6

Conservation of Linear Momentum

KEY IDEA

If no external forces act on a system, then the total momentum of the system is constant.

PHYSICS AROUND US . . . Dealing with Momentum

You get in your car, put your books on the seat next to you, and drive away. A squirrel runs across the road and you jam on your brakes to avoid hitting it. As you do so, the pile of books slides onto the floor of the car.

Later that day, in the cafeteria, you watch a fellow student hurrying across the floor with her tray. Stopping suddenly, she reaches out to grab her soft-drink cup to keep it from falling over and some of the drink sloshes over on her hand.

That night, you turn on your TV to watch a hockey game. The puck comes free and two players skate for it at top speed. They collide just as they reach the puck and bounce off each other, taking themselves out of the action and leaving the puck for another player to pick up.

All of these occurrences (and countless more) illustrate a quantity called momentum, whose properties follow from Newton's laws of motion. And believe it or not, in addition to being involved in everyday events, momentum also governs such large-scale processes as the formation of planets and stars. As a result, momentum is one of the most important physical attributes of an object in motion.

LINEAR MOMENTUM

In Chapter 4 we introduced the property of inertia, which represents the tendency of a moving object to keep moving or a stationary object to remain stationary. Because of inertia, the only way to change an object's motion is to apply a force, as Newton's laws tell us. Everyday experiences provide all of us with an intuitive understanding of this concept. For example, we sense that a massive object such as a large train is very hard to stop, requiring a lot of force, even if it is moving slowly. You certainly wouldn't want to try and stop a train by standing in front of it!

At the same time, a small object moving very fast—a rifle bullet, for example—is also very hard to stop. Thus, our everyday experience seems to be telling us that the tendency of a moving object to remain in motion depends both on the mass of the object and on its speed. The greater the mass or the speed, the more difficult it is to stop the object or change its direction of motion.

Physicists encapsulate these notions in a quantity called **momentum** (plural, momenta). Momentum can be defined both in words and as an equation.

1. In words:

The momentum of an object is the product of that object's mass and velocity.

That means an increase in either mass or velocity increases momentum proportionally.

2. In an equation with words:

$$\text{Momentum} = \text{Mass} \times \text{Velocity}$$

3. In an equation with symbols:

$$p = m \times v$$

Here we use the letter p to denote momentum.

As we see in this chapter, momentum defined in this way is an extraordinarily useful concept in physics, and one that is deeply embedded in Newton's laws of motion. To begin, however, this definition leads to three important consequences.

1. **Momentum is a vector quantity.** Like velocity, momentum has both a magnitude *and* a direction. Thus, the rules for adding the total momenta of several objects are the standard rules for vector addition (see Chapter 2).
2. **The definition of momentum matches our intuition about the tendency of objects to remain in motion.** The equation tells us that the larger the mass or the greater the velocity, the greater the momentum.
3. **The units of momentum are those of mass times velocity, or kg-m/s.** There is no special name for this unit; it is simply written as a combination of the three basic units for mass, distance, and time.

Note that this product of mass times velocity is sometimes called **linear momentum** to distinguish it from *angular momentum,* which we'll discuss in the next chapter. In normal conversation, the term *momentum* by itself is understood to refer to linear momentum because we are referring to an object moving in a straight line. You can see other examples of momentum in Looking at Momentum.

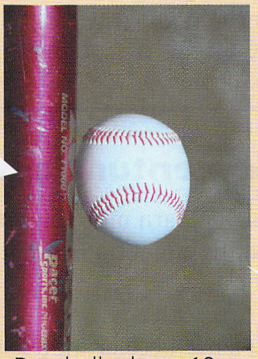
Looking at Momentum

If you've ever swatted at a bee buzzing around your head, you know you can brush it away pretty easily. The momentum of a thrown baseball is about 1000 times greater than that, and you can certainly feel the impact when you catch it or hit it with a bat. A charging rhinoceros has about 1000 times more momentum than a baseball and could trample you flat. So just imagine the impact of a fully loaded oil supertanker, the biggest moving objects ever built, or the effect of a meteor crashing into the Earth.

10^{-2} kg-m/s

Bee, about 0.01 kg-m/s

10^{1} kg-m/s



Baseball, about 10 kg-m/s

10^{4} kg-m/s

Rhinoceros, about 20,000 kg-m/s

10^{10} kg-m/s

Supertanker, about 10 billion kg-m/s

10^{13} kg-m/s

Meteor, about 5 trillion kg-m/s

Develop Your Intuition: Billiards Momentum

Two billiard balls of equal mass m roll toward each other at equal speeds, v. What is the total momentum of the two balls?

Since the masses are equal and the speeds are equal, the magnitudes of the individual momenta of the two balls are equal. However, momentum is a vector quantity and has a direction. In this case, one ball has a momentum of magnitude mv directed toward the right and the other has a momentum of magnitude mv directed toward the left. The two momenta cancel each other out, so the total momentum of this particular system with two billiard balls is zero. In general, the total momentum of a group of objects does not have to be larger than the momentum of each individual object. It can even be zero, as in this case.

Momentum and Newton's Laws

We can learn more about momentum by looking at Newton's second law of motion. In equation form, it reads

$$F = m \times a$$

But since acceleration is defined to be the change in velocity divided by the time it takes that change to occur, this equation can be rewritten

$$F = m\frac{\Delta v}{\Delta t}$$

where the Greek letter capital delta, Δ, should be read "the change in" (see Chapter 2).

Now we can play a little mathematical trick. If the mass of the object in motion doesn't change, then the mass multiplied by the change in velocity must be the same as the change in the product of mass times velocity.

$$m\Delta v = \Delta(mv)$$

(Can you convince yourself that this is true?) In this case, we can write Newton's second law as

$$F = \frac{\Delta(mv)}{\Delta t}$$

In other words, this variation of Newton's second law tells us that the net force applied to an object is equal to the change in that object's momentum divided by the time it takes the momentum to change. The concept of momentum, then, is actually an integral part of Newton's laws and not a concept that has to be added to it.

Although we have assumed here that the mass of the object is constant, the equation is actually true for the more general case in which the mass changes as well. This situation may arise, for example, during the launch of a rocket because the mass of the rocket decreases as fuel is burned during the ascent.

This version of Newton's second law turns out to be helpful in examining more about momentum, as we see in the next section. It also turns out that by examining changes in momentum, we can solve problems that are difficult or impossible to solve by considering only forces and accelerations.

IMPULSE

We can rearrange Newton's second law as written in the last section to get a better understanding of how forces act to change the momentum of a system.

1. In words:

The change in momentum is equal to the product of the net external force and the time during which it acts.

2. In an equation with words:

Change in momentum = Net force × Time interval

3. In an equation with symbols:

$$\Delta(mv) = F \times \Delta t$$

This form of Newton's second law tells us that a change in the momentum of a system is equal to the product of the net force that acts on it multiplied by the length of time that the force acts. In other words, a small force acting over a long time can produce the same change in momentum as a large force acting over a small time.

Physicists use the term **impulse** to refer to the product of the force multiplied by the time over which it acts. (This is another example of a common word in English given a very specific meaning in physics.) In terms of impulse, then, Newton's second law can be restated

Impulse = Change in momentum

This version is often called the **impulse–momentum relationship.**

FIGURE 6-1. When a tennis ball is hit, the force of the impact often deforms the ball briefly before the ball breaks contact with the racquet.

Large Forces Acting for Short Times

Think of a tennis ball moving through the air while the player brings the racquet around to start his swing (Figure 6-1). The ball and the head of the racquet are in contact for only a fraction of a second, but during that short period the force is quite high. The total impulse, then, is large and the ball has a large momentum when it leaves the racquet. (See Example 6-2 in the Problem-Solving Examples, page 135.)

Small Forces Acting over Long Times

Think about a complementary case, in which a huge cruise ship enters a harbor and is nudged into place by a tugboat (Figure 6-2). The tugboat may push on the ship for several minutes, exerting a (relatively) small force over a long time. The direction of motion (and therefore the velocity) of the ship changes slightly as a result, but its mass is so large that even a tiny change in velocity produces a large change in momentum. To produce a large change in momentum, a large impulse is required, which is why the tugboat pushes for so long. (See Example 6-3 in the Problem-Solving Examples, page 135.)

FIGURE 6-2. A tugboat moves a much larger cruise ship by pushing against it steadily for a period of time, changing the ship's momentum bit by bit.

ADDING MOMENTA

Have you played a game of pool recently? If so, you remember hitting the white cue ball with the cue stick so that the cue ball collided with a colored ball, propelling the colored ball toward a pocket. Before the collision, the colored ball was stationary, while the cue ball was moving with a (more or less) constant velocity. After the collision, both balls were moving, although less swiftly than the cue ball had been moving before. You can see the same sort of behavior in many games; bowling, marbles, and croquet are examples. And, as we see in Chapter 9, such collisions are constantly taking place between the atoms that make up material objects. In all these cases, a transfer of momentum takes place between two colliding objects.

Total Momentum

Billiard balls and collections of atoms are examples of systems made up of many particles, each of which has a mass and a velocity. The **total momentum** P of such a system is defined as the sum of the momenta of all the objects in it. For the moment, let's talk only about objects that are all moving along the same line, either to the right or to the left.

CASE 1 ▪ Two balls moving in the same direction.
If the balls have masses m_1 and m_2 and velocities v_1 and v_2, respectively, and both are moving to the right, then the total momentum of the system is

$$P = m_1v_1 + m_2v_2$$

The total momentum of this system (Figure 6-3*a*) is, like the velocities, directed to the right.

CASE 2 ▪ Two balls moving in opposite directions.
If the balls have masses m_1 and m_2 and velocities v_1 and v_2, respectively, but the first is moving to the right and the second is moving to the left, then the total momentum of the system is

$$P = m_1v_1 - m_2v_2$$

In this case, the two momenta are in opposite directions (Figure 6-3*b*). In the special case in which the balls have equal momenta (for example, if they have the same mass and speed), then the total momentum of the system is zero.

CASE 3 ▪ More than two balls.
In this case (Figure 6-3*c*), the total momentum is the sum of all of the momenta of the balls. See that m_1 and m_2 are moving to the right, so their momenta are added, while m_3, m_4, and m_5 are moving to the left, so their momenta are subtracted.

$$P = m_1v_1 + m_2v_2 - m_3v_3 - m_4v_4 - m_5v_5$$

Internal Forces

Newton's laws tell us that when two billiard balls (or two atoms) collide, they exert forces on each other. If we call F_{12} the force that the first ball exerts on the

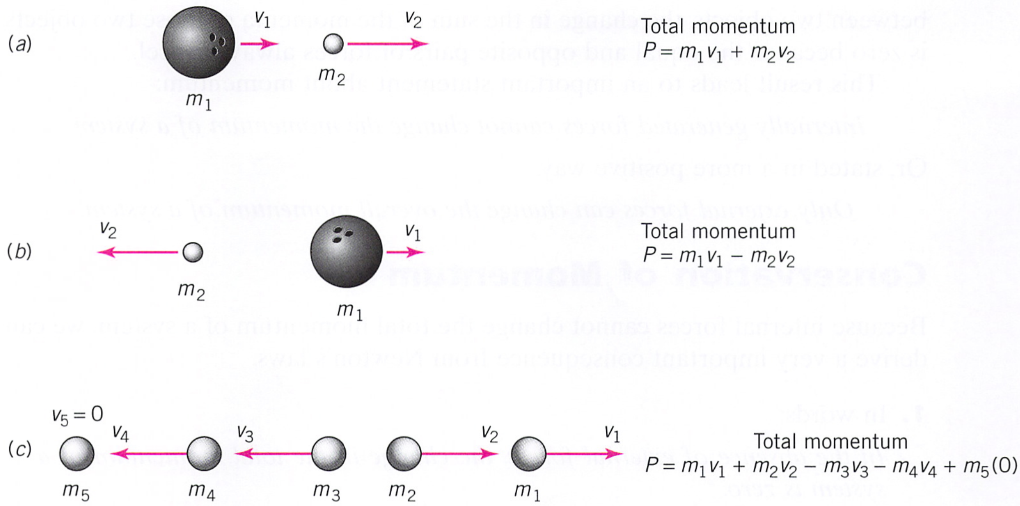


FIGURE 6-3. Balls of various masses move along the same line. When two balls are moving to the right (*a*), the total momentum is the sum of the individual momenta and is directed to the right. When all of the balls are not moving in the same direction (*b, c*), the total momentum is calculated by taking the difference between the momenta of the right-moving balls and of the left-moving balls.

second and F_{21} the force that the second ball exerts on the first, then Newton's third law tells us that

$$F_{12} = -F_{21}$$

We say that these kinds of forces are internally generated in the two-ball system. When we add up all the forces on the system, they cancel each other out. Thus, although each of the two balls feels an unbalanced force (and therefore accelerates), the entire system feels no net force.

Let's see how this works out in a simple example (Figure 6-4). Suppose we have two balls of mass m rolling toward each other with speeds v_1 and v_2. The impulse equation for the first ball says that

$$\Delta(mv_1) = F_{21}\Delta t$$

The equation for the second ball is

$$\Delta(mv_2) = F_{12}\Delta t$$

If we add these two equations together, we find that

$$\Delta(mv_1 + mv_2) = (F_{12} + F_{21})\,\Delta t = 0$$

The fact that the sum of the two forces is equal to zero follows from Newton's third law: these two forces must be equal and opposite.

A little thought should convince you that this result always holds no matter how many billiard balls or atoms are in a system. Whenever there is a collision

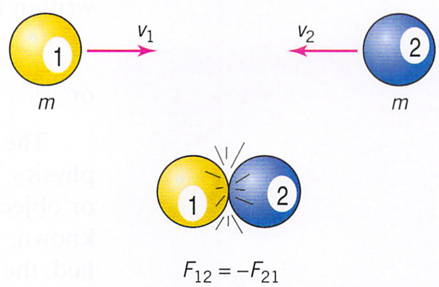


FIGURE 6-4. Two balls of mass m roll toward each other with speeds v_1 and v_2. The internal forces of the collision sum to zero.

between two objects, the change in the sum of the momenta of those two objects is zero because the equal and opposite pairs of forces always cancel.

This result leads to an important statement about momentum:

Internally generated forces cannot change the momentum of a system.

Or, stated in a more positive way,

Only external forces can change the overall momentum of a system.

Conservation of Momentum

Because internal forces cannot change the total momentum of a system, we can derive a very important consequence from Newton's laws.

1. In words:

In the absence of external forces, the change in the total momentum of a system is zero.

2. In an equation with words:

The change in momentum of an isolated system equals zero.

3. In an equation with symbols:

$$\Delta P = 0$$

When physicists find a quantity that does not change during an interaction, they say that the quantity is "conserved." The conclusion we have just reached, therefore, is called the **law of conservation of momentum** and is stated as

If no external forces act on a system,
then the total momentum of that system remains the same.

In most practical situations, the law of conservation of momentum can also be written as

$$\text{Initial momentum} = \text{Final momentum}$$

or

$$P_\mathrm{i} = P_\mathrm{f}$$

The law of conservation of momentum is of fundamental importance in physics. The outcome of almost every interaction between two or more particles or objects is determined in part by the conservation of momentum. As far as is known, whenever the conditions for momentum conservation have been satisfied, the law has never been violated.

THE NATURE OF CONSERVATION LAWS

Physicists have found several **conservation laws**—statements that a quantity is constant in nature—in addition to conservation of momentum. For example, we study conservation of angular momentum, energy, and electric charge in later chapters. Conservation laws are different in character from Newton's laws of motion and gravity, but they are just as fundamental, useful, and important. For example, the impulse–momentum form of Newton's second law says that if you apply a net force over an interval of time (an impulse), you cause a change in an object's momentum. Conservation of momentum says that if you don't apply an external force to a system, the total momentum of the system doesn't change—

momentum is conserved. That might sound like another way of saying the same thing, but there are important differences. In particular, the impulse/momentum form of Newton's second law says that some quantities change in an interaction; these changes are sometimes hard to measure. The principle of conservation of momentum identifies quantities that don't change in an interaction; these quantitites are easy to measure.

To understand better what we can learn from a conservation law, imagine you are shooting a game of pool with a friend. You start out with 15 colored balls and a cue ball and you take turns hitting the cue ball into the other balls, trying to knock them into one of the six pockets. We impose a law of conservation of pool balls: no balls may be created or destroyed, but all 16 balls must stay on the table or in its pockets at all times.

After playing for 15 minutes, you count up the number of balls on the table and see that there are 5 balls plus the cue ball. You can't see any other balls, but you know there are 10 balls in the table pockets. Why? Because conservation of pool balls states 15 colored balls must be in the system at all times. You don't need to check each pocket on the table and count up the balls as long as you know the conservation law applies. The conservation law does not tell you the details of the game, such as the great shot you hit that sank 2 balls at once, but the law does keep track of the general state of the system.

Conservation laws can provide useful information about a system. Near the end of the game, your friend lines up a shot with only two colored balls left on the table (Figure 6-5*a*). You turn away for a minute and hear the loud smack of

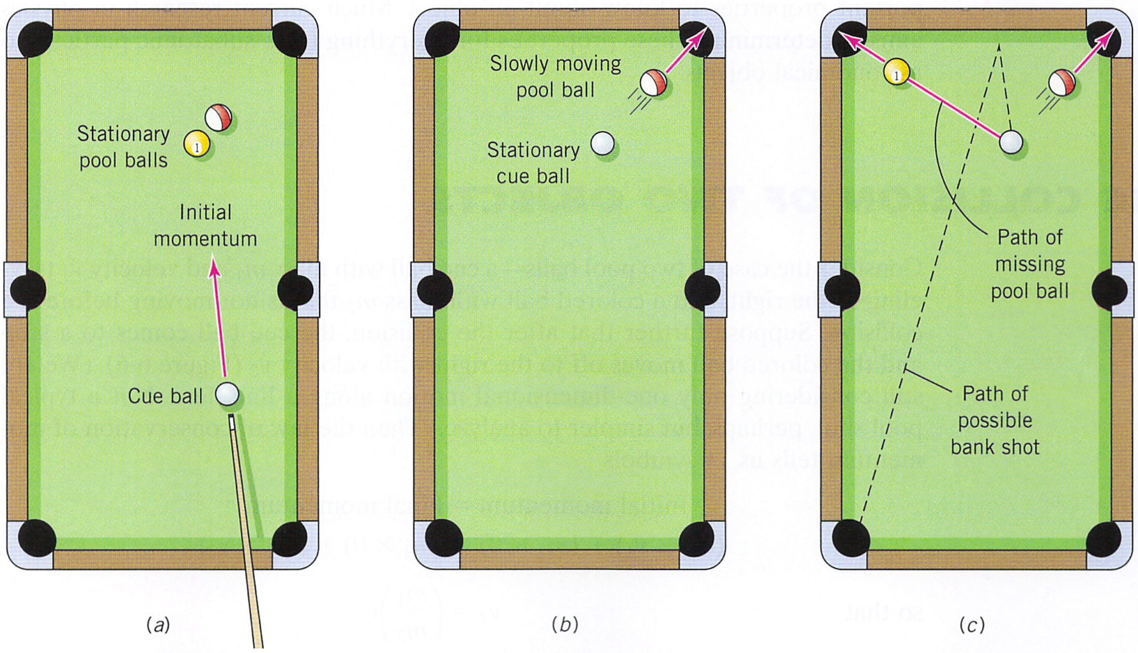


FIGURE 6-5. (*a*) The cue ball and two colored balls remain on a pool table. Your friend is about to shoot when you look away. (*b*) You hear the collision of the cue ball and look back to see the nearly stationary cue ball, and one colored ball moving slowly to the right. What happened to the other ball? (*c*) Conservation of colored balls tells you that the missing ball must be in one of the pockets, while conservation of momentum suggests that the ball was hit into the far left corner pocket, assuming that your friend didn't execute a difficult bank shot (dotted line) into the near left corner pocket!

the cue ball hitting the colored balls. When you turn back to the table, the cue ball is almost motionless on the table and one colored ball is rolling slowly toward the far right corner pocket (Figure 6-5*b*). What happened to the other ball? You can't see it, but you know from conservation of pool balls that since it's not on the table, it must be in one of the pockets. But which one? Here conservation of momentum applies. Once your friend hit the cue ball, no forces were exerted on the system of three balls. The cue ball started out fast, with a significant amount of momentum. The conservation law says that this momentum is still in the system: it can't be created or destroyed. So the missing ball must have gained momentum after being hit by the cue ball and must be in the far left corner pocket (Figure 6-5*c*). We examine the details of this transfer of momentum a little later in this chapter, but you don't have to calculate impulse or change of momentum to determine where the ball went. That's the beauty of conservation laws: you obtain information just from knowing that some quantity before an interaction is unchanged after the interaction.

Physicists often rely on conservation laws to determine the behavior of objects they can't observe directly. For example, they don't have to actually see atomic-scale particles to calculate forces and accelerations before and after a collision. Conservation laws apply to those particles, so we can still calculate information about them and about the interaction. It's not magic and it's not guesswork, it's simply applying known laws to a given situation.

Not all quantities in physics are conserved. There is no conservation of force or conservation of velocity, for instance. But those quantities that are conserved, such as momentum, angular momentum, energy, and electrical charge, are important properties to know about an object. Much current research in physics aims at determining these properties for everything from subatomic particles to astronomical objects.

THE COLLISION OF TWO OBJECTS

Consider the case of two pool balls—a cue ball with mass m_1 and velocity v_1 traveling to the right, and a colored ball with mass m_2 that is not moving before the collision. Suppose further that after the collision, the cue ball comes to a stop and the colored ball moves off to the right with velocity v_2 (Figure 6-6). (We are still considering only one-dimensional motion along a line here. Not a typical pool shot, perhaps, but simpler to analyze.) Then the law of conservation of momentum tells us, in symbols

$$\text{Initial momentum} = \text{Final momentum}$$

$$(m_1 \times v_1) + (m_2 \times 0) = (m_1 \times 0) + (m_2 \times v_2)$$

so that

$$v_2 = \left(\frac{m_1}{m_2}\right)v_1$$

This result matches our intuition, because if the masses of the two balls are equal, then the colored ball moves to the right with exactly the same velocity as the cue ball had before the collision.

We can gain more insight into the meaning of momentum by considering another example of a collision (Figure 6-7). Suppose that a speeding bullet (mass m_1, velocity v_1) and a powerful locomotive (mass m_2, velocity v_2 in the opposite

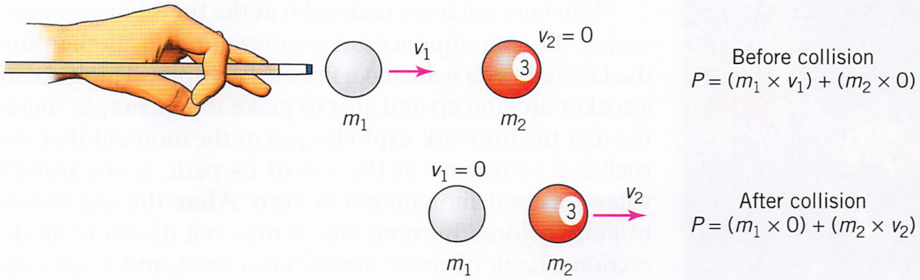


FIGURE 6-6. A cue ball with mass m_1 and velocity v_1 collides with a colored ball with mass m_2 that is not moving before the collision. Momentum is conserved.

direction) are approaching each other. The total momentum P of the system before impact is just

$$P = (m_1 \times v_1) - (m_2 \times v_2) = \text{Total momentum before collision}$$

The minus sign in this expression represents the fact that the bullet and the train are moving in opposite directions before the collision. Suppose, for the sake of argument, that after the collision the bullet recoils and is moving in the same direction as the train, but with velocity u_1, while the train keeps moving forward with velocity u_2 (where u_2 is not the same as u_1). The new total momentum is

$$P = (m_1 \times u_1) + (m_2 \times u_2) = \text{Total momentum after collision}$$

Conservation of momentum tells us that the total momenta before and after the collision have to be the same. Since the momentum of the bullet is reversed, the momentum of the train has to decrease and the train has to slow down (albeit by a very small amount).

You can, in fact, think of the collision as a transfer of momentum from the train to the bullet, with the train losing positive (say, rightward-directed) momentum and the bullet gaining this positive momentum as it reverses direction. In this case, the train with its larger mass has a momentum much larger than the bullet has, so it does not slow down very much. On the other hand, even a small amount of the train's momentum, when transferred to the bullet, produces a very large change in that object's momentum (in this case, it reverses the bullet's direction).

It's important to keep in mind that the law of conservation of momentum doesn't say that momentum can never change. The law just says that momentum won't change unless an outside force is applied. If a soccer ball is rolling across a field and a player kicks it, a force is applied to the ball as soon as the player's foot touches it. At that instant, the momentum of the ball changes, and that change is reflected in its change of direction and speed.

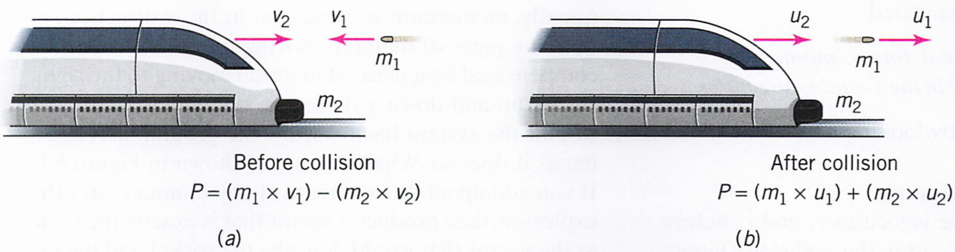


FIGURE 6-7. (*a*) A bullet (mass m_1, velocity v_1) collides with a powerful locomotive (mass m_2, velocity v_2 in the opposite direction). (*b*) The train slows down a little bit, but the bullet completely reverses direction; however, total momentum is conserved.

The symmetrical patterns of exploding fireworks demonstrate conservation of momentum.

You may not have realized it at the time, but you were seeing the consequences of conservation of momentum the last time you watched a fireworks display. Think about a rocket arching up and, just to make things simple, imagine that the firework explodes just at the moment that the rocket is stationary at the top of its path, at the instant when its total momentum is zero. After the explosion, brightly colored burning bits of material fly out in all directions. Each of these pieces has a mass and a velocity, so each piece has some momentum. Conservation of momentum, however, tells us that when we add up all the momenta of the pieces right after the explosion, they should cancel each other out and give a total momentum of zero. For example, if there is a 1-gram piece moving to the right at 10 meters per second, there has to be the equivalent of a 1-gram piece moving to the left at the same velocity. Thus conservation of momentum gives fireworks their characteristic symmetrical starburst pattern.

LOOKING DEEPER

Collisions in Two Dimensions

Up to this point we have been talking about situations in which colliding objects move in one dimension only. However, conservation of momentum works in more complicated cases as well. For example, consider two billiard balls that make a glancing collision as shown (Figure 6-8). We can analyze this situation by recalling that momentum, like velocity, is actually a vector quantity.

When the two billiard balls approach each other at an oblique angle, their momenta can be represented by vectors, as shown in Figure 6-8*a*. As we discussed in Chapter 2, each of these vectors can be thought of as the sum of two components, one in the x-direction and one in the y-direction. These components are shown in the figure. When momenta are represented in this way, the conservation law can be stated

In the absence of external forces, momentum is conserved independently in the x- and y-directions.

We can examine this idea by looking at two situations.

CASE 1 ■ No external forces.
If the two billiard balls have velocities v_1 and v_2 before the collision and u_1 and u_2 after the collision (Figure 6-8*b*), then the statement tells us that

Momentum in x-direction before collision
= Momentum in x-direction after collision

$$mv_{1x} - mv_{2x} = mu_{1x} - mu_{2x}$$

and

Momentum in y-direction before collision
= Momentum in y-direction after collision

$$mv_{1y} - mv_{2y} = -mu_{1y} + mu_{2y}$$

CASE 2 ■ External force in one direction.
Another common situation occurs when an external force such as gravity acts in one direction, but no external force acts in the other. In this case, momentum is conserved in the direction in which no force acts, and Newton's second law describes the change in momentum in the other direction.

In the example of the exploding fireworks display, gravity is acting in the vertical y-direction and there is no external force in the horizontal x-direction. Consequently, momentum is conserved in the x-direction, so that any piece of material moving to the left has to be compensated by a piece of material moving to the right. In the up-and-down y-direction, however, gravity acts to pull the system back toward the ground, accelerating as it does so. What happens is shown in Figure 6-9. If you add up all the momenta at any moment after the explosion, they produce a vector that is exactly the same as the vector that would describe the rocket had the explosion never occurred.

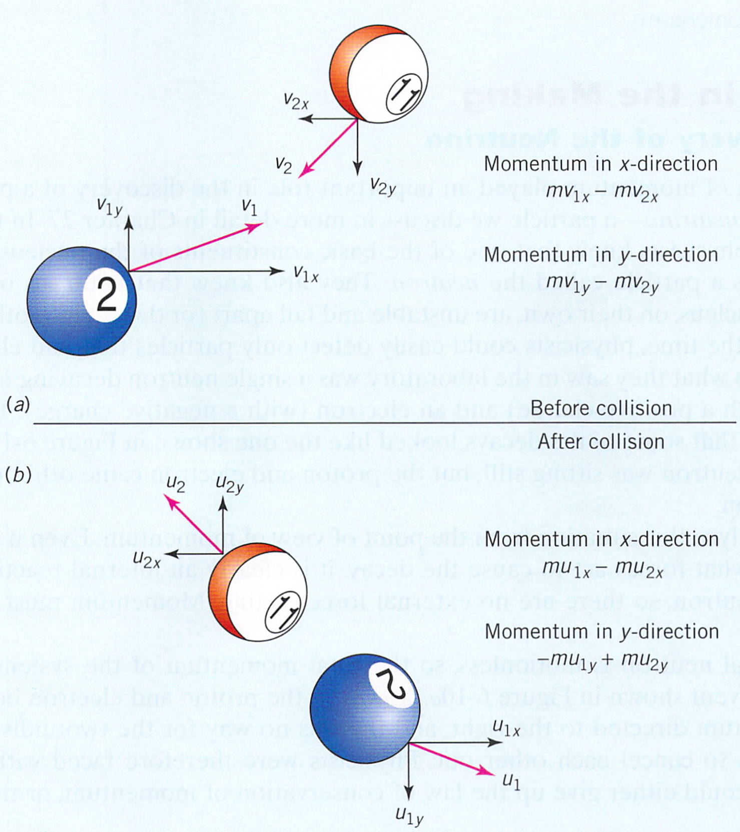


FIGURE 6-8. Two billiard balls make a glancing collision. Their momenta can be represented by vectors. Momentum is conserved in the x-direction and the y-direction separately.

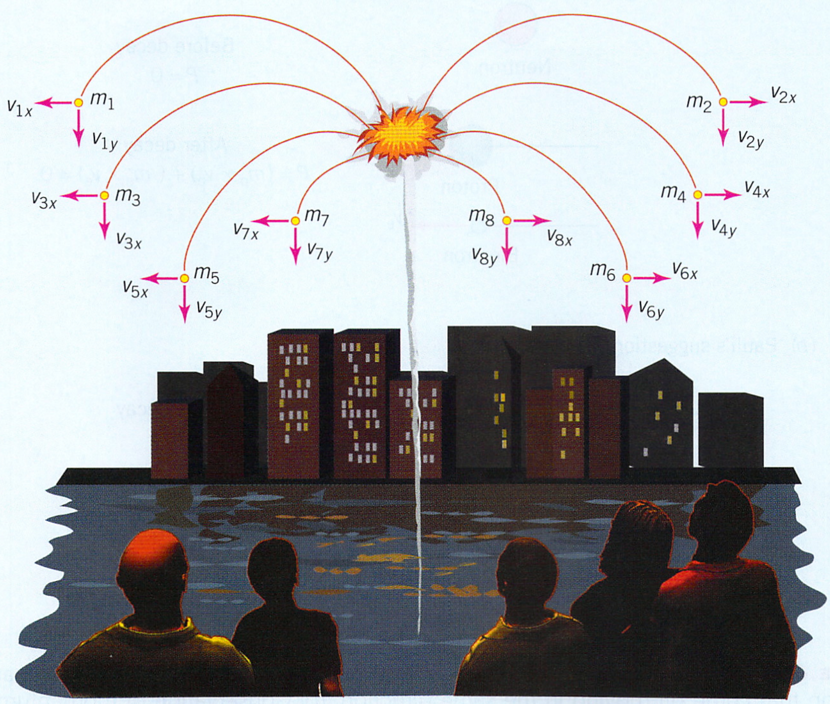


FIGURE 6-9. If you add up all the momenta of individual fragments at any moment after a fireworks explosion, they produce a vector that is exactly the same as the vector that would describe the momentum of the rocket had the explosion never occurred.

Physics in the Making

The Discovery of the Neutrino

Conservation of momentum played an important role in the discovery of a particle called a *neutrino*—a particle we discuss in more detail in Chapter 27. In the early 1930s, physicists knew that one of the basic constituents of the nucleus of the atom was a particle called the *neutron*. They also knew that neutrons outside of the nucleus, on their own, are unstable and fall apart (or decay) into other particles. At the time, physicists could easily detect only particles that had electric charge, so what they saw in the laboratory was a single neutron decaying into a proton (with a positive charge) and an electron (with a negative charge). The problem was that some of the decays looked like the one shown in Figure 6-10*a:* initially the neutron was sitting still, but the proton and electron came off in the same direction.

Let's analyze this situation from the point of view of momentum. Even if we don't know what forces act to cause the decay, it is clearly an internal reaction within the neutron, so there are no external forces acting. Momentum must be conserved.

The initial neutron is motionless, so the total momentum of the system is zero. In the event shown in Figure 6-10*a,* however, the proton and electron both have momentum directed to the right, and there is no way for the two individual momenta to cancel each other out. Physicists were therefore faced with a choice: they could either give up the law of conservation of momentum, or they

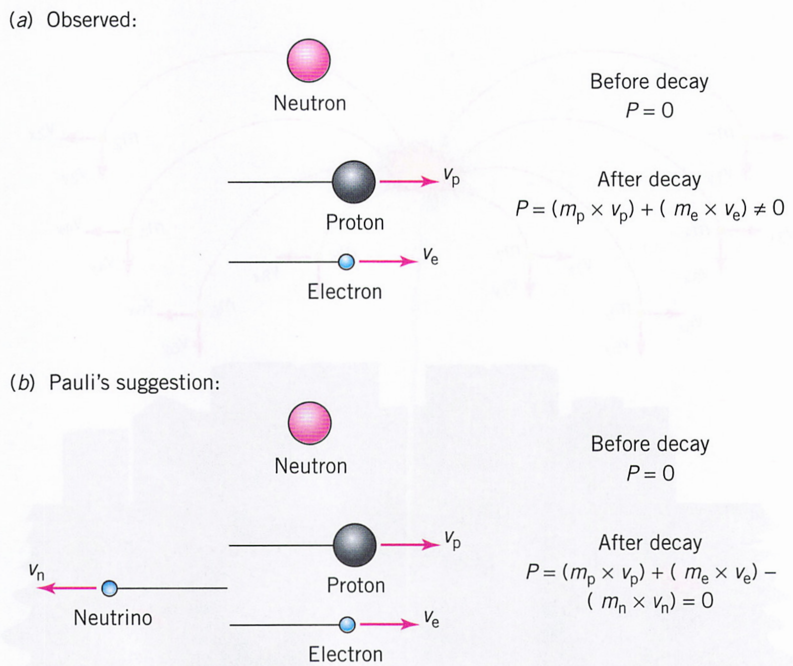
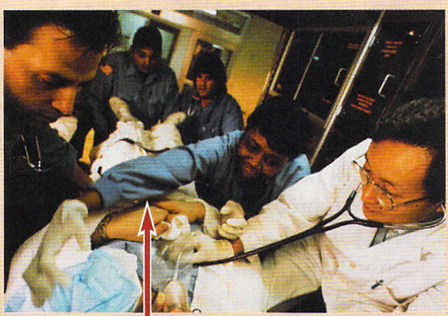


FIGURE 6-10. (*a*) The decay of a stationary neutron occasionally displays a proton and electron that come off moving in the same direction. (*b*) Conservation of momentum requires that another particle must balance the momentum of the proton and electron.

Physics and Daily Life–Momentum

Momentum depends on mass and velocity, so any time something moves, it has momentum. When you hit a volleyball, you give it momentum; when you walk across a lawn you have momentum—even moving your arm gives it momentum.



could assume that another particle was emitted in the reaction, a particle they could not detect, which was carrying momentum directed to the left. With some reluctance, the German physicist Wolfgang Pauli took the latter route. He suggested that there was a particle (eventually called the neutrino, or "little neutral one") that, because it had no electrical charge, was invisible to experimenters, but that could balance the momentum of the electron and proton, as shown in Figure 6-10*b*. The hypothetical particle also solved other problems that had been encountered by experimenters.

For several decades, then, physicists accepted the existence of the undetectable neutrino because it allowed them to hold onto central laws of nature such as the conservation of momentum. When the neutrino was finally detected in 1956, this faith in the laws of nature was amply rewarded. ●

Thinking More About

Momentum: Why Isaac Newton Would Wear His Seat Belt

One of the authors (JT) knows a highway patrolman who makes a point about auto safety by saying, "I never pulled a dead man from behind a seat belt." Although some people do indeed die in car crashes while wearing seat belts, there is no question that seat belts greatly improve your chances of walking away from a crash. Given what you know about momentum and impulse, why should this be so?

Look at it this way: when you are sitting in a car, your momentum is your mass multiplied by the speed of the car. Call this quantity *P*. In a crash, the speed of the car—and of you—is rapidly reduced to zero. Therefore, the change in your momentum during the crash is *P*—your momentum before the crash minus your momentum after the crash. The impulse–momentum relationship tells us that a force must act to supply the impulse needed to bring about this change.

If you're not wearing a seat belt, you keep moving forward when the car stops, and your motion stops when your head hits the steering wheel or the windshield. Because these surfaces are hard, the time of the impact is short—a fraction of a second. Consequently, the force needed to stop you must be large. In addition, this force is applied to a small area of your skull, greatly increasing the pressure. Such a large focused force can be deadly.

If you're wearing a flexible seat belt, however, a smaller force is applied over a longer time interval to produce the same change in momentum. What's more, the broad, flat seat belt distributes that force over a much larger area of your body. The chances of injury are greatly reduced. This is one reason why seat belts save lives.

Given the physics of this situation, should states require that drivers and passengers wear seat belts? Is such legislation an intrusion on personal freedom? In what other ways do scientific principles influence our laws?

Other examples of momentum around us appear in Physics and Daily Life on page 131.

Summary

The **momentum** of an object is defined as the product of that object's mass and its velocity. Like velocity, momentum is a vector. In terms of momentum, Newton's second law says that the rate of change in the momentum of a system is equal to the net force.

Impulse is defined as the product of the net force and the time interval over which that force acts. Another way of stating Newton's second law, called the **impulse–momentum relationship,** is to say that the change in momentum of a system is equal to the impulse applied to it.

The **total momentum** of a system with several objects is defined as the sum of the momenta of all of those objects. Only external forces applied to a system can change its total momentum. Internal forces may change the momentum of one part of a system, but these changes always cancel out when they are added up over the entire system.

In the absence of a net external force, the total momentum of a system does not change. This statement, known as the **law of conservation of momentum,** is one example of a **conservation law.**

Key Terms

conservation law Statement that a quantity is constant in nature. (p. 124)

impulse The product of a force multiplied by the time over which it acts. (p. 121)

impulse–momentum relationship Restatement of Newton's second law, in which impulse equals the change in momentum. (p. 121)

law of conservation of momentum Statement that if no external forces act on a system, then the total momentum of that system remains the same. (p. 124)

linear momentum Another term used for *momentum* (the product of mass times velocity) when the object is understood to move in a straight line. (p. 118)

momentum The product of an object's mass and velocity. (p. 118)

total momentum The sum of the momenta of all the objects in a system. (p. 122)

Key Equations

Momentum = mass × velocity

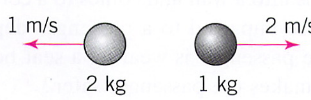
Impulse = change in momentum
= net force × time (force is applied)

Review

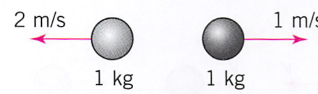
1. What is momentum?
2. How is inertia related to linear momentum? Consider Newton's laws.
3. What does it mean to say that momentum is a vector quantity? Give an example of this.
4. What is an impulse? How is it derived from Newton's second law $F = ma$?
5. Which produces the greater impulse, a large force acting for a short time or a small force acting over a long time? Explain.
6. What is meant by the total momentum of a system? Give an example.
7. What effect do internally generated forces have on the total momentum of a system? Explain.
8. When adding up a system's total momentum, do we need to account for the direction of the individual particles that comprise the system? How so?
9. What do scientists mean when they say something is conserved?
10. What is the conservation of momentum?
11. Identify a physical quantity that is not conserved.
12. What happens when an external force is applied to a system? Is momentum conserved?
13. What is meant by a collision in two dimensions? Give an example.
14. In a two-dimensional collision, is momentum in the x-dimension conserved? How about the y-dimension?

Questions

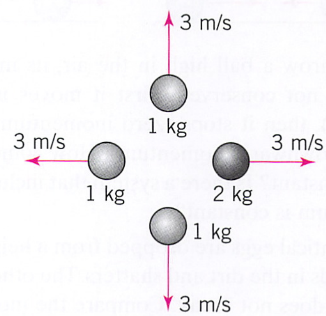
1. Can a system of multiple objects moving in different directions have a total momentum of zero? How can this be?
2. Which has more momentum, an 18-wheel truck that is parked on a street or a mosquito buzzing around your ears?
3. A large truck that is moving to the right collides head-on with a stationary compact car. Which vehicle (if either) exerts the larger force on the other? Which force imparts a greater impulse? Assuming the collision forces are the only forces acting on the vehicles during the collision, which vehicle's momentum changes more? Compare the direction of the momentum change of the truck to the momentum change of the car.
4. Two balls are moving in opposite directions as shown in the figure. What is the direction of the total momentum of the system?



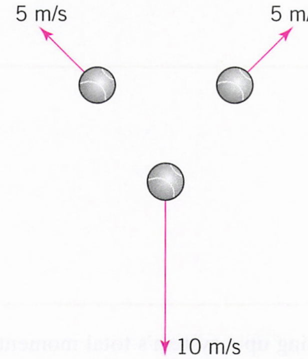
5. Two balls are moving in opposite directions as shown in the figure. What is the direction of the total momentum of the system?



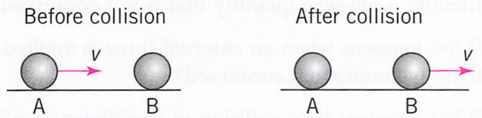
6. What is the direction of the total momentum of the system of objects shown in the figure? Explain your answer.



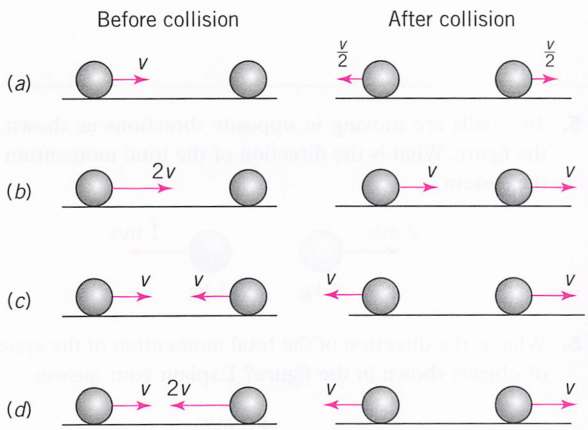
7. What is the direction of the total momentum of the system of identical tennis balls shown in the figure? Explain your answer.



8. Two identical billiard balls, A and B, collide head-on. Before the collision, A is moving and B is stationary. After the collision, A is stationary and B is moving. Compare the speed of A before the collision to the speed of B after the collision. Explain how you reach your conclusion.



9. Which of the collisions in the figure are possible and which are impossible? The objects have identical masses. (*Hint:* Which collisions violate conservation of momentum?)



10. If you throw a ball high in the air, its momentum is apparently not conserved. First it moves up (upward momentum), then it stops (zero momentum), then it moves down (downward momentum). How come its momentum is not constant? Is there a system that includes a ball whose momentum is constant?

11. Two identical eggs are dropped from a height of 10 meters. One lands in the dirt and shatters. The other lands on a pillow and does not break. Compare the momentum change of each egg after each lands. In terms of collision time and forces, why does one egg break and the other does not?

12. Several years ago in New Orleans, a large ocean liner was unable to stop in time and crashed into a riverwalk dock, causing immense damage. Explain why a ship of such size would have such difficulty stopping, and describe the collision with the pier in terms of an impulse.

13. What are some reasons that you frequently see truck turnouts on mountain highways? (A truck turnout is a level or upward-sloping area alongside a steep downward highway where a truck can come to a stop and let its brakes cool.) Would truck brakes have more of a tendency to fail than passenger car brakes? Why? Consider momentum and Newton's laws.

14. Bungee jumpers use cords that are elastic. Explain in terms of impulse why a metal cord is *never* used in place of the elastic cord, even though it might be less likely to break. Explain this in terms of force, momentum, and impulse.

15. Why are people who have to jump from any appreciable height, such as parachuters or stunt people, taught to land with their knees bent and to roll on impact?

16. What is the purpose of a good 'follow-through' when swinging a golf club or a baseball bat?

17. Explain the theory behind air bags in cars. What are the advantages and disadvantages of their use? Why are airbags on occasion lethal to the occupant?

18. Explain to yourself in some detail just how conservation of momentum may be used to understand how a rocket moves. Can the motion of a rocket be completely explained by this conservation law? Why?

19. Conservation of momentum is really a consequence of Newton's second law. Explain the connection.

20. When new comets enter the solar system and move toward the Sun, they often display erratic motion, moving first one way and then another as large chunks of material evaporate in the heat. Use the concept of momentum to explain this behavior.

21. Modern fireworks displays include dramatic explosions in which flares perform spiral, twisting, or wiggling motions. How might these distinctive motions be produced?

22. While playing a game of pool, you line up a shot, strike the cue ball, and watch as it knocks the 9 ball into the corner pocket. How was momentum transferred in this situation? What happened to the momentum when the 9 ball entered the pocket? Was momentum conserved? If not, why not?

23. Car accidents are dangerous because the car may experience a very high acceleration (stopping fast in a head-on collision, for example). Consider a specific accident where a car slams into a wall and comes to a complete stop. Does the impulse imparted to a passenger depend on whether or not the passenger is wearing a seat belt? How come a seat belt makes the passenger safer?

Problem-Solving Examples

EXAMPLE 6-1

Large and Small Momenta

Compare the momentum of a baseball (mass 0.3 kg) with the momentum of a blue whale (mass 150,000 kg). Suppose the baseball is moving to the right at the speed of a good fastball (30 m/s). How fast would the whale have to be going to have the same momentum as the baseball?

SOLUTION: Momentum is defined as

$$p = m \times v$$

If the whale were swimming at the same speed as the baseball is thrown, the whale would have much more momentum. However, an object of large mass moving slowly can have the same momentum as an object of small mass moving quickly. How slowly would the whale have to move? Let's take a look at the numbers.

We can first find the momentum of the baseball by substituting the numbers for mass and velocity.

$$\begin{aligned} p &= m \times v \\ &= 0.3 \text{ kg} \times 30 \text{ m/s} \\ &= 9 \text{ kg-m/s} \end{aligned}$$

We are told that this is equal to the momentum of the whale. We also know the whale's mass and we want to find its velocity. We can rearrange the definition of momentum to solve for the speed v.

$$p = mv$$

$$v = \frac{P}{m}$$

$$= \frac{9 \text{ kg-m/s}}{1.5 \times 10^6 \text{ kg}} = 0.000006 \text{ m/s}$$

This is a *very* slow speed. It would take almost 50 hours for the whale to move 1 meter. ●

EXAMPLE 6-2

A Long Drive

What is the total impulse imparted to a 10-g golf ball that flies off the tee at 40 m/s (about 90 miles per hour)? Strobe photographs have indicated that in a golf drive, the club stays in contact with the ball for only about 1 millisecond (= 0.001 s). Assuming this time interval occurs for the given golf drive, how much force does the golfer exert?

REASONING: First we find the impulse by the impulse–momentum relationship. Then we use the results and the given time interval to calculate the force from the definition of impulse.

SOLUTION: By the impulse–momentum relationship, impulse equals the change in momentum, $\Delta(mv)$. In a golf drive the mass of the golf ball doesn't change, but its velocity increases, in this case from 0 to 40 m/s. The initial momentum of the golf ball is 0, because it is at rest on the tee. Therefore, the change in momentum is equal to the final momentum.

$$\text{Impulse} = \text{Change in momentum}$$

$$\begin{aligned} \Delta(mv) &= m \times \Delta v \\ &= 10 \text{ g} \times 40 \text{ m/s} \\ &= 0.4 \text{ kg-m/s} \end{aligned}$$

Once we know the impulse, we can use its definition to determine the net force applied to the ball for the given time interval.

$$\text{Impulse} = F \times \Delta t$$

$$F = \frac{\text{Impulse}}{\Delta t}$$

$$= \frac{0.4 \text{ kg-m/s}}{0.001 \text{ s}}$$

$$= 400 \text{ kg-m/s}^2 = 400 \text{ N}$$ ●

EXAMPLE 6-3

Nudging a Cruise Ship

1. What is the impulse imparted to a cruise ship with mass 10^7 kg and velocity 5 m/s that is brought to rest by a tugboat?

SOLUTION: Here the final momentum is 0, because the ship is brought to rest. The change in momentum thus equals the initial momentum.

$$\begin{aligned} m \times v &= 10^7 \text{ kg} \times 5 \text{ m/s} \\ &= 5 \times 10^7 \text{ kg-m/s} \end{aligned}$$

2. If it takes 10 minutes to stop the ship, what is the average force exerted by the tugboat?

SOLUTION: First we have to convert units from minutes to seconds.

$$10 \text{ minutes} = 600 \text{ seconds}$$

From the impulse–momentum relationship, the change in momentum is equal to the impulse. This means that

$$\text{Impulse} = F \times \Delta t = \text{change in momentum}$$

$$F \times 600 \text{ s} = 5 \times 10^7 \text{ kg-m/s}$$

or

$$\begin{aligned} F &= \frac{5 \times 10^7 \text{ kg-m/s}}{600 \text{ s}} \\ &= 8.3 \times 10^5 \text{ kg-m/s}^2 \\ &= 8.3 \times 10^5 \text{ N} \end{aligned}$$

This force is modest compared to the weight of the ship. It corresponds to the weight of an object with a mass of about 80,000 kg, which is less than 1% of the ship's mass. ●

Problems

1. Calculate the momenta of the following:

a. A 200-gram rifle bullet traveling 300 m/s.
b. A 1000-kg automobile traveling 0.1 m/s (a few miles per hour).
c. A 70-kg person running10 m/s (a fast sprint).
d. A 10,000-kg truck traveling 0.01 m/s (a slow roll).

2. Allison (30 kg) is coasting in her wagon (10 kg) at a constant velocity of 5 m/s. She passes her mother, who drops a bag of toys (5 kg) into the wagon.

a. Do you expect the wagon to speed up or slow down? Why?
b. What is the initial momentum of Allison and the wagon before her mother drops the toys in?
c. What is the final momentum of Allison, the wagon, and the toys?
d. What is the final speed of the wagon after Allison's mother drops the toys in?

3. What is the total momentum of each of the following systems?

a. Two 1-kg balls move away from each other; one travels 5 m/s to the right, the other 5 m/s to the left.
b. Two balls move away from each other, both traveling at 7 meters per second. One has a mass of 2 kg and the other has a mass of 3 kg.
c. Two 1000-kg cars drive east; the first moves at 20 m/s, the second at 40 m/s.
d. One of the 1000-kg cars moves west at 40 m/s, while the second moves east from the same starting point at a constant velocity of 30 m/s.

4. A 20-metric-ton train moves south at 50 m/s.

a. At what speed must it travel to have twice its original momentum?
b. At what speed must it travel to have a momentum of 500,000 kg-m/s?
c. If there were a speed limit for this train as it traveled through a city, but not a weight limit, what mass in kilograms must be added to the train to slow it down to 20 m/s, while at the same time keeping the momentum the same as in part b?

5. A. Calculate the impulse imparted to the object in the following collisions.

a. A 0.5-kg hockey puck moving at 35 m/s hits a straw bale, stopping in 1 second.
b. A T-ball with a mass of 0.2 kg travels in the air at 15 m/s until it is stopped in the glove of a shortstop over a period of 0.1 seconds.
c. A 12,000-kg tank moving at 4 m/s is brought to a halt in 2 seconds by a reinforced-steel tank barrier.

B. What is the average net force exerted by these objects on the objects they collide with?

C. Which is more important in determining the amount of damage an object sustains in a collision: the total momentum change (impulse) or the momentum change per unit time? Is the total area over which this force is applied important in determining how much damage is done?

6. A racing car with a mass of 1400 kg hits a slick spot and crashes head-on into a concrete wall at 90 km/hour, coming to a halt in 0.8 s. An ambulance weighing 3000 kg comes racing to the rescue, hits the same slick spot, and then collides with a padded part of the wall at 80 km/hr, coming to a halt in 2 seconds.

a. What is the impulse exerted on each vehicle?
b. What was the force exerted by each vehicle on the wall? What was the force exerted by the wall on each of the vehicles?
c. What was the deceleration of each vehicle, from the time it contacted the wall to the time it completely stopped?

7. In Problem 6 in Chapter 4, you were asked to solve the following problem using the knowledge of Newton's laws that you had accumulated up to that point.

Margie (45 kg) and Bill (65 kg), both with brand new roller blades, are at rest facing each other in the parking lot. They push off each other and move in opposite directions, Margie moving at a constant speed of 14 ft/s. At what speed is Bill moving?

a. Use what you have now learned about momentum to answer this problem in a different way.
b. Which method was easier for you to use to solve this problem, the Chapter 4 method or this one? How do the approaches compare? Are they really that different? Explain.

8. You are ice sailing in a boat on a very large, perfectly flat, piece of the Arctic. All of a sudden the wind dies and you cannot steer, but you have a boatload of frozen oranges that you have brought with you to eat. How can you try to stop yourself before your ice boat, heading for a deep crevice, goes over the edge? Use conservation of momentum to provide a solution.

9. Which object has a greater momentum, a 0.1-kg bullet traveling at 300 m/s or a 3000-kg truck moving at 0.01 m/s?

10. What is the total momentum of a two-particle system composed of a 1000-kg car moving east at 50 m/s and a second 1000-kg car moving west at 25 m/s?

11. Tony (60 kg) coasts on his bicycle (10 kg) at a constant speed of 5 m/s, carrying a 5-kg pack. Tony throws his pack forward, in the direction of his motion, at 5m/s relative to the speed of the bicycle just before the throw.
    a. What is the initial momentum of the system (Tony, the bicycle, and the pack)?
    b. What is the final momentum of the system immediately after the pack leaves Tony's hand?
    c. Is there a change in the speed of Tony's bicycle? If so, what is the new speed?

Investigations

1. The next time you are at an amusement park, go to the bumper cars. Observe what occurs in various collisions. What happens when a car occupied by a large heavy person hits a car with a very small person inside? What happens when cars with people of equal mass collide? Also, look at the effects of the angles of the collisions. Are your observations consistent with what you learned in this chapter?
2. Go to a pool hall or to a friend's house where there is a pool table. Set up different shots and observe whether momentum is conserved in these collisions. Ignore the various spins that may be applied to these balls as you try to further understand the nature of linear momentum and collisions. (The effect of spin is covered in the next chapter.)
3. Look up the masses and normal speeds for the balls used in several of your favorite sports. How much momentum do the balls typically have? What kind of impulses result when they are hit by or in turn collide with different solid objects such as bats, clubs, or human flesh?
4. Using the balls you studied in 3 above, design an experiment to measure the precise momentum of the different balls, along with the impulses generated in their normal use. Detail the methods that you could use to do this.
5. Research the history of weaponry. Examine the role momentum and impulse have played in the design and development of military machines and weaponry. Examine such objects as catapults, battering rams, cannonballs, tanks, and any others that you can think of. Has momentum been more important at different times? What types of trade-offs were involved in the design and manufacture of these machines?
6. To some physicists, particularly in the nineteenth century, conservation laws have represented more than just useful descriptions of nature. These scientists have seen a profound esthetic beauty and mathematical simplicity in such statements. James Prescott Joule, who helped to establish the law of conservation of energy (see Chapter 12), said, "Nothing is destroyed, nothing is ever lost, but the entire machinery, complicated as it is, works smoothly and harmoniously. . . . Everything may appear complicated in the apparent confusion and intricacy of an almost endless variety of causes, effects, conversions, and arrangements, yet is the most perfect regularity preserved—the whole being governed by the sovereign will of God." Imagine a universe in which momentum is not conserved and describe phenomena that might seem strange or different from our everyday experience. Is such a universe plausible? Is it less esthetic than our own?
7. Numerous serious scientific studies have been devoted to the threat posed to the Earth by collisions with asteroids. (It was just such a collision 65 million years ago that is believed to have caused the extinction of the dinosaurs.) Asteroids can be several kilometers wide traveling at speeds of hundreds of meters per second. Given what you know about momentum, what strategies do you think might be employed to save the Earth if a collision were found to be imminent?

WWW Resources

See the *Physics Matters* home page at **www.wiley.com/college/trefil** for valuable web links.

1. **zebu.uoregon.edu/nsf/mo.html** Animated Java simulation laboratories on the conservation of linear momentum from the Department of Physics, University of Oregon.
2. **www.physicsclassroom.com/mmedia/momentum/cba.html** A tutorial on impulse, momentum, and collisions from physicsclassroom.com including many animated problems and applets.
3. **www.nhtsa.dot.gov/index.html** Home of the US National Highway Traffic Safety Administration containing many resources regarding auto crash safety and testing.