of every atom in the universe and we had infinite computational power, then we could predict the position and velocity of every atom in the universe for all time. He made no distinction between an atom in a rock and an atom in your hand. According to the argument, everyone's movements are completely determined by the laws of physics to the end of time. You cannot choose your future. What is to be was determined from the very beginning.

While this argument raises many interesting questions, it has been rendered moot by two modern developments in science. One of these, the Heisenberg uncertainty principle (see Chapter 22), tells us that at the level of the atom it is impossible to know simultaneously and exactly both the position and velocity of any particle. (Heisenberg showed that any measurement of a particle's position alters its velocity, and vice versa.) Thus, you can never get all the information the *Divine Calculator* needs to begin working.

More recently, scientists working with computer models have discovered that there are many systems in nature that can be described in simple Newtonian terms but whose futures are extremely sensitive to initial conditions, making them, to all intents and purposes, unpredictable. These are called "chaotic systems," and the field of study devoted to them is called *chaos theory.*

Whitewater on a mountain stream provides a familiar example of a chaotic system. Imagine putting two chips of wood in water on the upstream side of the rapids. No matter how small you make the chips, or how close together they are at the beginning, those chips (and the water on which they ride) may be widely separated by the time they get to the end. If you knew the exact initial position of a chip and every detail of the waterway's shape and other characteristics with complete mathematical precision, you could, in principle, predict where the chip will come out downstream. But if there is the slightest uncertainty in your initial description, no matter how small, the actual position of the chip and your prediction will differ, often wildly. Every measurement in the real world has some uncertainty associated with it, so it is never possible to determine the exact position of the chip at the start of its trip. For all practical purposes, you cannot predict where it will come out even if you know all the forces acting on it.

The existence of chaos, then, tells us that there are some systems in nature in which the Newtonian vision of a completely predictable universe simply doesn't apply. Some important aspects of our lives, ranging from next week's weather to next year's health, are inherently unpredictable. Does science's inability to answer such important questions diminish its importance to society? In what ways does society prepare itself for the unpredictable aspects of the physical world?

Summary

Newton's law of universal gravitation describes **gravity,** the most prevalent force in our daily lives. At the Earth's surface, the gravitational force exerted on an object is called its **weight.** The same force that pulls a falling apple to Earth supplies the **centripetal force** that causes the Moon to curve around the Earth in its orbit. Indeed, the force of gravity (with the same **gravitational constant,** G) operates everywhere, with pairs of forces between every pair of masses in the universe. Newton's laws of motion, together with the law of universal gravitation, describe the orbits of planets, moons, comets, and satellites. They also allow us to derive Galileo's results and Kepler's laws of planetary motion (Chapter 3), thereby unifying the sciences of mechanics and astronomy.

Key Terms

centripetal force The force in circular motion, directed toward the center of the circle, that keeps an object following a curved or circular path. (p. 103)

gravitational constant (G) The exact numerical relation between the masses of two objects and the distance between them, on the one hand, and the gravitational force between them, on the other. (p. 98)

gravity The attractive force that acts between any two objects in the universe. (p. 97)

Newton's law of universal gravitation Newton's law that between any two objects in the universe there is an attractive force (gravity) that is proportional to the masses of the objects and inversely proportional to the square of the distance between them. (p. 98)

weight The force of gravity on an object. (p. 100)

Key Equations

$$\text{Force} = G \times \frac{\text{First mass} \times \text{Second mass}}{\text{Distance}^2}$$

$$\text{Force} = \text{Mass} \times g = \text{Weight}$$

$$(\text{Velocity of a satellite})^2 = G \times \frac{\text{Mass of central body}}{\text{Radius of orbit}}$$

Constants

$g = 9.8 \text{ m/s}^2$

$G = 6.674 \times 10^{-11} \text{ m}^3/\text{s}^2\text{-kg}$

Review

1. What similarity did Newton see between the Moon and an apple?
2. Why is gravity called a universal force?
3. State the law of universal gravitation.
4. Why is the gravitational constant, G, called a constant of proportionality? In Newton's equation for gravity, what is proportional to what?
5. How did Henry Cavendish determine the value of the gravitational constant, G?
6. What is the difference between weight and mass?
7. Does a bathroom scale measure weight or mass? Explain your answer.
8. What is centripetal force? Give an example of this force in action.
9. What supplies the centripetal force that keeps the planets in their orbits?
10. What is the relation between the velocity of a satellite, the radius of its orbit, and the mass of the central body?

11. According to the orbital equation, what factors determine the velocity of a satellite in orbit?

12. What is a geosynchronous orbit? Why are such orbits important to modern technology?

13. How did the work of Edmond Halley support Newton's theories?

14. What is a chaotic system?

15. What is the main idea of Newton's universal law of gravitation?

16. The gravitational constant is now known to 1 part in 10,000, yet physicists are still trying to measure this constant. Why?

17. Why did scholars of the sixteenth century distinguish between "terrestrial gravity" and "celestial gravity?"

18. What are the differences between the gravitational constant, G, and the acceleration due to gravity, g? Why is g not considered to be a universal constant?

19. Newton's equation for gravity incorporates an inverse square relationship between the force of gravity and the distance between two objects. What other familiar phenomena exhibit an inverse square relationship? (*Hint:* See Chapter 2.)

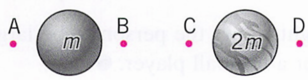
Questions

1. If this textbook is sitting on a table, the force of gravity is pulling it down. Why doesn't it fall?

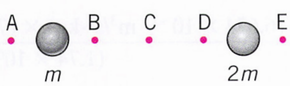
2. Which of the following objects does not exert a gravitational force on you?

   a. this book c. the nearest star
   b. the Sun d. a distant galaxy

3. Two planets with the same diameter are close to each other, as shown in the figure. One planet has twice as much mass as the other planet. At which locations (A, B, C, or D) would both planets' gravitational force pull on you in the same direction? From among these four locations, where would you stand so that the force of gravity on you is a maximum i.e., at which point would you weigh the most?



4. Two iron spheres, of mass m and $2m$, respectively, are shown in the figure. At which location (A, B, C, D, or E) would the net gravitational force on an object due to these two spheres be a minimum?



Questions 4, 5

5. Two iron spheres, of mass m and $2m$, respectively, are shown in the figure. At which location (A, B, C, D, or E) would the net gravitational force on an object be at a maximum due to these two spheres?

6. If you moved to a planet that has the same mass as the Earth but twice the diameter, how would your weight be affected?

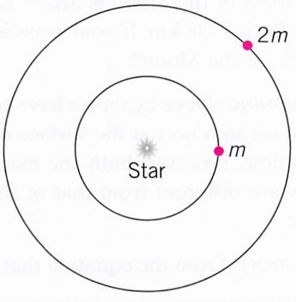
7. If you moved to a planet that has twice the mass of the Earth and also twice the diameter, how would your weight be affected?

8. The Earth exerts an 800-N gravitational force on a man. What gravitational force, if any, does the man exert on the Earth?

9. The environment in a satellite or space station orbiting the Earth is often referred to as a *weightless* environment. However, we have defined *weight* as the force of gravity on an object. Do you agree that objects on board orbiting satellites are *weightless?* Explain.

10. A bungee jumper feels weightless as she falls toward the Earth. Obviously the force of gravity has not disappeared simply because she has jumped off a high platform. What accounts for the weightless feeling people get when they fall freely?

11. The Earth's radius is about 3.7 times larger than the Moon's radius. If the Earth and the Moon had the same mass, which would have the greater acceleration due to gravity? Explain. Since we know that the Earth has a greater acceleration due to gravity, what does this tell you about the mass of the Earth compared to the mass of the Moon?



Questions 12, 13

12. Consider two planets of mass m and $2m$, respectively, orbiting the same star in circular orbits. The more massive planet is twice as far from the star as the less massive planet. Which, if either, planet experiences a stronger gravitational attraction to the star? Explain.

13. Consider two planets of mass m and $2m$, respectively, orbiting the same star in circular orbits. The more massive planet is twice as far from the star as the less massive planet. Which planet is moving faster? Which planet has the shorter orbital period?

14. If our own Sun were twice as massive as it is, would the Earth have to move faster or slower in order to remain in the same orbit?

15. When Galileo first observed the four largest moons orbiting the planet Jupiter, he quickly determined the time it took for each moon to complete one orbit. Why won't this measurement allow us to determine the masses of the moons? Could such a measurement allow us to determine the mass of Jupiter?

16. How is weight related to mass?

17. How is Newton's law of gravitation related to Kepler's third law of planetary motion?

18. Does the Moon fall toward the Earth? Explain.

19. In *Star Trek* and other science fiction sagas, you often encounter a fictional device called a tractor beam, capable of pulling objects into the starship. Suppose that an object is falling under the influence of gravity and drag. In addition, imagine that a tractor beam on the ground is pulling the object down. If there is a limit to the force the tractor beam can exert, will the object still attain a terminal velocity?

20. If you fill a bucket partially with water and then swing it fast enough in a circle over your head, the water will stay in the bucket even when it is upside down. Since gravity is pulling the water down, why doesn't it spill out?

21. Why do you weigh less on the Moon than on Earth?

22. Would your mass change if you took a trip to the Space Station? Why or why not?

23. According to some nineteenth-century geological theories (now largely discredited), the Earth has been shrinking as it gradually cools. If so, would g have changed over geological time? Would G have changed over geological time?

Problem-Solving Examples

EXAMPLE 5-3 Football Player Mass

Suppose a football player has a mass of 120 kg. What is his weight in newtons? What is his weight in pounds?

REASONING AND SOLUTION: To find the weight in newtons, use the same equation as in Example 5-2.

$$\begin{aligned}\text{Weight} &= \text{Mass} \times g \\ &= 120 \text{ kg} \times 9.8 \text{ m/s}^2 \\ &= 1176 \text{ N}\end{aligned}$$

From the *Table of Conversion Factors* in Appendix A we know that multiplying pounds by 4.45 will give newtons, so

$$\begin{aligned}\text{Weight (in pounds)} &= \frac{\text{Weight (in newtons)}}{4.45} \\ &= \frac{1176}{4.45} \text{ pounds} \\ &= 264 \text{ pounds}\end{aligned}$$

This weight shows the person to be large, but not unusually so for a football player. ●

EXAMPLE 5-4 Weight on the Moon

The mass of the Moon is $M_M = 7.18 \times 10^{22}$ kg and its radius R_M is 1738 km. If your mass is 60 kg, what would you weigh on the Moon?

REASONING: Once again we have to calculate the force exerted on an object at the surface of an astronomical body. This time, however, both the mass and the radius of the body are different from that of the Earth, while G is the same.

SOLUTION: From the equation that defines weight, we have

$$\begin{aligned}\text{Weight} &= \text{Force due to gravity} \\ &= \frac{G \times M_M \times 60 \text{ kg}}{R_M^2} \\ &= \frac{(6.674 \times 10^{-11} \text{ m}^3/\text{s}^2\text{-kg}) \times (7.18 \times 10^{22} \text{ kg}) \times 60 \text{ kg}}{(1.74 \times 10^6 \text{ m})^2} \\ &= 95 \text{ newtons}\end{aligned}$$

This weight is about one-sixth of the weight the same object would have on the Earth, *even though its mass is the same in both places.* ●

EXAMPLE 5-5 The Space Shuttle

We can get an idea of how the orbit equation works by considering the space shuttle. A typical shuttle orbit is 200 kilometers (about 120 miles) above the Earth. How fast does a satellite at this distance have to move to stay in orbit?

SOLUTION: The equation for a satellite's speed, derived earlier in this chapter, is

$$v^2 = \frac{GM}{r}$$

As we saw earlier, the mass of the Earth is 6×10^{24} kg and its radius is 6.4×10^6 meters. The shuttle orbit, 200,000

($= 0.2 \times 10^6$) meters above the surface, therefore corresponds to a value of r of about 6.6×10^6 meters. If we put these values into the equation, we find that the square of the speed of the shuttle in its orbit is

$$v^2 = (6.674 \times 10^{-11}\ \text{m}^3/\text{s}^2\text{-kg}) \times \frac{6 \times 10^{24}\ \text{kg}}{6.6 \times 10^6\ \text{m}}$$
$$= 6 \times 10^7\ \text{m}^2/\text{s}^2$$

Taking the square root of this value, we see that the speed is

$$v = 7.8 \times 10^3\ \text{m/s}$$

This speed, equal to almost 8 km (5 miles) per second, is many times faster than commercial jet aircraft, which cruise at about 0.3 km (0.2 miles) per second.

One way of getting an idea of this speed is to ask how long it would take the shuttle to complete one orbit of the Earth. We can determine this time by rearranging the definition of speed into the form: time equals distance divided by speed. The length of the orbit is just the circumference of a circle of radius r, or $2\pi r$. Thus, the orbital period, or time for one revolution, is

$$\text{Orbital period} = \frac{2\pi r}{v}$$
$$= 2\pi \frac{6.6 \times 10^6\ \text{m}}{7.8 \times 10^3\ \text{m/s}}$$
$$= 5300\ \text{s}$$
$$= 89\ \text{minutes}$$

The space shuttle completes an orbit of the Earth every hour and a half. ●

Problems

1. What do you weigh in pounds? What do you weigh in newtons?

2. What is your mass in kilograms?

3. What would you weigh if the Earth were four times as massive as it is and its radius were twice its present value?

4. How long would our year be if our Sun were half its present mass and the Earth's orbit was in the same place that it is now?

5. How much would you weigh if you were standing on a mountain 200 km tall (*i.e.*, if you were standing still at about the altitude of a space shuttle orbit)? How much does this differ from your weight on the surface of the Earth? Would you be able to detect this weight difference on an ordinary bathroom scale?

6. **A.** Calculate the force of gravity on a 65-kg person in the following:
   a. at the surface of the Earth ($R = 6400$ km)
   b. at twice the Earth's radius
   c. at four times the Earth's radius

   B. Plot this gravitational force with distance. What pattern or relationship do you expect to obtain? Does your plot conform to your expectations?

7. Compare the gravitational force on a 1-kg mass at the surface of the Earth with that on the surface of the Moon ($M_M = 1/81.3$ mass of the Earth; $R = 0.27$ Earth radius).

8. How much less would you weigh on the top of Mount Everest than at sea level?

9. Calculate the weight in pounds and newtons of the following items:
   a. the Statue of Liberty (205 tons)
   b. a 40-ounce softball bat
   c. a solid rocket booster of the space shuttle (5.9×10^5 kg)

10. Calculate the weight in pounds and newtons of the three objects in Problem 9 if they were: a. on the surface of the Moon (see Problem 7); b. on the surface of Mars ($M = 0.11$ mass of the Earth; $R = 0.53$ Earth radius).

11. Calculate the speed and period of a ball tied to a string of length 0.3 meters making 2.5 revolutions every second.

12. Calculate the average speed of the Moon in kilometers per second around the Earth. The Moon has a period of revolution of 27.3 days and an average distance from the Earth of 3.84×10^8 meters.

13. Calculate the speed at the edge of a compact disc (radius = 6 cm) that rotates 3.5 revolutions per second.

14. Calculate the centripetal force exerted on the Earth by the Sun. Assume that the period of revolution for the Earth is 365.25 days and the average distance is 1.5×10^8 km.

15. The height of a mountain is limited by the ability of the atoms at the bottom to sustain the weight of the materials above them. Assuming that the tallest mountains on Earth are near this limit, how tall could a mountain be on the Moon? On Mars?

Investigations

1. In Chapter 1, we talk about astrology and whether a planet or star can influence our lives. Calculate the gravitational force on a newborn infant exerted by a star the size of the Sun 1 light year (9.5×10^{15} m) away. Compare it to the gravitational force exerted by a 100-kg physician 0.1 m away.
2. One objection that Copernicus's contemporaries raised to his theory was that if the Earth were really turning, we would all be thrown off the way that clay is thrown off a spinning potter's wheel. Use Newton's laws of motion and the law of universal gravitation to counter this argument.
3. In what sense is the Newtonian universe simpler than Ptolemy's? Suppose observations had shown that the two did equally well at explaining the data. Construct an argument you would make to say that Newton's universe should still be preferred.
4. If Kepler had been transported to another solar system, what would he have had to do in order to show that his laws applied there? What would Newton have had to do?
5. Some astronomers have proposed that Newton's law of gravitation may have to be modified over very large distances—that the gravitational "constant" varies over the immense scale of galaxies. What evidence do we have that gravitation is a universal force? How might you test this assumption? (*Hint:* Search the Internet for information on Modified Newtonian Dynamics or MOND.)
6. In what ways does gravity affect the form and function of living things? Relate this to both plants and animals.
7. Use the web to investigate some of the ongoing experiments to determine the value of G with greater precision and accuracy. Why is it so difficult to measure G to better than three decimal places, when most other physical constants are known to as many as 10 places?
8. Read a biography of Pierre Simon Laplace, who was one of history's most influential scientists. What were his major achievements? What major historical events occurred during his lifetime? How did his research influence his philosophical ideas?

WWW Resources

See the *Physics Matters* home page at **www.wiley.com/college/trefil** for valuable web links.

1. **www.curtin.edu.au/curtin/dept/phys-sci/gravity/** An online gravity tutorial at the Department of Applied Physics, Curtin University of Technology.
2. **www.physics.purdue.edu/class/applets/NewtonsCannon/newtmtn.html** A humorous Java applet animating a woodcut from Newton's *Principia* that demonstrates satellite motion.
3. **www.physicsclassroom.com/Class/circles/circtoc.html** An animated tutorial from physicsclassroom.com discussing circular motion, planetary motion and universal gravitation.
4. **liftoff.msfc.nasa.gov/toc.asp?s=Satellites** Contains tutorials on types of satellites (including a section on geosynchronous satellites) and an extensive section on tracking current Earth-orbiting spacecraft live via the web.