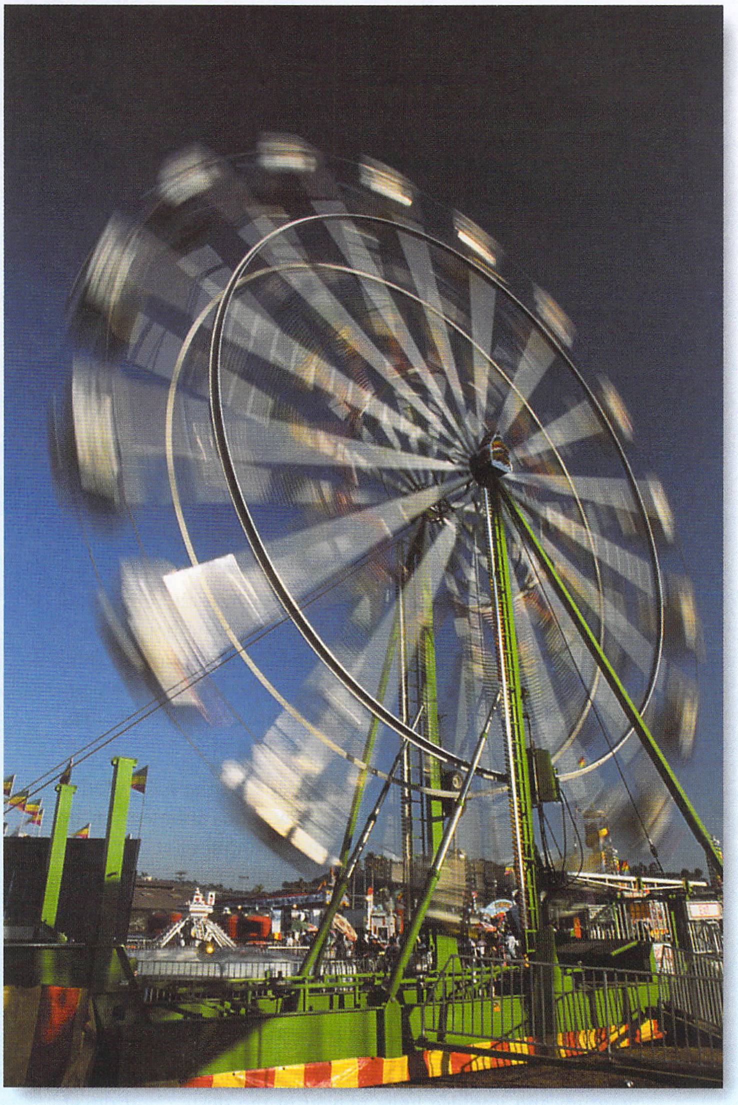
7

Rotational Motion of an Object

KEY IDEA

In the absence of external torques, the angular momentum of any system is constant.



PHYSICS AROUND US . . . Spinning

An ice skater speeds up as she goes into a turn. Suddenly, she swings gracefully into a spin, both arms and one leg extended to the side. As she pulls her arm and leg in closer to her body, the rate of spin increases until, at the end, her features turn into a blur.

You are watching a news program at home when the anchor starts talking to a colleague half a world away. High above the Earth's atmosphere, a satellite relays the signals back and forth between the two reporters. Inside the satellite, meanwhile, a set of small spinning gyroscopes allow the onboard computers to keep track of which way the satellite's antennae are pointing so that the signals are aimed in the right direction.

It's winter. You wrap your coat tightly around you in the blustering wind. You know, however, that in six months you'll be walking around in shorts. You may even complain to a friend about how pleasant it would be if you could average the two seasons out, but you know it will never happen.

Amazingly, all three of these phenomena—skater, satellite, and seasons—are related to a quantity that physicists call *angular momentum*. All rotating objects have angular momentum, so in this chapter we also examine the physics of rotational motion.

ROTATIONAL MOTION

So far we have discussed the motion of an object along a line, both straight lines and curved lines. We have not looked at any motion the object might have with respect to itself, such as spinning, as it moves along a line. However, real objects can spin around as they move, and they demonstrate different properties than objects that don't spin.

Many of the laws of mechanics for objects that move in a line have analogs for objects that spin. We see in this chapter that rotating objects have rotational speed analogous to linear speed and obey a rotational version of Newton's second law. We study rotational analogs for force and momentum as well. Much of the physics in this chapter will seem new and yet somewhat familiar.

All spinning objects display **rotational motion.** We can analyze this motion in terms of two properties: an axis of rotation and a rotational velocity.

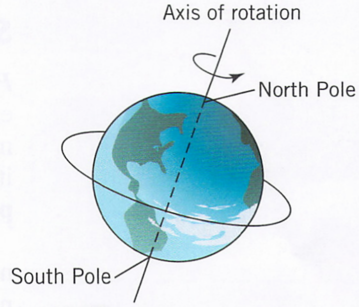


FIGURE 7-1. Each point on an imaginary line that passes through the center of the Earth, connecting the two poles, is stationary with respect to the Earth. Every other point in the planet turns around this axis of rotation.

Axis of Rotation

Imagine the Earth spinning in space. Everything on the surface is moving except for two points—the points we call the North and South Poles. These points remain stationary with respect to the object while everything else turns around them. You can imagine a line through the center of the Earth connecting the two poles (Figure 7-1). Each point on this line is also stationary with respect to the Earth, while every other point in the planet turns around it. This line, around which everything else rotates, is called the **axis of rotation** of the Earth. It is this axis on which an ordinary desktop globe spins.

Every rotating object has an axis of rotation. For example, the wheels of your car rotate around an axis at their center, along the car's axle. The Sombrero galaxy rotates around an axis perpendicular to its plane (Figure 7-2). Of course, your car's wheels turn many times each second, while the outer parts of a galaxy take hundreds of millions of years to make just one rotation. Nevertheless, the basic idea in each situation is the same. An overall rotation occurs around an axis, which itself remains stationary with respect to the object.

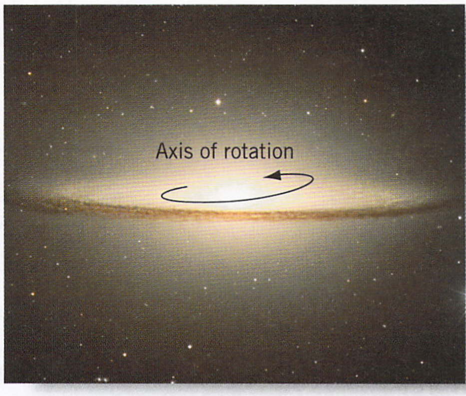


FIGURE 7-2. The Sombrero galaxy is a typical spiral galaxy, much like our own Milky Way, rotating about an axis through its center.

Develop Your Intuition: The Pizza Galaxy

A pizza maker rolls out the dough for the crust, then flips it spinning into the air to thin it out (and to show off) (Figure 7-3). What is the axis of rotation of the pizza dough while it's in the air?

Like the galaxy, the pizza dough spins (roughly) in a flat disk. At the center of the disk, an imaginary vertical line remains stationary with respect to the pizza dough. This line moves up and down with the dough, but no point on the line moves in a circle. This imaginary line through the disk of dough is the axis of rotation. Every rotating object has an axis of rotation, even if the axis is sometimes moving.

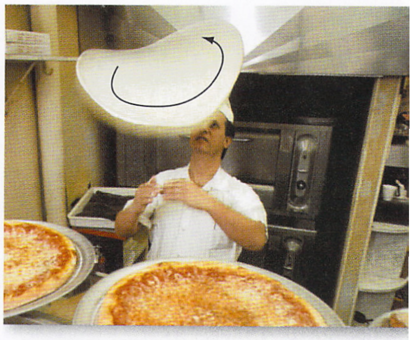


FIGURE 7-3. Pizza dough thins out into a circular disk as it spins, something like how a spiral galaxy forms.

Speed of Rotation

Period and Frequency The Earth spins about its axis of rotation about once every 24 hours. This daily cycle is one of the most familiar rotational motions for many of us. However, *any* rotational motion can be characterized by the time it takes for the object to make one complete rotation. This time is called the **period of rotation,** which is usually represented by T.

Another common way of talking about rotational motion is to specify the number of times an object completes a rotation in a given amount of time—a number called the **frequency of rotation,** which is usually represented by f. The frequency of Earth's rotation, for example, is 1 cycle per 24 hours or about 365 cycles per year.

Frequency can be defined in terms of any time period (a day, a year, etc.), but in many scientific applications frequency is defined as the number of rotations completed in 1 second. In the case of the Earth, only a small fraction of a rotation is completed in 1 second. In other cases (your car's wheels at high speeds, for example), many rotations may be completed in 1 second, so the frequency is some number larger than 1.

Rotational frequency is measured in a unit called the **hertz,** named after the German physicist Heinrich Rudolf Hertz (1857–1894), who discovered radio waves (see Chapter 19). A body that completes one rotation in 1 second has a frequency of 1 hertz (1 Hz).

Develop Your Intuition: How Often Does the Electric Current in Your Hair Dryer Change Direction?

Pick up any electric appliance in your house, such as a toaster or hair dryer, and find where its properties are specified. You will probably see a note that reads "60 Hz." What does this mean?

"Hz" is the abbreviation for hertz. This designation on the appliance means that it is intended for use with electric currents in which the direction of current changes 60 times each second (see Chapter 17), as in the United States. In Europe, the standard current is 50 Hz, which is one reason why it's not always possible to use American appliances in Europe.

Relation Between Period and Frequency Period and frequency are related to each other. An object that spins with a high frequency has a short period, while an object with a low frequency has a long period. An object that has a long rotational period (such as the Earth) completes only a small fraction of a revolution in a second, and hence has a low frequency. In fact, the relationship between the period and the frequency of a rotation is simple to state.

1. In words:

The longer the period, the lower the frequency, and vice versa.

2. In equations with words:

The period is equal to one divided by the frequency.

$$\text{Period} = \frac{1}{\text{Frequency}}$$

and

The frequency is equal to one divided by the period.

$$\text{Frequency} = \frac{1}{\text{Period}}$$

3. In equations with symbols:

$$T = \frac{1}{f} \quad \text{and} \quad f = \frac{1}{T}$$

Our Spinning Planet

EXAMPLE 7-1

What is the frequency of the Earth's rotation in hertz?

SOLUTION: We know that the period of the Earth's rotation is 1 day or 24 hours. We want frequency recorded in units of hertz, or cycles per second, so we first must calculate the number of seconds in 1 day. There are 60 seconds in a minute, 60 minutes in an hour, and 24 hours in a day. Consequently, the number of seconds in 1 day (the period of rotation) is

$$T = 60 \text{ s/min} \times 60 \text{ min/h} \times 24 \text{ h/day}$$
$$= 86{,}400 \text{ s/day}$$

Then, from the relationship between period and frequency,

$$\text{Frequency} = \frac{1}{\text{Period}}$$
$$f = \frac{1}{T} = \frac{1}{86{,}400} \text{ Hz}$$
$$= 1.16 \times 10^{-5} \text{ Hz}$$ ●

Angular Speed Think about children on a moving carousel. The children on the inner horses, closer to the axis of rotation, travel a shorter distance during each rotation than children on the outer horses. Children on the inner horses must move at a slower speed than children on the outer horses. Yet we know that everyone on the carousel makes one circuit in the same amount of time. That shared aspect of rotational motion is called **angular speed.**

Imagine drawing a line that starts at the center of the carousel and passes through two children who are at different distances from the center before the rotation starts (Figure 7-4 on page 142). As the carousel turns, we can characterize the change in position of the children by the angle θ (the Greek letter theta), as shown. The point is that both children have moved through this same angle, even though they have traveled different distances along their respective paths.

1. In words:

Angular speed is the angle through which an object has moved about the axis of rotation divided by the time it takes it to go through that angle.

2. In an equation with words:

$$\text{Angular speed} = \frac{\text{Rotation angle}}{\text{Time}}$$

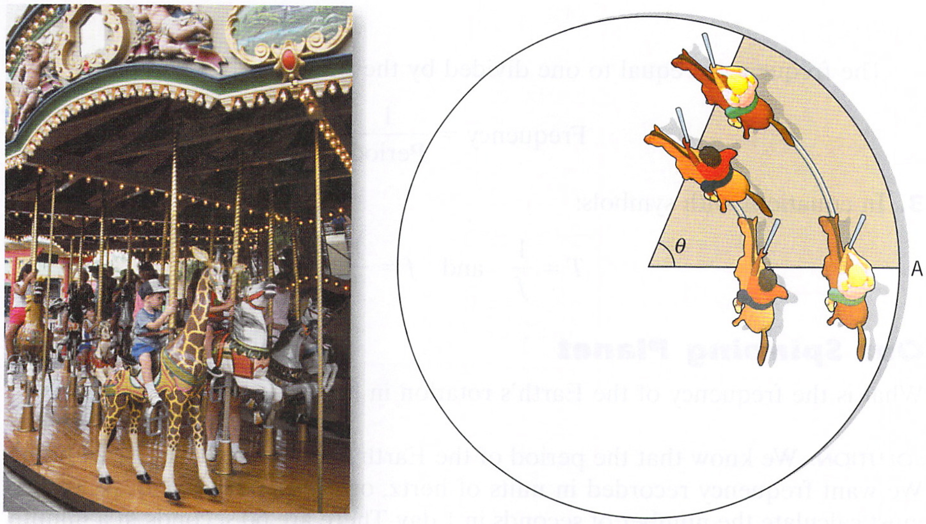


FIGURE 7-4. (*a*) Children riding near the outside of a rotating carousel move faster than children near the center, but they all move with the same angular speed. (*b*) A line (A) from the center of a carousel passes through two children at different distances from the center. As the carousel turns, the change in position of the children is given by the angle θ, as shown. Both children have moved through this same angle, even though they have traveled different distances along their respective paths.

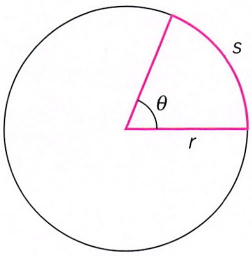


FIGURE 7-5. Angles involved in the measurement of angular speed are often given in terms of radians. If an object travels through an arc of length *s* (in meters) along a circle of radius *r* (also in meters), then the angle θ (measured in radians) is defined as $\theta = \frac{s}{r}$.

3. In an equation with symbols:

$$\omega = \frac{\theta}{t}$$

This definition of angular speed, ω (the Greek letter omega), is similar to the definition of linear speed as the distance traveled divided by the time it takes to travel that distance. Note, however, that while the two children on the carousel have the same *angular* speed, they have different *linear* speeds. In particular, the child farther from the center is moving faster.

It is customary to define the angle in the definition of angular speed in terms of a unit called the *radian*. If, as in Figure 7-5, an object travels through an arc of length *s* (in meters) along a circle of radius *r* (also in meters), then the angle θ (measured in radians) is defined as:

$$\theta = \frac{s}{r}$$

EXAMPLE 7-2

Coming Full Circle

If an object travels all the way around a circle, what is θ in degrees? In radians?

SOLUTION: A full circle is 360 degrees, so that is the value of θ in degrees. To get the value in radians, we note that if the object goes all the way around a circle, it will have traveled a distance equal to the circumference of the circle. Thus,

$$s = 2\pi r$$

From the definition of the radian, then, the angle through which the object travels is

$$\theta = \frac{s}{r} = \frac{(2\pi r)}{r} = 2\pi$$

Thus, 360 degrees is equal to 2π radians.

Angular Frequency Each time a rotating object completes one revolution it traverses an angle of 2π radians. Sometimes it is useful to measure rotational motion in terms of the *angular frequency*, which is defined as the number of radians traversed in 1 second:

$$\text{Angular frequency} = 2\pi f$$

where f is the frequency of rotation in cycles per second, or hertz. Note that frequency f and angular frequency are measures of exactly the same physical phenomenon—namely, how rapidly an object rotates. The only difference is whether the units are cycles per second, or hertz (rotation frequency), or radians per second (angular frequency).

TORQUE

Suppose you turned your bicycle upside down so that the wheels were in the air and you wanted to get the wheels spinning. What would you do? Most likely you would place your hand firmly on the wheel and move it downward—a process that would get the wheel turning. If you wanted the wheel to go faster, you would repeat the operation until you achieved the desired speed.

Let's look at this simple example from the point of view of the forces being applied (Figure 7-6). The axis of rotation is the axle that attaches the wheel to the bike. To get the wheel spinning, you have to apply a force that satisfies two criteria. First, the force must have a component that is tangent to the wheel; that is, in the plane of rotation and parallel to the edge of the wheel. Second, this force has to be applied some distance away from the axis. In this example the force is applied a perpendicular distance r away from the axis, where r is the wheel's radius. (This distance is sometimes called the "lever arm," a term that arises from the common use of torques in the operation of a simple lever.) A force applied in this way, satisfying these two criteria, is said to produce a **torque.**

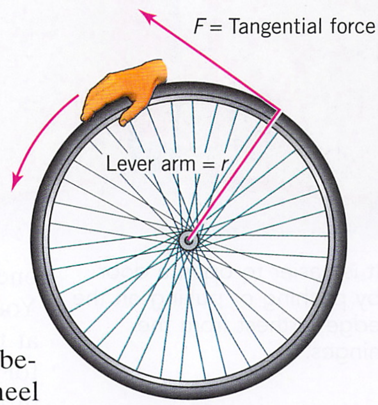


FIGURE 7-6. To get a wheel spinning about its axis, you have to apply a torque. The force that generates the torque must satisfy two criteria. First, the force must have a component that is tangent to the wheel; that is, in the plane of rotation and parallel to the edge of the wheel. Second, this force has to be applied some distance away from the axis. This distance is sometimes called the "lever arm," a term that arises from the common use of torques in the operation of a simple lever.

If a tangential force F is applied a distance r from the axis of rotation of an object, then the torque τ (the Greek letter tau) is defined as

1. In words:

The torque is the tangential force being applied times the perpendicular distance from the axis of rotation.

2. In an equation with words:

Torque equals tangential force times perpendicular distance.

3. In an equation with symbols:

$$\tau = F \times r$$

Torque plays a role in rotational motion analogous to the role of force in linear motion. If we want to change the motion of an object moving in a line, we have to apply a force. In the same way, if we want to change the way something is rotating, we have to apply a torque. In our example, we applied a torque to the bicycle wheel to start and then speed up its rotation.

It's important to remember that in order to produce a change in rotation, a force must have a component that is tangent (in a direction parallel to the edge of the wheel). If you grabbed the bicycle wheel and pushed inward toward the center or to the side (perpendicular to the plane of rotation), the wheel wouldn't start to spin. In these cases, you'd be applying a force without producing a torque.

It is easier to open a door by pushing or pulling on the edge farthest from the hinges.

Torque is needed to change the rotational motion of any object, not just a wheel. For example, when you open a door by pushing on the handle, you are exerting a force at a distance (the width of the door) from the axis of rotation (the hinges). The handle is placed at the edge of the door farthest from the hinges to increase the torque produced by a given force (your push). You probably know from experience that it's much harder to push a door open if you push on the side near the hinges.

The use of a torque underlies the operation of a simple lever. In this application, a beam is placed over a sharp edge, called the "fulcrum," with the length of the beam on one side of the fulcrum longer than the length on the other side (Figure 7-7). You apply a downward force to the long side of the beam in order to lift a load at the short side of the beam. Here the fulcrum serves as the axis of rotation for the beam. The load on the short side of the beam produces a torque in one direction that must be overcome by the torque in the opposite direction, produced by your applied force on the long side of the beam (your lever arm). Since your lever arm is longer, you can lift the load by exerting a smaller force than the weight of the load, but you must move the arm through a greater distance.

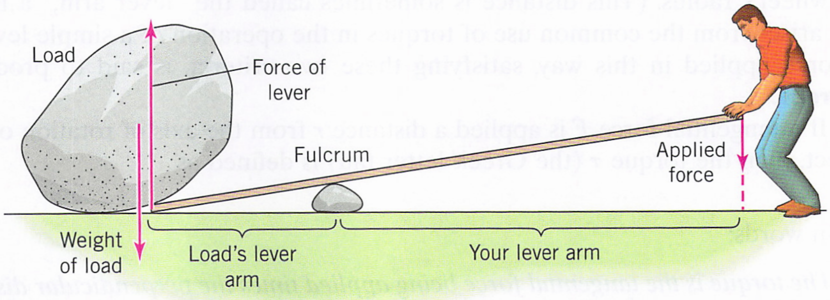


FIGURE 7-7. The use of a torque underlies the operation of a simple lever. A beam is placed over the fulcrum, with the length of the beam on one side of the fulcrum longer than the length on the other side. Applying a downward force to the long side of the beam allows you to lift a heavy object on the short side of the beam. The fulcrum serves as the axis of rotation for the beam.

Braking

From Newton's laws of motion, it follows that any change in rotational motion, whether speeding up or slowing down, requires a force—a torque. When you are riding your bike and want to stop, you squeeze a lever attached to the handlebars. As you do this, two small plastic or rubber brake pads clamp down on the metal frame of the tire and the bike slows down (Figure 7-8). Where are the torques in this process?

The brake pads squeezing down on the metal frame generate a frictional force. This force acts in a direction tangent to the wheel and in a direction opposite to the direction of the wheel's motion. Because the frictional force is applied away from the axis of rotation, it produces a torque, in this case a torque that slows down the rotation rather than one that speeds it up.

The brakes in your car work the same way, although you can't see them in operation. When you put your foot on the brake pedal, tough ceramic pads clamp down on spinning metal parts of the wheel, producing a frictional torque that slows down the wheel.

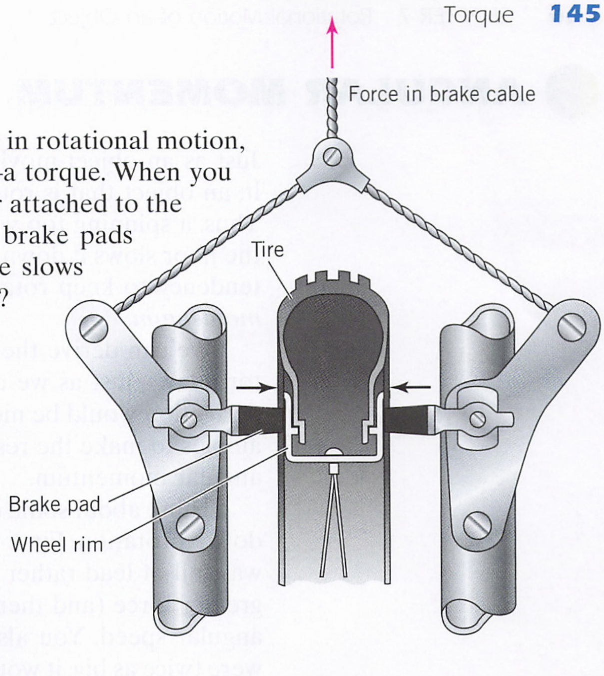


FIGURE 7-8. Brake pads exert a frictional force against the rim of a bicycle wheel to slow down the wheel's rotation and bring the bike to a stop.

Develop Your Intuition: Torque to the Rescue

When you change a tire on your car, it may happen that the nut holding the tire to the axle is rusted and difficult to turn. In this case, experienced mechanics sometimes put a length of pipe around the handle of the wrench, in such a way that the pipe extends farther out than the original handle of the wrench (Figure 7-9). Use the concept of torque to explain why this might be a useful thing to do.

The problem is that you can exert only so much force on the wrench and therefore produce only so much torque to turn the recalcitrant nut. However, torque depends on two factors: the amount of applied tangential force and the distance of that force from the axis of rotation. The pipe, in effect, makes the handle of the wrench longer—it increases r in the torque equation—and therefore allows you to exert a greater torque while applying the same force.

A word of caution: Sometimes this technique can generate more torque than you want. The authors have seen tire changers use such a long pipe that the steel nut was completely sheared off, requiring a trip to a mechanic to set things right.

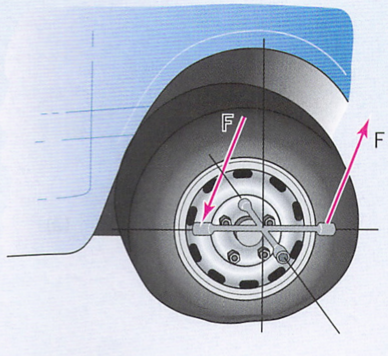


FIGURE 7-9. Tire wrench in use; an extender would increase the length of the lever arm so you could exert a larger torque.

ANGULAR MOMENTUM

Just as an object moving in a straight line keeps moving unless a force acts on it, an object that is rotating keeps rotating unless a torque acts to make it stop. Thus, a spinning top will spin until the friction between its point of contact and the floor slows it down. A wheel will turn until friction in its bearing stops it. This tendency to keep rotating is often stated in terms of a quantity called *angular momentum.*

We can derive the formula for angular momentum step by step from Newton's laws, just as we derived the formula for linear momentum. However, the derivation would be more complicated, so we'll just point out some everyday examples to make the result seem reasonable. Afterward, we'll state a definition of angular momentum.

Think about some common experiences in which a torque speeds up or slows down a rotation. First, in the example of the bicycle wheel, imagine that the tire was full of lead rather than air. Common sense tells you that it would require a greater force (and therefore a greater torque) to spin the wheel up to the same angular speed. You also know from experience that if the radius of the wheel were twice as big, it would require a greater force (and therefore a greater torque) to get it rotating. The relation between torque and the change of rotation, then, must involve both the mass of the rotating object and the distance between the axis of rotation and the location of that mass.

Moment of Inertia

The quantity that describes the distribution of mass around an axis of rotation is called the **moment of inertia** (I). In general, the farther away the mass is from the axis, the greater the moment of inertia (Figure 7-10). For example, if you pick up a dumbbell with two weights on it, as shown, you have to apply a certain amount of torque to get it to rotate. If you move the weights out so that they are twice their original distance from the axis of rotation, it takes a greater torque to get the same amount of rotational speed—four times as much torque in this

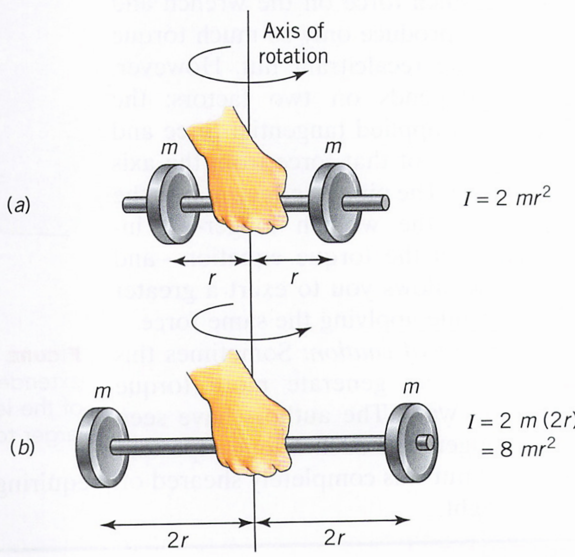


FIGURE 7-10. Moment of inertia (I) describes the distribution of mass around an axis of rotation. The farther away the mass is from the axis, the greater the moment of inertia.

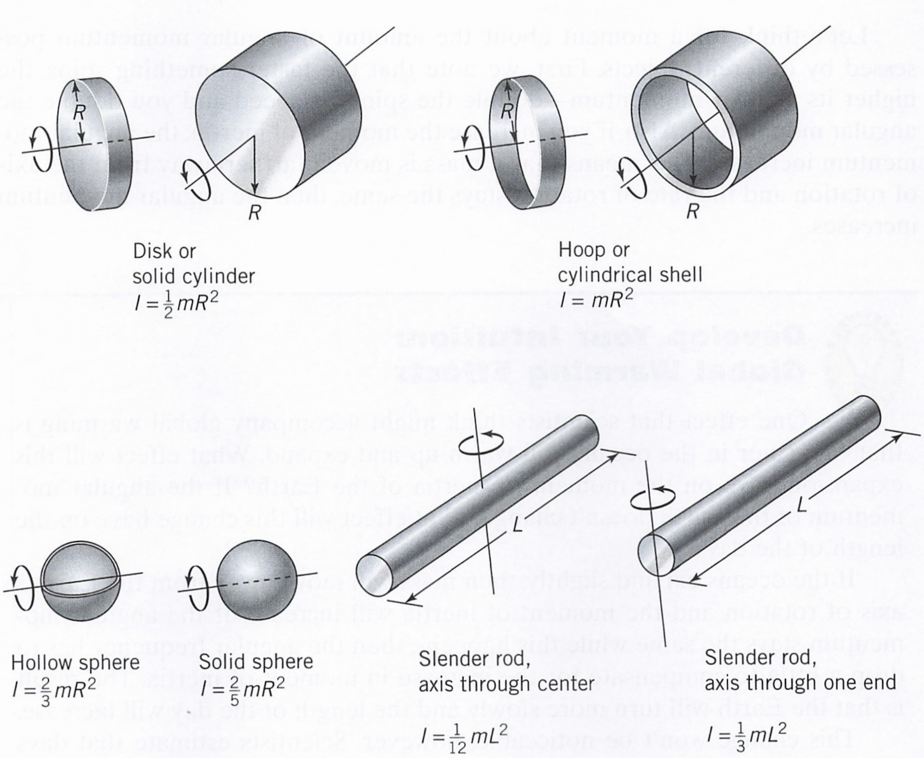


FIGURE 7-11. Moments of inertia for several common objects rotating around various axes. All of these moments of inertia increase with the mass (m) of the object (double the mass and you double the moment of inertia), while the moment of inertia increases as the square of the dimension of the object (double the radius of a sphere, for example, and you increase its moment of inertia by a factor of four).

example. The torque must increase because moving the weights farther out increases the moment of inertia of the dumbbell.

In Figure 7-11, we show several common objects rotating around various axes, along with their moments of inertia. Notice that all the moments of inertia increase with the mass of the object—double the mass and you double the moment of inertia. Note also that the moment of inertia increases as the square of the dimension of the object—double the radius of a sphere, for example, and you increase the moment of inertia by a factor of four.

Definition of Angular Momentum

We are now ready to give a technical definition of **angular momentum.**

1. In words:

Angular momentum of an object depends on how the mass of the object is distributed and on its rate of rotation.

2. In an equation with words:

Angular momentum = Moment of inertia times angular speed.

3. In an equation with symbols:

$$L = I \times \omega$$

where L is the angular momentum and I is the moment of inertia.

Let's think for a moment about the amount of angular momentum possessed by different objects. First, we note that the faster something spins, the higher its angular momentum—double the spinning speed and you double the angular momentum. Also, if you increase the moment of inertia, the angular momentum increases. This means that if mass is moved farther away from the axis of rotation and the rate of rotation stays the same, then the angular momentum increases.

Develop Your Intuition: Global Warming Effects

One effect that scientists think might accompany global warming is that the water in the oceans will warm up and expand. What effect will this expansion have on the moment of inertia of the Earth? If the angular momentum of the Earth doesn't change, what effect will this change have on the length of the day?

If the oceans expand slightly, then mass will move away from the Earth's axis of rotation and the moment of inertia will increase. If the angular momentum stays the same while this happens, then the angular frequency has to drop a little to compensate for the increase in moment of inertia. The result is that the Earth will turn more slowly and the length of the day will increase.

This change won't be noticeable, however. Scientists estimate that days might increase in length by a few millionths of a second over the next few centuries.

Angular Momentum and Torque

Newton's second law describes the effect of a force on the rate of change of linear momentum (see Chapter 6). An analogous law for rotating bodies defines the effect of torque (a tangential force) on the rate of rotation (a change in angular momentum).

1. In words:

The rate of change in angular momentum of an object equals the net external torque on that object.

2. In an equation with words:

Net external torque = Change in angular momentum divided by the change in time

3. In an equation with symbols:

$$\tau = \frac{\Delta L}{\Delta t}$$

The net external torque on an object is defined as the sum of the torques generated by all external forces acting on that object. Can you see the similarity between this equation and the equation in Chapter 6 for linear motion? In both cases, the force (or net external torque) on an object causes a change in momentum (or angular momentum) over a time interval, Δt.

CONSERVATION OF ANGULAR MOMENTUM

If a net external torque on a system is zero, the preceding equation tells us that the change in angular momentum must be zero. In other words, without the action of a force acting at a distance from the axis of rotation, the total angular momentum of a system cannot change. Like its counterpart, linear momentum, angular momentum is conserved in the absence of outside influences. This principle is known as the **conservation of angular momentum.**

1. In words:

If the net external torque is zero, the angular momentum of any system must stay constant over time.

2. In an equation with words:

The change in angular momentum of an isolated system equals zero.

3. In an equation with symbols:

$$\Delta L = 0$$

The consequences of the conservation of angular momentum that you're most likely to experience have to do with situations in which something happens to change an object's moment of inertia. In this case, the angular velocity must change as well so that the product of I and ω will stay the same. In most practical situations of this kind, you may find it helpful to express conservation of angular momentum in the form:

$$\text{Initial angular momentum} = \text{Final angular momentum}$$

or

$$I_i \times \omega_i = I_f \times \omega_f$$

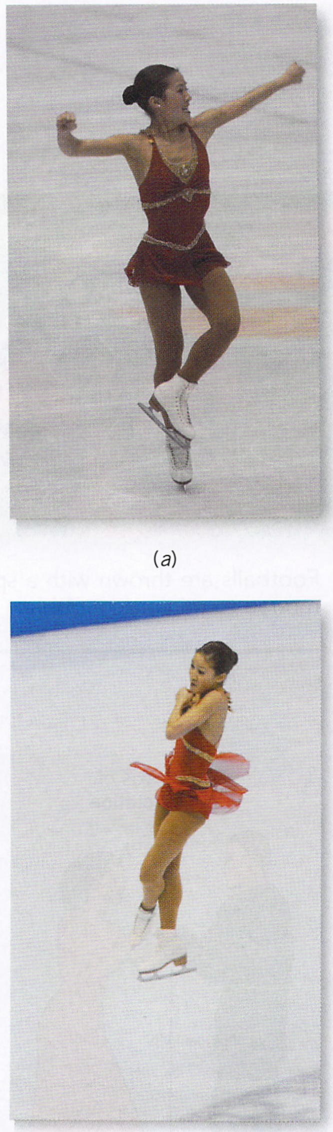
A striking illustration of this point can be seen in figure skating competitions, as described in the Physics Around Us section. As the skater goes into the spin with her arms spread wide, her moment of inertia is high because an appreciable amount of mass (her arms) is located far from the axis of rotation (Figure 7-12*a*). As she pulls her arms in over her head, her moment of inertia drops (Figure 7-12*b*). Since no outside force is acting to affect the spin, her angular momentum must remain the same. The only way for this to happen is for her angular velocity (that is, her rate of spin) to increase.

We can make this same point by looking at the equation for angular momentum:

$$L = I \times \omega$$

If the skater pulls in her arms, her moment of inertia decreases. The only way the angular momentum L can stay the same is for her angular frequency ω to increase in a compensating way. So, for example, if I decreases by $\frac{1}{2}$, ω has to increase by a factor of 2 to keep L the same. Hence, her rotational speed doubles.

Can you can use this same reasoning to explain why the spin slows down when she puts her arms back out?



(a)

(b)

FIGURE 7-12. (*a*) Figure skater in a spin. (*b*) She can increase her rotation rate by pulling her arms and legs closer to her body, decreasing her moment of inertia.

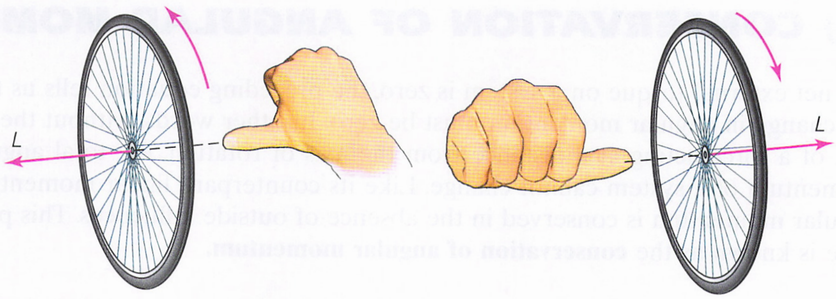


FIGURE 7-13. The right-hand rule for finding the direction of angular momentum for a spinning object. Curl the fingers of your right hand in the direction of the rotation; your right thumb will point in the direction of the angular momentum.

Footballs are thrown with a spiral motion to minimize wobble.

The Direction of Rotation

Like linear momentum, angular momentum has a direction. The so-called "right-hand rule" for finding this direction for a spinning object is simple: if you curl the fingers of your right hand in the direction of the rotation, your right thumb will be pointing in the direction of the angular momentum (Figure 7-13). For example, the angular momentum of the Earth points upward along the axis of rotation to the North Pole.

The conservation of angular momentum implies that *both* the size and direction of the angular momentum of an object remain fixed in the absence of torques. This fact explains why the orientation of an isolated spinning object, such as a football thrown in a spiral, is generally more stable than that of a nonspinning object, such as a football bouncing on the ground.

Develop Your Intuition: Waltzers

Dancers doing a waltz normally hold each other at arm's length to avoid stepping on each other's feet. If they have to make a fast turn, however, you will often see them move together, with each partner putting his or her feet between or even behind those of the other. Why do they risk calamity this way?

The answer has to do with the conservation of angular momentum. Under normal circumstances, the couple's mass is located far from the axis of rotation (which is located between the dancers). By moving closer together, they reduce their moment of inertia and, like the ice skater, increase their angular velocity (Figure 7-14). This change in position helps them get through the turn, after which they can separate and slow down the rotation.

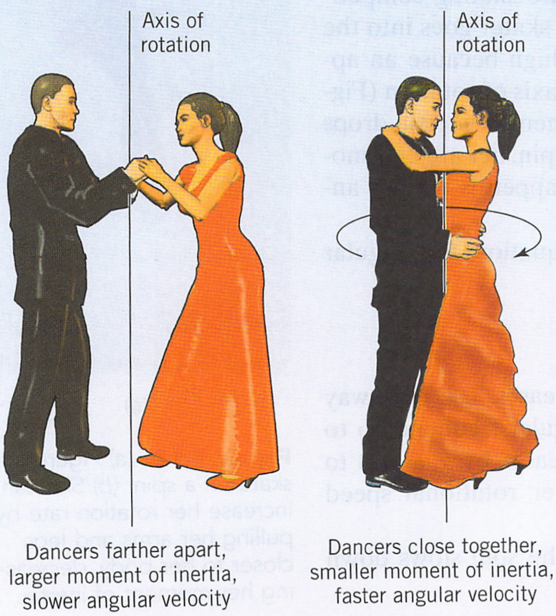


FIGURE 7-14. By moving closer together, dancers reduce their moment of inertia and increase their angular velocity. This change in position helps them get through fast turns, after which they can separate and slow down the rotation.

LOOKING DEEPER

The Collapse of Stars

Stars often end their life cycle by undergoing a rapid collapse. The Sun, for example, which now has a radius of 7×10^8 meters, will collapse to a type of star called a "white dwarf," approximately the same size as the Earth. (The radius of the Earth is about 6.4×10^6 meters—less than a hundredth that of the Sun.) The Sun currently rotates about its axis once every 26 days. How will its rotation change when it becomes a white dwarf?

No outside forces will act on the Sun during its collapse, so its angular momentum must be conserved. As the Sun contracts to the size of the Earth, its moment of inertia will decrease dramatically because, as with a skater pulling in her arms, the mass will be located much closer to the axis of rotation. Consequently, the shrunken Sun's rate of rotation will have to increase dramatically to compensate. In other words, after the collapse, the Sun will rotate much faster than it does now.

We work out the numbers for this in Example 7-3 at the end of the chapter. The result turns out to be 3.1 minutes. In fact, some stars collapse to even smaller objects than white dwarves and they rotate hundreds of times *each second.*

Connection

Inertial Guidance Systems

The conservation of angular momentum plays an important role in the inertial guidance systems for navigation in airplanes and satellites. The idea behind such systems is very simple. A massive object such as a sphere or a flat circular disk is set into rotation inside a device in which very little resistance (that is, almost no torque) is exerted by the bearings. Once such an object, called a "gyroscope," is set into rotation, its angular momentum continues to point in the same direction, regardless of how the aircraft or rocket moves around it. By sensing the constant rotation and seeing how it is related to the orientation of the satellite, engineers can tell which way the satellite is pointed.

Toy gyroscopes and tops work in the same way (Figure 7-15). In the case of a spinning top, its weight starts to topple it over as it spins slightly away from perfectly vertical. However, when the weight is acting at a slight distance from the vertical axis, it produces a torque. This torque changes the angular momentum of the top; but what changes is the direction of the angular momentum, not its size. The angular momentum changes direction in such a way as to bring the top back to spinning vertically. However, the changing direction of angular momentum continues to cause a torque even as the top tries to get back to vertical. The result is that the top wobbles as it spins. The axis of rotation traces out the sides of an imaginary cone that gets larger as the top slows down from friction at the contact point.

Other spinning objects show this wobbling motion, which goes by the technical name "precession." You can certainly see the wobble in a thrown football if it is not thrown with a tight spiral. The Earth, too, wobbles in its motion around the Sun, as do all the other planets. ●

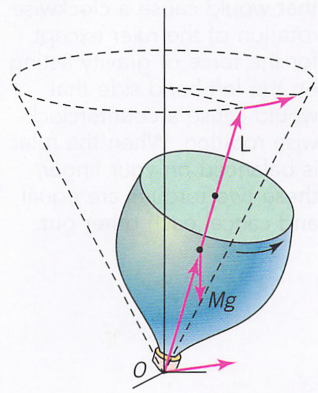


FIGURE 7-15. A toy top or gyroscope maintains its orientation as it spins. Inertial guidance systems rely on rapidly spinning gyroscopes that maintain orientation very accurately.

MOTION IN A PLANE WITH ROTATION

In Chapter 6 we learned how to analyze situations involving the momentum of an object moving in a straight line or in a plane under the influence of a force such as gravity. In this chapter, we have learned how to deal with a body that is rotating around an axis. The most general kind of motion involves a combination of these two types of motion. Think about the movements of a boomerang in flight or a Frisbee sailing through the air. In these cases, the flying objects are moving under the influence of gravity, but they are also rotating around an axis. We now have all the background we need to talk about these sorts of complicated motions.

Center of Mass

Imagine taking a ruler and balancing it on one finger. You know that the ruler will balance if you support it right in the middle. As shown in Figure 7-16, we can understand this situation in terms of the torques exerted on the ruler by gravity. The downward pull of gravity on the right-hand side produces a torque that, if it were the only force acting, would produce a clockwise rotation of the ruler. Similarly, the force of gravity acting on the left-hand side creates a torque that would produce a counterclockwise rotation. When the ruler is balanced on your finger, these two torques are equal and cancel each other out. The ruler sits stationary, not rotating at all. The point of support on which an object can be balanced like this is called its **center of mass** (or, sometimes, its **center of gravity**).

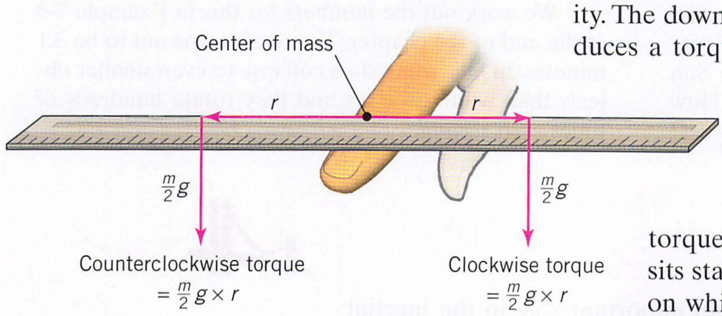


FIGURE 7-16. The downward pull of gravity on the right-hand side of a balancing ruler produces a torque that would cause a clockwise rotation of the ruler except for the force of gravity acting on the left-hand side that would cause a counterclockwise rotation. When the ruler is balanced on your finger, these two torques are equal and cancel each other out.

You can think of the center of mass of an object as being the average of the positions of the object's mass. For a regular geometrical object such as a cube or a sphere with a uniform mass distribution, the center of mass is at the geometrical center of the object. The center of mass of a smooth disk, for example, is at the exact center of the disk.

The center of mass of a system doesn't have to be at its geometrical center, however. Every object, no matter how complicated its shape or structure, has a single point where all the torques due to gravity cancel, a single point where it could be supported without rotation. For example, if one side of the ruler weighs more than the other side, the center of mass would be located more toward the heavy side.

Complex Motion

In Chapter 3 we saw that the discussion of motion in two dimensions could be greatly simplified by the law of compound motion, which allows us to break the problem up into a connected pair of simpler one-dimensional problems. In the same way, motions that include rotations can be broken up into a series of simpler problems. The analog of the law of compound motion for rotating objects is:

Motion that involves rotation can be thought of as the motion of the center of mass (treated as if the object were a single particle) plus rotation about the center of mass.

Consider, as an example, a springboard diver performing a somersault in the air. Using the rule we have just stated, we could talk about her motion in two stages: (1) the motion of her center of mass and (2) the rotation around her center of mass. The motion of the diver's center of mass is simply that of a point particle under the influence of the force of gravity. As we saw in Chapter 3, the

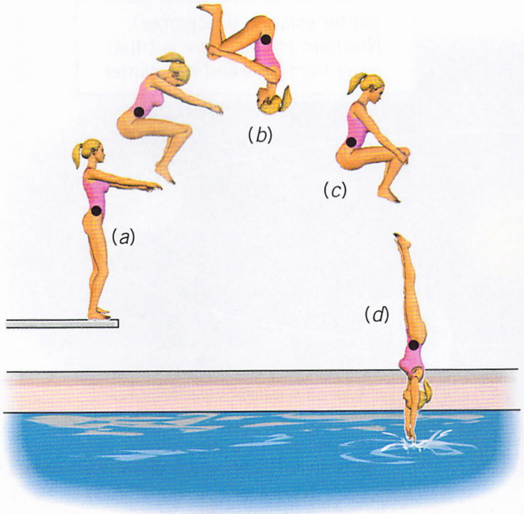


FIGURE 7-17. When the diver leaves the diving board (*a*) her body is vertical, but at the top of her arc (*b*), her hips are now horizontal. Thus the board applied a torque to her body when she jumped off, giving her angular momentum and causing her to rotate around her center of mass as the dive progresses. Once the diver is launched, there are no further torques acting on her. When she grabs her knees (*c*), which lowers her moment of inertia, she starts rotating faster to keep the angular momentum the same. When she straightens out again (*d*), the moment of inertia increases and the angular velocity drops.

path followed by such a particle is a parabola that rises to a peak and then falls, as shown in Figure 7-17.

To understand the motion around the center of mass, imagine traveling along the parabola with the diver. When she leaves the diving board, her body is vertical, as shown in Figure 7-17. At the top of her parabolic arc, her hips are now horizontal. When she reaches the water, she is again vertical but with her head down. In other words, the board applies a torque to her body when she jumps off, causing her to rotate around her center of mass as the dive progresses. At the beginning of the dive, she is rotating and therefore has a certain amount of angular momentum.

A springboard diver always follows a parabolic path through the air.

Once the diver is launched, however, there are no further torques acting on her, a fact that means that whatever angular momentum she had leaving the board remains constant. Thus, when she grabs her knees, an act that lowers her moment of inertia, she starts rotating faster to keep the angular momentum the same. This tucking action is what produces the spectacular rotation during the high part of the dive. When she straightens out again, the moment of inertia increases and the angular velocity drops. She enters the water cleanly.

Develop Your Intuition: Not a Swan Dive

How can a springboard diver enter the water so that there is no rotation about his center of mass? What would such a dive look like?

No matter what he does, the diver's center of mass will still move under the force of gravity through the same parabola. If he leaves the springboard without torque, however, his body must remain in a heads-up vertical position throughout the dive. There will be no rotation around his center of mass, and he will enter the water feet first. While such a dive is not likely to win an Olympic medal, he can make quite a splash by pulling his knees up with his arms to perform a "cannonball."

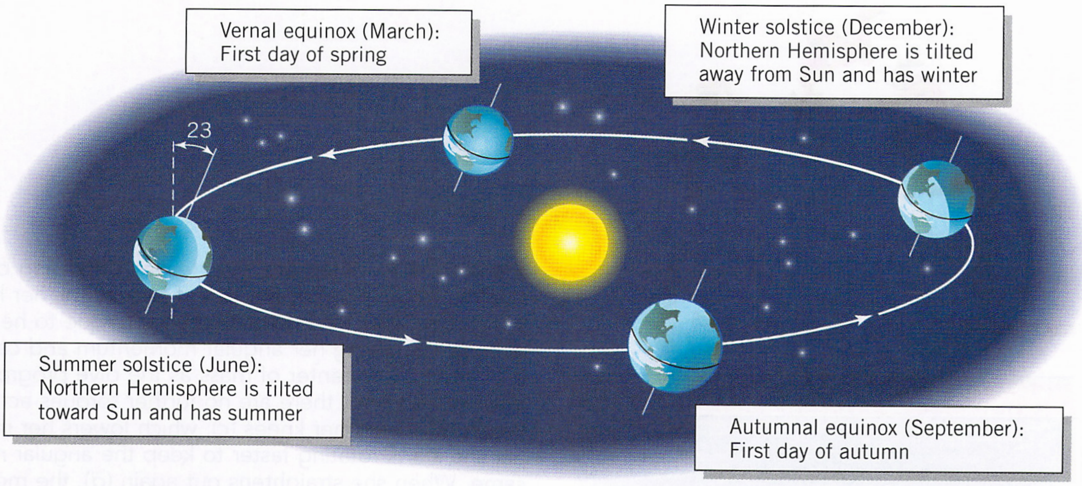


FIGURE 7-18. In the months of July and August, the Earth is tilted so that the Northern Hemisphere leans toward the Sun. During January and February, the Northern Hemisphere is tilted away from the Sun.

Connection

The Seasons

The continual progression of seasons on our planet is related to the conservation of angular momentum. As shown in Figure 7-18, the Earth's axis of rotation is tilted by an angle of 23 degrees to the plane of the Earth's orbit around the Sun. Over time scales of a few years, we can treat the Earth as if no significant torques act on it, so Earth's angular momentum must be constant. Both the size and direction of the Earth's angular momentum remain fixed, so that the direction of the axis remains the same over the course of a year. (The axis does change direction slightly due to precession, as we mentioned in the Connection section on inertial guidance systems, page 151, but this effect is small enough that we can neglect it in the present discussion.)

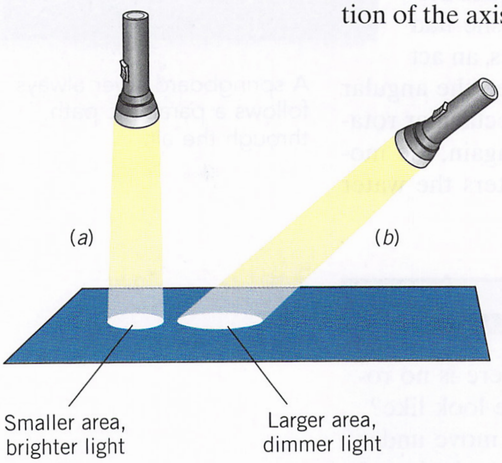


FIGURE 7-19. When a flashlight shines on the floor from directly overhead (*a*), the light is brighter and it covers a smaller surface area than when the light shines at an angle (*b*).

In the months of July and August, the Earth is tilted so that the Northern Hemisphere leans toward the Sun, as shown in Figure 7-18. (The tilt is oriented most directly toward the Sun at the summer solstice, around June 21 of each year.) This orientation means that more sunlight falls on each square foot of the Earth's Northern Hemisphere surface during this period than during January and February, when the Northern Hemisphere is tilted away from the Sun. The effect is similar to shining a flashlight on the floor from directly overhead or from an angle; the light is brighter when it covers the smaller area of surface from directly overhead (Figure 7-19). Earth's tilt is why the Northern Hemisphere experiences summer from June to August and why it experiences winter from December to February—*even though the Earth is closest to the Sun during those winter months.* We have seasons because the conservation of angular momentum ensures that the Earth's axis of rotation tilts in the same direction over the course of the year. ●

THINKING MORE ABOUT

Angular Momentum: The Spinning Solar System

The solar system, which includes the Earth, the Moon, the other planets, and lots of smaller objects that are gravitationally bound to the Sun, such as asteroids and comets, has a lot of angular momentum. The planets swing around the Sun in their stately orbits, while the Sun spins on its axis once every 26 days. But we know that, in the absence of an external force, angular momentum is conserved—it doesn't spontaneously increase or decrease. So where did all the solar system's angular momentum come from originally?

According to the most widely accepted theory, called the "nebular hypothesis," the Sun and planets formed almost 5 billion years ago from an immense, swirling, irregular cloud of dust and gas, called a "nebula," that extended across trillions of kilometers of space. Like any swirling cloud, the nebula had its own angular momentum—it was slowly spinning about its axis of rotation. Eventually, ever so slowly, the cloud began to spiral inward from the force of gravity due to the concentration of mass at its center (Figure 7-20). As the nebula became more dense and compact, it also began to spin faster. Ultimately, gravity pulled most of the dust and gas into the great central mass that we now call the Sun. Smaller clumps of matter called "planetesimals," which were orbiting too fast to fall into the Sun, formed the planets—Earth, Mars, Jupiter and so on.

The French mathematician and physicist Pierre Simon Laplace first proposed the nebular hypothesis in 1796. His work incorporated the earlier results of the German philosopher Immanuel Kant, who showed in 1755 that a contracting cloud of gas would form a disk in a plane perpendicular to the axis of rotation. Laplace went through calculations similar to those in the Looking Deeper box on the collapse of stars (see page 151), showing how the rotational rate of the Sun would speed up over time. However, he came up with a major problem. The calculations predicted that the Sun's period of rotation should be only a

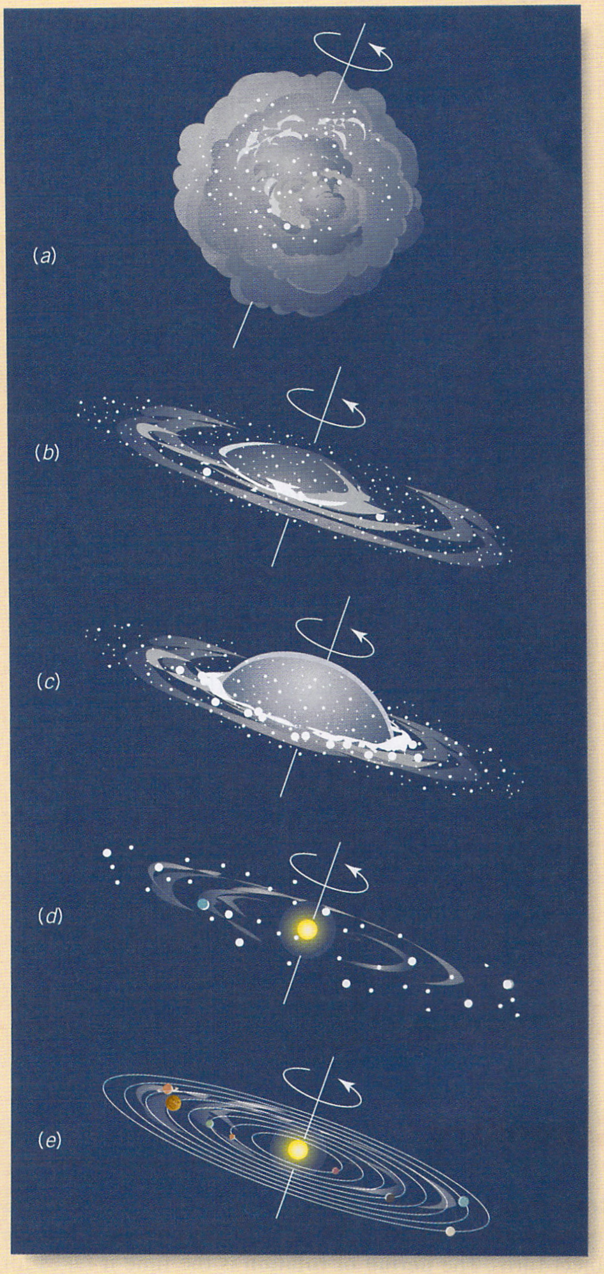


FIGURE 7-20. Astronomers today believe the solar system formed from a large cloud of gas and dust (*a*). As the cloud contracted under the influence of gravity, it started to spin faster and formed a flattened disk (*b*). Eventually, the Sun formed from the central part of the disk (*c*), while the outer parts formed into small rocky planetesimals (*d*) that over the course of time collided and grew into today's planets (*e*).

few hours, not a month. It seemed that the nebular hypothesis worked a little too well.

For a long time, the nebular hypothesis was discredited because of this result. Astronomers turned to other theories to explain the formation of the solar system, most of them based on various kinds of catastrophes. One idea was that a massive comet came so close to the Sun that it pulled out a long stream of material from the Sun, which eventually produced the planets. This theory was thrown out when astronomers learned that comets have nowhere near enough mass to cause such a disruption.

Another idea popular at one time was that the Sun was once part of a two-star or even three-star system. According to this idea, the system was unstable and eventually one of the stars collided with the Sun, causing the system to scatter apart and producing a stream of gas that became the planets. Beginning in the 1930s, physicists began to find problems with these catastrophe theories. For instance, calculations showed that a hot stream of matter from the Sun would dissipate, rather than condense to form planets. Other observations showed that the chemical makeup of the planets was not consistent with material pulled from the outer surface of the Sun, but must have formed under cooler conditions. This finding led astronomers to reconsider the nebular hypothesis, wondering what had gone wrong with Laplace's calculations.

Eventually, scientists realized a solution to the problem: the Sun has a strong magnetic field. As the early planets orbited the Sun, the Sun exerted magnetic forces on them, as well as gravitational forces. This magnetic force would have acted to sweep the planets along in faster orbits. However, by Newton's third law, the planets would have exerted a force back on the Sun, slowing its rotation. When physicists ran the calculations taking this into account, they found pretty close agreement with observations.

This history of a scientific theory is not unusual; it often happens that theories are abandoned because of a seemingly insurmountable problem, only to be resuscitated when a way to solve the problem is found. The important point to realize is that the success or failure of a scientific theory depends on how well it matches and explains testable observations.

Today, astronomers accept the nebular hypothesis as the most likely scenario for the origin of the solar system. The total angular momentum of the spinning Sun and orbiting planets, as well as the orientation of the axis of rotation of the entire solar system, is conserved from that swirling nebular cloud almost 5 billion years ago.

Given this acceptance, do you think that the original rejection of the nebular hypothesis was a failure of the scientific method? Do such changes of opinion suggest that scientists are fickle in the theories that they support? The physicist Richard Feynman once said "We are trying to prove ourselves wrong as quickly as possible, because only in that way can we find progress." What do you think he meant by that? How does that relate to the theory of solar system formation?

Summary

Rotational motion is exhibited whenever an object spins about an **axis of rotation,** which is the line about which the object turns. The **period of rotation** is the time it takes for a body to complete one entire cycle, and the **frequency of rotation** is the number of completed rotations per unit time. Frequency is customarily measured in **hertz,** which is defined such that 1 Hz corresponds to one complete rotation each second.

The **angular speed** of a rotating object is the angle through which the object rotates divided by the time it takes to go through that angle. The angular frequency measures the number of times a rotation goes through one radian each second.

When a tangential force is applied away from the axis of rotation, that force produces a **torque.** The magnitude of the torque is given by $\tau = rF$, where F is the tangential force and r is the distance from the axis to the point of application of the force.

The **moment of inertia** of an object measures the distribution of its mass. The more mass there is and the farther away it lies from the axis of rotation, the greater the moment of inertia. The **angular momentum** of a rotating body is defined as the product of its angular speed and its moment of inertia. The rate of change of angular momentum in any system is equal to the net external torque, a result that follows from the rotational form of Newton's second law. In the absence of a net external torque, the angular momentum of a rotating body cannot change—a result called the law of **conservation of angular momentum.**

The **center of mass** or **center of gravity** of an object is the average position of its mass and can be thought of as the point at which the object can be balanced without rotation. Motion that involves both rotation and ordinary movement through space can be broken down into two simple processes: motion of the center of mass dictated by Newton's laws of motion and rotation around the center of mass.

Key Terms

angular momentum The moment of inertia of a body, times its angular velocity. (p. 147)

angular speed The angle through which an object has moved about the axis of rotation, divided by the time it takes it to go through that angle. (p. 141)

axis of rotation The line through the center of an object, around which everything else rotates. (p. 139)

center of mass (center of gravity) The point of support on which an object can be balanced. (p. 152)

conservation of angular momentum If the net external torque is zero, the angular momentum of any system must stay constant over time. (p. 149)

frequency of rotation The number of times an object completes a rotation in a given amount of time. (p. 140)

hertz The unit of measure of frequency, corresponding to one complete rotation every second. (p. 140)

moment of inertia The quantity that describes the distribution of mass around an axis of rotation. (p. 146)

period of rotation The time it takes for an object to make one complete rotation. (p. 140)

rotational motion The spinning motion that occurs when an object rotates about an axis located within it, usually an axis through its center of mass. (p. 139)

torque The force applied perpendicular to a line from the axis of rotation, multiplied by the distance from the axis of rotation. (p. 143)

Key Equations

$$\text{Frequency of rotation} = \frac{1}{\text{Period of rotation}}$$

$$\text{Angular speed} = \frac{\text{Angle traversed}}{\text{Time it takes to traverse the angle}}$$

$$\text{Torque} = \text{Force applied} \times \text{Distance from the axis of rotation}$$

$$\text{Net external torque} = \frac{\text{Change in angular momentum}}{\text{Change in time}}$$

$$\text{Angular momentum} = \text{Moment of inertia} \times \text{Angular velocity}$$

Review

1. What is rotational motion? Give an example.
2. What is an axis of rotation? Give an example.
3. What is the period of a rotating body? In what units is the period described?
4. Define the frequency of rotation.
5. What is the period of the Earth's rotation? Its frequency?
6. What is the frequency in hertz of a disk that makes one complete revolution every second? Every 2 seconds?
7. Describe the relationship between frequency and period. What is the mathematical equation?
8. What is the angular speed of a rotating body?
9. How does angular speed differ from linear speed for a point on a rotating body? (Think in terms of actual displacement in meters vs. degrees.)
10. What is a radian?
11. For one complete rotation of a rotating body, what is the angular displacement in degrees? In radians?
12. Define the angular frequency of an object. What are its units?
13. What is a torque?
14. How can you increase the torque on an object without increasing the force applied?

15. Compare the role of torque in rotational motion to the role of force in linear motion. When you apply a torque to a rotating object, how does this affect the rate of rotation?

16. What is angular momentum?

17. How does the mass of an object affect its angular momentum?

18. What is a moment of inertia? How does the distribution of mass affect this?

19. What is the conservation of angular momentum? Give an example.

20. Compare and contrast angular momentum to linear momentum.

21. If angular momentum is conserved, does the moment of inertia have to stay the same? How about the angular speed? The product of these two quantities?

22. What is meant by the direction of angular momentum? Is the direction of angular momentum the same as the direction of an object's rotation about an axis? Compare this to the direction of linear momentum.

23. What is the center of mass of an object?

24. Is the center of mass always located at the geometric center of an object? Explain.

25. What is the law of compound motion for rotating objects?

26. How is the conservation of angular momentum responsible for the difference between summer and winter?

Questions

1. What is the frequency of the minute hand of a clock in hertz?

2. When you push on an object such as a wrench, a steel pry bar, or even the outer edge of a door, you are producing a torque equal to the force applied times the lever arm. At what angle to the lever arm should a force be applied to produce maximum torque and why?

3. A children's seesaw is essentially a plank balanced on a fulcrum. Explain its operation in terms of torque and angular momentum. What happens when one person is much heavier? Does it matter where on the seesaw each person sits? Explain.

4. From what you learned in this chapter, why was the invention of rifling in a long gun or cannon barrel so important? (*Rifling* is a series of screw-like grooves etched into the interior of a rifle barrel that imparts a spin to the bullet.)

5. Why does a helicopter have a tail rotor? Some of the largest helicopters have two rotors on top; do these two rotors spin in the same direction?

6. How does conservation of angular momentum affect the stability of a bike?

7. What are some of the reasons that people initially have a difficult time staying upright when they are learning to ride a bike? How does turning the bicycle wheel act to stabilize a cyclist when a bike is stationary?

8. How might a pole-vaulter pass over a 14-foot bar if she were only able to get her center of mass to reach 13.5 feet?

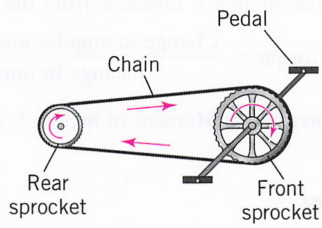
9. How would you describe the path of a flock of birds or a school of fish using the center of mass of the combined masses of the individuals in these populations? Why might you do this?

10. The Earth, the Sun, and most other objects in space are not uniform spheres. Instead, they tend to be much denser toward the center (the core) than on the outside (the crust). For a body of a given mass and size, which will have the greater moment of inertia—a uniform sphere or a nonuniform sphere with a dense core? Why? (*Hint:* Think about the distribution of mass relative to its distance from the axis of rotation.)

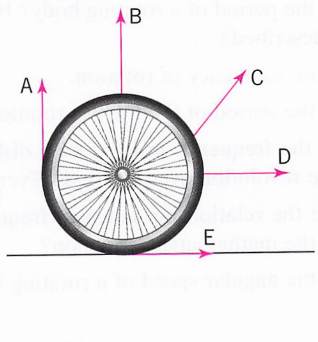
11. If you were in a spaceship with an inertial guidance system (see page 151) and if an outside observer saw that the spaceship was rotating clockwise, what motion would you see in the gyroscope inside the ship?

12. Which planet has a longer period of rotation around the Sun, Mercury or the Earth? (*Hint:* Mercury is the closest planet to the Sun.) Which planet has a higher frequency?

13. While you're riding a bicycle (see figure), which has a higher rotation frequency, the front sprocket or the rear sprocket?

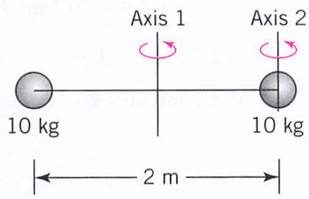


14. Five forces act on the outside of a wheel, as shown in the figure. Which of the five forces exert a torque about the center of the wheel?

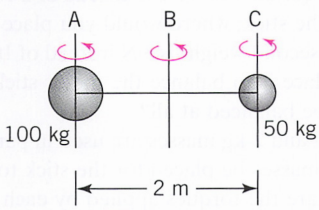


15. Consider two forces acting on the front tire of a bicycle wheel as the rider is braking: the road pushing up on the tire, and friction pushing back on the tire. Which of these forces exerts a torque about the center of the wheel? Explain.

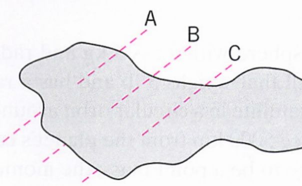
16. Consider the simple dumbbell shown in the figure. It consists of two 10-kilogram spheres separated by a 2-meter light rod. Consider rotating it about two axes: one axis through the middle of the rod and one axis passing through one of the spheres. In which case will the dumbbell have a higher moment of inertia?



17. Consider the asymmetrical dumbbell shown in the figure. It consists of two spheres separated by a 2-meter light rod. Rotation about which axis (A, B, or C) involves the lowest moment of inertia?

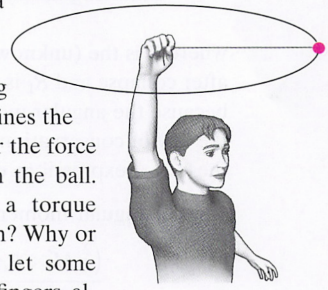


18. Consider the irregularly shaped object shown in the figure. Through which axis (A, B, or C) will the moment of inertia be greatest? Through which axis will it be least? Explain your reasoning.

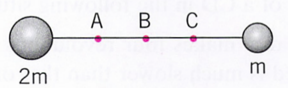


19. If everyone in the world moved to the equator, what would happen to the moment of inertia of the Earth? What would happen to the angular momentum of the Earth? What would happen to the angular speed of the Earth?

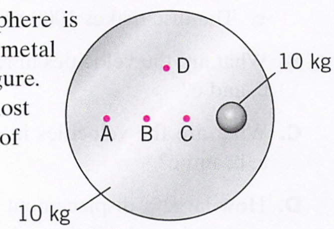
20. You have a ball tied to a string, and you spin it around in a horizontal circle. Your arm, sticking straight up in the air, defines the axis of rotation. Consider the force that the string exerts on the ball. Does that force exert a torque about the axis of rotation? Why or why not? Suppose you let some string slip through your fingers, allowing the ball to spin at a greater distance from the axis. What happens to the angular speed of the rotation? Explain.



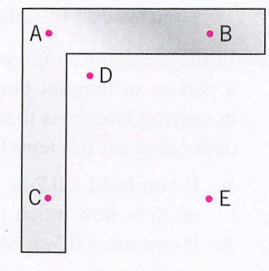
21. In the following figure, which location is most likely to be the center of mass of the dumbbell?



22. A 10-kilogram metal sphere is welded to a 10-kilogram metal disk as shown in the figure. Which location is most likely to be the center of mass of the object?



23. Which location is most likely to be the center of mass of the L-shaped object shown in the figure?



Problem-Solving Examples

EXAMPLE 7-3

The Sun's Collapse

In the Looking Deeper section, we discuss how stars spin faster as they collapse inward with time. We gave the starting and ending diameters of the Sun as 7×10^8 meters collapsing down to 6.4×10^6 meters. How fast will it be rotating when it becomes a white dwarf?

REASONING AND SOLUTION: The steps to follow for solving this problem are described in the Looking Deeper section. From Figure 7-11, the moment of inertia of a uniform sphere is ($\frac{2}{5} M \times R^2$). The Sun's angular frequency is $2\pi f$ or $2\pi/T$ where T is the period of rotation. Therefore, the angular momentum of the Sun today is:

$$\text{Angular momentum} = \frac{\text{Moment of inertia} \times 2\pi}{\text{Period}}$$

$$\text{Initial angular momentum} = \frac{(\frac{2}{5} M \times R_i^2)2\pi}{26 \text{ days}}$$

where R_i is the initial radius of the Sun before the collapse. The angular momentum of the Sun after the collapse will be:

$$\text{Final angular momentum} = \frac{(\frac{2}{5} M \times R_f^2)2\pi}{T}$$

where T is the (unknown) time it takes to do one rotation after collapse and R_f is the radius after collapse. However, because the angular momentum of the Sun is constant, by the law of conservation of angular momentum, we can set these two expressions equal:

$$\text{Initial angular momentum} = \text{Final angular momentum}$$

$$\frac{(\frac{2}{5} M \times R_i^2)2\pi}{26 \text{ days}} = \frac{(\frac{2}{5} M \times R_f^2)2\pi}{T}$$

Canceling the $\frac{2}{5}$, the mass of the sun, and 2π, we find

$$\frac{R_i^2}{26 \text{ days}} = \frac{R_f^2}{T}$$

so

$$T = 26 \text{ days} \times \left(\frac{R_f^2}{R_i^2}\right)$$

$$= 26 \text{ days} \times \frac{(6.4 \times 10^6 \text{ m})^2}{(7 \times 10^8 \text{ m})^2}$$

$$= 26 \times \left(\frac{4.1 \times 10^{13} \text{ m}^2}{4.9 \times 10^{17} \text{ m}^2}\right) \text{days}$$

$$= 2.2 \times 10^{-3} \text{ days}$$

$$= 3.1 \text{ minutes}$$ ●

Problems

1. A. What is the rotational speed in revolutions per second (hertz) of a CD in the following situations?

a. The disc makes four revolutions in 48 seconds (this speed is much slower than that of a normal CD).
b. The disc rotates six times in 240 seconds.
c. The disc makes 1000 revolutions in 2 minutes.

B. What are the velocities in radians per second for parts a, b, and c?

C. What are the velocities in degrees per second for parts a, b, and c?

D. How large a displacement in degrees occurs in each of parts a, b, and c if these discs spin for 30 seconds at the same speed? In radians?

2. In the mechanical and plumbing trades, many tools come in a variety of lengths. One of the reasons they are available in varying lengths is that different torques can be generated depending on the lengths of these tools.

a. If you hold a 0.2-m wrench at its end and exert a force of 30 N, how much torque will you generate?
b. If you use a 0.5-m wrench and exert a force of 45 N, how much torque can you generate?
c. If a plumber needs to generate a torque of 160 N-m to unscrew a rusted pipe and can only generate a maximum force that day of 20 N because she has been out too late the night before, what length wrench is the smallest that she can use?
d. What effect does the length of the handle of a wrench have on the torque that can be generated by it?
e. To achieve the maximum torque from a given applied force to a lever arm such as a wrench, at what angle should the force be applied to the lever or wrench?

3. A meter stick is balanced perfectly on a fulcrum at the 0.5-m mark.

a. If a 5-N weight is added to the very tip of the zero side of the meter stick, where do you need to place a 10-N weight to rebalance the stick?
b. If you use an 8-N weight instead of a 10-N weight to balance the stick, where would you place it?
c. If the second weight is 3 N instead of 10 N, where should you place it to balance the meter stick? Can the meter stick be balanced at all?
d. If 1-kg and 2-kg masses are used in part a, where should these masses be placed for the stick to balance?
e. What are the torques applied by each of the weights in parts a, b, and c?
f. Repeat parts a–d with the fulcrum placed at 0.7 m.
g. If the torques do not balance, what happens? Is angular momentum conserved?

4. What is the moment of inertia of each of the following objects?

a. A hollow sphere with mass 5 kg and radius 0.5 m
b. A solid ball that weighs 3 lb and has a radius of 1 foot
c. A 200-kg satellite in a circular orbit around a small planet at a distance 5000 km from the planet's center (Consider the satellite to be a point mass; the moment of inertia of a single particle is MR^2.)
d. A large truck tire of 0.75-m radius and mass 20 kg (assume all the mass is concentrated on the outer edge)

5. What is the angular momentum of the rotating objects in Problem 4 under the following circumstances?

a. When the spheres in parts a and b rotate at 2 revolutions per second?
b. When the spheres in parts a and b rotate at 1 radian per second?
c. When the satellite in part c makes 1 revolution every 90 minutes?
d. When the tire in part d spins at a rate of 1 revolution per second?

6. In what direction does the angular momentum vector point for the following situations (remember the right-hand rule)?
   a. A Ferris wheel spinning clockwise as you look at it
   b. A CD that spins counterclockwise as you look at it
   c. A bicycle wheel as the bike moves straight in a forward direction
   d. The left rear tire of a car moving straight backward in reverse
   e. The right rear tire of a car moving straight backward in reverse
7. If the direction of the angular momentum vector is pointed straight at you, in what direction does an object rotate?
8. Several children are playing on a merry-go-round in a park. Initially four of them, each weighing 20 kg, sit on the edge, 3 m from the center.
   a. If you neglect the weight of the merry-go-round, what is the initial angular momentum if it spins at a rate of 6 revolutions per minute?
   b. Not comfortable sitting on the edge of a spinning disk, the four children decide to walk to the center and sit halfway between the center and the edge, at 1.5 m. Will the angular velocity of the merry-go-round change? If so, what is the new angular velocity?
   c. Was angular momentum conserved when the children moved?
9. Astronomers know of collapsed stars called "pulsars" that rotate hundreds of times per second. The Sun (radius 7×10^8 m) now rotates once every 26 days. What would its radius have to be for it to rotate 100 times each second? Compare that radius to the size of the town you live in.

Investigations

1. Make a list of objects in your everyday life that rotate as part of their function. Think about the distribution of mass in these items and make notes on this. Based on your estimate of their mass distribution, rank the objects by how much torque is needed to cause them to experience an angular acceleration. Consider the distance the force is applied from the axis of rotation.
2. Research the development of gyroscopes. What are the principles behind them? What uses do they have?
3. Go to a pool hall or to a friend's house who has a pool table. Try putting different spins on the balls. Observe what happens to a spinning ball as it collides with another ball. Analyze this in terms of angular momentum. Is it conserved? Why or why not?
4. If you have the opportunity to work with a mechanic, plumber, or carpenter, ask to be shown how to use different hand tools and experiment with the effect of using tools of varying lengths. For example, try a long versus a short wrench, screwdriver, or hammer. Does your experience concerning how much force is needed to do your chosen task comform with what you have learned in this chapter?
5. Using the library, the Internet, or both, research how the helicopter was developed. What were some of the problems that had to be solved to ensure stable flight, and how were they solved?
6. Investigate how a pitcher can throw a curve ball. How does the spin of the ball differ for the different kinds of curve balls?
7. Use the web to investigate the discovery of dark matter by Vera Rubin of the Carnegie Institution of Washington. How did her observations of the rotation of galaxies reveal that much of the mass of the universe is invisible?
8. Occasionally two galaxies collide so that hundreds of billions of stars coalesce into one giant galaxy. Use the web to investigate this process. What happens to the angular momentum of the new combined galaxy compared to the original two galaxies?

WWW Resources

See the *Physics Matters* home page at **www.wiley.com/college/trefil** for valuable web links.

1. **www.physics.brocku.ca/faculty/sternin/120/applets/CircularMotion/** An applet showing position, velocity and acceleration for uniform circular motion from the Department of Physics at Brock University.
2. **www.physics.uoguelph.ca/tutorials/torque/Q.torque.html** The Rotational Motion Tutorial at the Department of Physics, University of Guelph.
3. **web.hep.uiuc.edu/home/g-gollin/dance/dance_physics.html** Dedicated to the physics of dance for aficionados of both.
4. **www.windows.ucar.edu/tour/link=/cool_stuff/tour_evolution_ss_1.html** Discusses solar system evolution, including current theories of system formation.