Electric charges exert a force on each other. If two charges are of opposite types, one positive and one negative, they each exert an attractive force on the other. If the two charges are the same type, each repels the other.

The magnitude of the force one point charge exerts on another is proportional to the product of their charges, and inversely proportional to the square of the distance between them:

$$F = k\frac{Q_1 Q_2}{r^2}; \tag{16–1}$$

this is **Coulomb's law**. In SI units, k is often written as $1/4\pi\epsilon_0$.

We think of an **electric field** as existing in space around any charge or group of charges. The force on another charged object is then said to be due to the electric field present at its location.

The *electric field*, $\vec{\mathbf{E}}$, at any point in space due to one or more charges, is defined as the force per unit charge that would act on a positive test charge q placed at that point:

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q}. \tag{16–3}$$

The magnitude of the electric field a distance r from a point charge Q is

$$E = k\frac{Q}{r^2}. \tag{16–4a}$$

The total electric field at a point in space is equal to the vector sum of the individual fields due to each contributing charge (**principle of superposition**).

Electric fields are represented by **electric field lines** that start on positive charges and end on negative charges. Their direction indicates the direction the force would be on a tiny positive test charge placed at a point. The lines can be drawn so that the number per unit area is proportional to the magnitude of E.

The static electric field inside a good conductor is zero, and the electric field lines just outside a charged conductor are perpendicular to its surface.

[*The **electric flux** passing through a small area A for a uniform electric field $\vec{\mathbf{E}}$ is

$$\Phi_E = E_{\perp}A, \tag{16–7}$$

where $E_{\perp}$ is the component of $\vec{\mathbf{E}}$ perpendicular to the surface. The flux through a surface is proportional to the number of field lines passing through it.]

[***Gauss's law** states that the total flux summed over any closed surface (considered as made up of many small areas ΔA) is equal to the net charge Q_{encl} enclosed by the surface divided by ϵ_0:

$$\sum_{\substack{\text{closed}\\ \text{surface}}} E_{\perp}\,\Delta A = \frac{Q_{encl}}{\epsilon_0}. \tag{16–9}$$

Gauss's law can be used to determine the electric field due to given charge distributions, but its usefulness is mainly limited to cases where the charge distribution displays much symmetry. The real importance of Gauss's law is that it is a general and elegant statement of the relation between electric charge and electric field.]

[*In the replication of DNA, the electrostatic force plays a crucial role in selecting the proper molecules so that the genetic information is passed on accurately from generation to generation.]

Questions

1. If you charge a pocket comb by rubbing it with a silk scarf, how can you determine if the comb is positively or negatively charged?
2. Why does a shirt or blouse taken from a clothes dryer sometimes cling to your body?
3. Explain why fog or rain droplets tend to form around ions or electrons in the air.
4. A positively charged rod is brought close to a neutral piece of paper, which it attracts. Draw a diagram showing the separation of charge and explain why attraction occurs.
5. Why does a plastic ruler that has been rubbed with a cloth have the ability to pick up small pieces of paper? Why is this difficult to do on a humid day?
6. Contrast the *net charge* on a conductor to the "free charges" in the conductor.
7. Figures 16–7 and 16–8 show how a charged rod placed near an uncharged metal object can attract (or repel) electrons. There are a great many electrons in the metal, yet only some of them move as shown. Why not all of them?
8. When an electroscope is charged, its two leaves repel each other and remain at an angle. What balances the electric force of repulsion so that the leaves don't separate further?
9. The form of Coulomb's law is very similar to that for Newton's law of universal gravitation. What are the differences between these two laws? Compare also gravitational mass and electric charge.
10. We are not normally aware of the gravitational or electric force between two ordinary objects. What is the reason in each case? Give an example where we are aware of each one and why.
11. Is the electric force a conservative force? Why or why not? (See Chapter 6.)
12. When a charged ruler attracts small pieces of paper, sometimes a piece jumps quickly away after touching the ruler. Explain.
13. Explain why the test charges we use when measuring electric fields must be small.
14. When determining an electric field, must we use a *positive* test charge, or would a negative one do as well? Explain.
15. Draw the electric field lines surrounding two negative electric charges a distance l apart.
16. Assume that the two opposite charges in Fig. 16–31a are 12.0 cm apart. Consider the magnitude of the electric field 2.5 cm from the positive charge. On which side of this charge—top, bottom, left, or right—is the electric field the strongest? The weakest? Explain.

17. Consider the electric field at points A, B, and C in Fig. 16–48. First draw an arrow at each point indicating the direction of the net force that a positive test charge would experience if placed at that point, then list the points in order of *decreasing* field strength (strongest first).

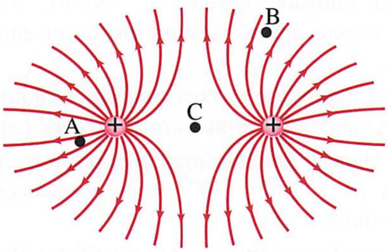

FIGURE 16–48 Question 17.

18. Why can electric field lines never cross?

19. Show, using the three rules for field lines given in Section 16–8, that the electric field lines starting or ending on a single point charge must be symmetrically spaced around the charge.

20. Given two point charges Q and $2Q$, a distance l apart, is there a point along the straight line that passes through them where $E = 0$ when their signs are (*a*) opposite, (*b*) the same? If yes, state roughly where this point will be.

21. Consider a small positive test charge located on an electric field line at some point, such as point P in Fig. 16–31a. Is the direction of the velocity and/or acceleration of the test charge along this line? Discuss.

22. Sketch the electric field lines for a uniform line of charge which is infinitely long. (*Hint*: Use symmetry.) Is the electric field uniform in strength?

* **23.** If the electric flux through a closed surface is zero, is the electric field necessarily zero at all points on the surface? Explain. What about the converse: If $\vec{\mathbf{E}} = 0$ at all points on the surface is the flux through the surface zero?

* **24.** A point charge is surrounded by a spherical gaussian surface of radius r. If the sphere is replaced by a cube of side r, will Φ_E be larger, smaller, or the same? Explain.

Problems

16–5 and 16–6 Coulomb's Law

[$1\text{ mC} = 10^{-3}\text{ C}$, $1\ \mu\text{C} = 10^{-6}\text{ C}$, $1\text{ nC} = 10^{-9}\text{ C}$.]

1. (I) Calculate the magnitude of the force between two 3.60-μC point charges 9.3 cm apart.

2. (I) How many electrons make up a charge of $-30.0\ \mu$C?

3. (I) What is the magnitude of the electric force of attraction between an iron nucleus ($q = +26e$) and its innermost electron if the distance between them is 1.5×10^{-12} m?

4. (I) What is the repulsive electrical force between two protons 5.0×10^{-15} m apart from each other in an atomic nucleus?

5. (I) What is the magnitude of the force a $+25\ \mu$C charge exerts on a $+3.0$ mC charge 35 cm away?

6. (II) Two charged dust particles exert a force of 3.2×10^{-2} N on each other. What will be the force if they are moved so they are only one-eighth as far apart?

7. (II) Two charged spheres are 8.45 cm apart. They are moved, and the force on each of them is found to have been tripled. How far apart are they now?

8. (II) A person scuffing her feet on a wool rug on a dry day accumulates a net charge of $-42\ \mu$C. How many excess electrons does she get, and by how much does her mass increase?

9. (II) What is the total charge of all the electrons in 1.0 kg of H_2O?

10. (II) Compare the electric force holding the electron in orbit ($r = 0.53 \times 10^{-10}$ m) around the proton nucleus of the hydrogen atom, with the gravitational force between the same electron and proton. What is the ratio of these two forces?

11. (II) Two positive point charges are a fixed distance apart. The sum of their charges is Q_T. What charge must each have in order to (*a*) maximize the electric force between them, and (*b*) minimize it?

12. (II) Particles of charge +75, +48, and $-85\ \mu$C are placed in a line (Fig. 16–49). The center one is 0.35 m from each of the others. Calculate the net force on each charge due to the other two.

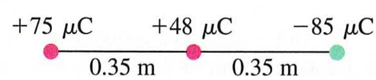

FIGURE 16–49 Problem 12.

13. (II) Three positive particles of equal charge, $+11.0\ \mu$C, are located at the corners of an equilateral triangle of side 15.0 cm (Fig. 16–50). Calculate the magnitude and direction of the net force on each particle.

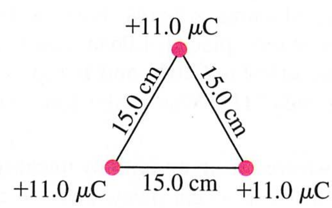

FIGURE 16–50 Problem 13.

14. (II) A charge of 6.00 mC is placed at each corner of a square 0.100 m on a side. Determine the magnitude and direction of the force on each charge.

15. (II) Repeat Problem 14 for the case when two of the positive charges, on opposite corners, are replaced by negative charges of the same magnitude (Fig. 16–51).

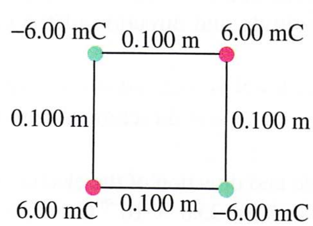

FIGURE 16–51 Problem 15.

16. (II) At each corner of a square of side l there are point charges of magnitude Q, $2Q$, $3Q$, and $4Q$ (Fig. 16–52). Determine the force on (*a*) the charge $2Q$, and (*b*) the charge $3Q$, due to the other three charges.

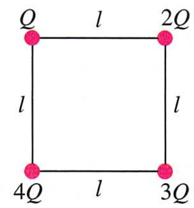

FIGURE 16–52 Problem 16.

17. (II) Three charged particles are placed at the corners of an equilateral triangle of side 1.20 m (Fig. 16–53). The charges are $+4.0\,\mu C$, $-8.0\,\mu C$, and $-6.0\,\mu C$. Calculate the magnitude and direction of the net force on each due to the other two.

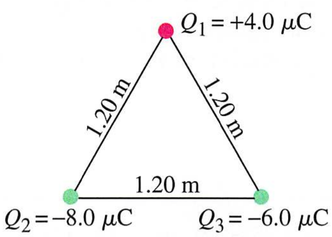

FIGURE 16–53 Problem 17.

18. (III) Two point charges have a total charge of $560\,\mu C$. When placed 1.10 m apart, the force each exerts on the other is 22.8 N and is repulsive. What is the charge on each?

19. (III) Two charges, $-Q_0$ and $-3Q_0$, are a distance l apart. These two charges are free to move but do not because there is a third charge nearby. What must be the charge and placement of the third charge for the first two to be in equilibrium?

20. (III) A $+4.75\,\mu C$ and a $-3.55\,\mu C$ charge are placed 18.5 cm apart. Where can a third charge be placed so that it experiences no net force?

21. (III) Two small nonconducting spheres have a total charge of $90.0\,\mu C$. (*a*) When placed 1.06 m apart, the force each exerts on the other is 12.0 N and is repulsive. What is the charge on each? (*b*) What if the force were attractive?

22. (III) A charge Q is transferred from an initially uncharged plastic ball to an identical ball 12 cm away. The force of attraction is then 17 mN. How many electrons were transferred from one ball to the other?

16–7 and 16–8 Electric Field, Field Lines

23. (I) What are the magnitude and direction of the electric force on an electron in a uniform electric field of strength 2360 N/C that points due east?

24. (I) A proton is released in a uniform electric field, and it experiences an electric force of 3.75×10^{-14} N toward the south. What are the magnitude and direction of the electric field?

25. (I) A downward force of 8.4 N is exerted on a $-8.8\,\mu C$ charge. What are the magnitude and direction of the electric field at this point?

26. (I) What are the magnitude and direction of the electric field 20.0 cm directly above an isolated 33.0×10^{-6} C charge?

27. (II) What is the magnitude of the acceleration experienced by an electron in an electric field of 750 N/C? How does the direction of the acceleration depend on the direction of the field at that point?

28. (II) What are the magnitude and direction of the electric field at a point midway between a $-8.0\,\mu C$ and a $+7.0\,\mu C$ charge 8.0 cm apart? Assume no other charges are nearby.

29. (II) Draw, approximately, the electric field lines about two point charges, $+Q$ and $-3Q$, which are a distance l apart.

30. (II) What is the electric field strength at a point in space where a proton $(m = 1.67 \times 10^{-27}\,\text{kg})$ experiences an acceleration of 1 million "g's"?

31. (II) An electron is released from rest in a uniform electric field and accelerates to the north at a rate of $115\,\text{m/s}^2$. What are the magnitude and direction of the electric field?

32. (II) The electric field midway between two equal but opposite point charges is 745 N/C, and the distance between the charges is 16.0 cm. What is the magnitude of the charge on each?

33. (II) Calculate the electric field at the center of a square 52.5 cm on a side if one corner is occupied by a $+45.0\,\mu C$ charge and the other three are occupied by $-27.0\,\mu C$ charges.

34. (II) Calculate the electric field at one corner of a square 1.00 m on a side if the other three corners are occupied by 2.25×10^{-6} C charges.

35. (II) Determine the direction and magnitude of the electric field at the point P in Fig. 16–54. The charges are separated by a distance $2a$, and point P is a distance x from the midpoint between the two charges. Express your answer in terms of Q, x, a, and k.

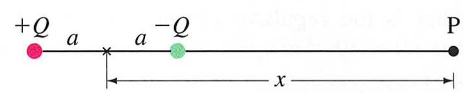

FIGURE 16–54 Problem 35.

36. (II) Two point charges, $Q_1 = -25\,\mu C$ and $Q_2 = +50\,\mu C$, are separated by a distance of 12 cm. The electric field at the point P (see Fig. 16–55) is zero. How far from Q_1 is P?

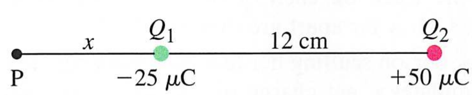

FIGURE 16–55 Problem 36.

37. (II) (*a*) Determine the electric field $\vec{\mathbf{E}}$ at the origin 0 in Fig. 16–56 due to the two charges at A and B. (*b*) Repeat, but let the charge at B be reversed in sign.

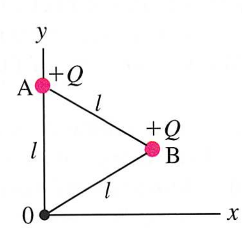

FIGURE 16–56 Problem 37.

38. (II) Use Coulomb's law to determine the magnitude and direction of the electric field at points A and B in Fig. 16–57 due to the two positive charges ($Q = 7.0\,\mu C$) shown. Are your results consistent with Fig. 16–31b?

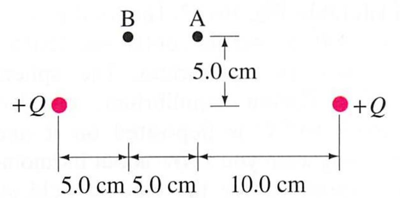

FIGURE 16–57 Problem 38.

39. (II) You are given two unknown point charges, Q_1 and Q_2. At a point on the line joining them, one-third of the way from Q_1 to Q_2, the electric field is zero (Fig. 16–58). What is the ratio Q_1/Q_2?

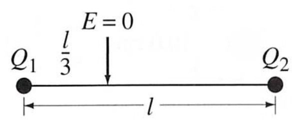

FIGURE 16–58 Problem 39.

40. (III) Determine the direction and magnitude of the electric field at the point P shown in Fig. 16–59. The two charges are separated by a distance of $2a$. Point P is on the perpendicular bisector of the line joining the charges, a distance x from the midpoint between them. Express your answers in terms of Q, x, a, and k.

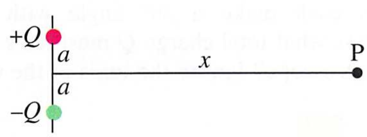

FIGURE 16–59 Problem 40.

41. (III) An electron (mass $m = 9.11 \times 10^{-31}$ kg) is accelerated in the uniform field $\vec{\mathbf{E}}$ ($E = 1.45 \times 10^4$ N/C) between two parallel charged plates. The separation of the plates is 1.10 cm. The electron is accelerated from rest near the negative plate and passes through a tiny hole in the positive plate, Fig. 16–60. (*a*) With what speed does it leave the hole? (*b*) Show that the gravitational force can be ignored.

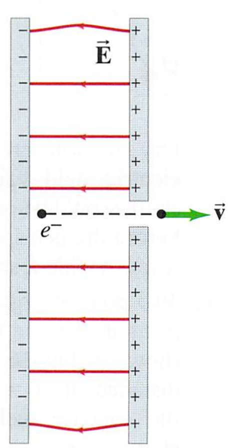

FIGURE 16–60 Problem 41.

42. (III) An electron moving to the right at 1.0% the speed of light enters a uniform electric field parallel to its direction of motion. If the electron is to be brought to rest in the space of 4.0 cm, (*a*) what direction is required for the electric field, and (*b*) what is the strength of the field?

* 16–10 Gauss's Law

* **43.** (I) The total electric flux from a cubical box 28.0 cm on a side is $1.45 \times 10^3\,N \cdot m^2/C$. What charge is enclosed by the box?

* **44.** (II) A flat circle of radius 18 cm is placed in a uniform electric field of magnitude 5.8×10^2 N/C. What is the electric flux through the circle when its face is (*a*) perpendicular to the field lines, (*b*) at 45° to the field lines, and (*c*) parallel to the field lines?

* **45.** (II) In Fig. 16–61, two objects, O_1 and O_2, have charges $+1.0\,\mu C$ and $-2.0\,\mu C$, respectively, and a third object, O_3, is electrically neutral. (*a*) What is the electric flux through the surface A_1 that encloses all three objects? (*b*) What is the electric flux through the surface A_2 that encloses the third object only?

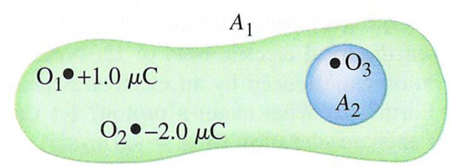

FIGURE 16–61 Problem 45.

* **46.** (II) A cube of side l is placed in a uniform field $E = 6.50 \times 10^3$ N/C with edges parallel to the field lines. (*a*) What is the net flux through the cube? (*b*) What is the flux through each of its six faces?

* **47.** (II) The electric field between two square metal plates is 130 N/C. The plates are 1.0 m on a side and are separated by 3.0 cm. What is the charge on each plate (assume equal and opposite)? Neglect edge effects.

* **48.** (II) The field just outside a 3.50-cm-radius metal ball is 2.75×10^2 N/C and points toward the ball. What charge resides on the ball?

* **49.** (II) A solid metal sphere of radius 3.00 m carries a total charge of $-3.50\,\mu C$. What is the magnitude of the electric field at a distance from the sphere's center of (*a*) 0.15 m, (*b*) 2.90 m, (*c*) 3.10 m, and (*d*) 6.00 m? (*e*) How would the answers differ if the sphere were a thin shell?

* **50.** (III) A point charge Q rests at the center of an uncharged thin spherical conducting shell. (See Fig. 16–33.) What is the electric field E as a function of r (*a*) for r less than the inner radius of the shell, (*b*) inside the shell, and (*c*) beyond the shell? (*d*) Does the shell affect the field due to Q alone? Does the charge Q affect the shell?

* 16–11 DNA

* **51.** (III) The two strands of the helix-shaped DNA molecule are held together by electrostatic forces as shown in Fig. 16–44. Assume that the net average charge (due to electron sharing) indicated on H and N atoms is $0.2e$ and on the indicated C and O atoms is $0.4e$. Assume also that atoms on each molecule are separated by 1.0×10^{-10} m. Estimate the net force between (*a*) a thymine and an adenine; and (*b*) a cytosine and a guanine. For each bond (red dots) consider only the three atoms in a line (two atoms on one molecule, one atom on the other). (*c*) Estimate the total force for a DNA molecule containing 10^5 pairs of such molecules.

General Problems

52. How close must two electrons be if the electric force between them is equal to the weight of either at the Earth's surface?

53. A 3.0-g copper penny has a positive charge of $38\,\mu C$. What fraction of its electrons has it lost?

54. A proton $(m = 1.67 \times 10^{-27}\,kg)$ is suspended at rest in a uniform electric field $\vec{E}$. Take into account gravity at the Earth's surface, and determine $\vec{E}$.

55. Measurements indicate that there is an electric field surrounding the Earth. Its magnitude is about 150 N/C at the Earth's surface and points inward toward the Earth's center. What is the magnitude of the electric charge on the Earth? Is it positive or negative? [*Hint*: the electric field outside a uniformly charged sphere is the same as if all the charge were concentrated at its center.]

56. (*a*) Given the local electric field of 150 N/C, what is the acceleration experienced by an electron near the surface of the Earth? (*b*) What about a proton? (*c*) Calculate the ratio of each acceleration to $g = 9.8\,m/s^2$.

57. A water droplet of radius 0.018 mm remains stationary in the air. If the downward-directed electric field of the Earth is 150 N/C, how many excess electron charges must the water droplet have?

58. Estimate the net force between the CO group and the HN group shown in Fig. 16–62. The C and O have charges $\pm 0.40e$, and the H and N have charges $\pm 0.20e$, where $e = 1.6 \times 10^{-19}\,C$. [*Hint*: do not include the "internal" forces between C and O, or between H and N.]

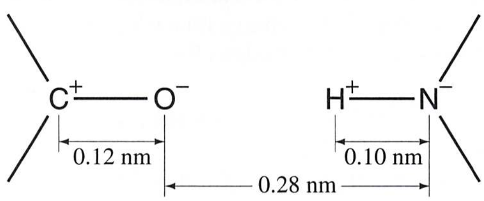

FIGURE 16–62 Problem 58.

59. In a simple model of the hydrogen atom, the electron revolves in a circular orbit around the proton with a speed of $1.1 \times 10^6\,m/s$. Determine the radius of the electron's orbit. [*Hint*: see Chapter 5 on circular motion.]

60. Suppose that electrical attraction, rather than gravity, were responsible for holding the Moon in orbit around the Earth. If equal and opposite charges Q were placed on the Earth and the Moon, what should be the value of Q to maintain the present orbit? Use these data: mass of Earth $= 5.98 \times 10^{24}\,kg$, mass of Moon $= 7.35 \times 10^{22}\,kg$, radius of orbit $= 3.84 \times 10^8\,m$. Treat the Earth and Moon as point particles.

61. An electron with speed $v_0 = 21.5 \times 10^6\,m/s$ is traveling parallel to an electric field of magnitude $E = 11.4 \times 10^3\,N/C$. (*a*) How far will the electron travel before it stops? (*b*) How much time will elapse before it returns to its starting point?

62. A positive point charge $Q_1 = 2.5 \times 10^{-5}\,C$ is fixed at the origin of coordinates, and a negative charge $Q_2 = -5.0 \times 10^{-6}\,C$ is fixed to the x axis at $x = +2.0\,m$. Find the location of the place(s) along the x axis where the electric field due to these two charges is zero.

63. A small lead sphere is encased in insulating plastic and suspended vertically from an ideal spring ($k = 126\,N/m$) above a lab table, Fig. 16–63. The total mass of the coated sphere is 0.800 kg, and its center lies 15.0 cm above the tabletop when in equilibrium. The sphere is pulled down 5.00 cm below equilibrium, an electric charge $Q = -3.00 \times 10^{-6}\,C$ is deposited on it and then it is released. Using what you know about harmonic oscillation, write an expression for the electric field strength as a function of time that would be measured at the point on the tabletop (P) directly below the sphere.

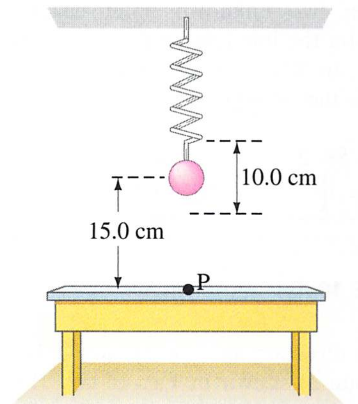

FIGURE 16–63 Problem 63.

64. A large electroscope is made with "leaves" that are 78-cm-long wires with tiny 24-g spheres at the ends. When charged, nearly all the charge resides on the spheres. If the wires each make a 30° angle with the vertical (Fig. 16–64), what total charge Q must have been applied to the electroscope? Ignore the mass of the wires.

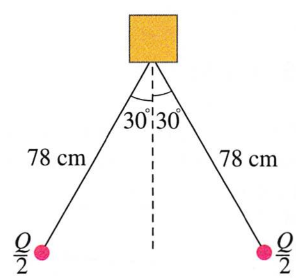

FIGURE 16–64 Problem 64.

65. Dry air will break down and generate a spark if the electric field exceeds about $3 \times 10^6\,N/C$. How much charge could be packed onto a green pea (diameter 0.75 cm) before the pea spontaneously discharges? [*Hint*: Eqs. 16–4 work outside a sphere if r is measured from its center.]

66. Two point charges, $Q_1 = -6.7\,\mu C$ and $Q_2 = 1.8\,\mu C$ are located between two oppositely charged parallel plates, as shown in Fig. 16–65. The two charges are separated by a distance of $x = 0.34\,m$. Assume that the electric field produced by the charged plates is uniform and equal to $E = 73{,}000\,N/C$. Calculate the net electrostatic force on Q_1 and give its direction.

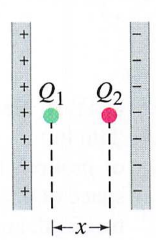

FIGURE 16–65 Problem 66.

67. A point charge ($m = 1.0$ g) at the end of an insulating string of length 55 cm is observed to be in equilibrium in a uniform horizontal electric field of 12,000 N/C, when the pendulum's position is as shown in Fig. 16–66, with the charge 12 cm above the lowest (vertical) position. If the field points to the right in Fig. 16–66, determine the magnitude and sign of the point charge.

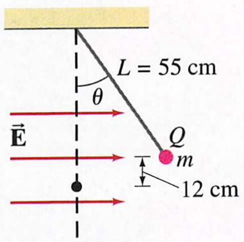

FIGURE 16–66 Problem 67.

68. A point charge of mass 0.210 kg, and net charge $+0.340\,\mu C$, hangs at rest at the end of an insulating string above a large sheet of charge. The horizontal sheet of uniform charge creates a uniform vertical electric field in the vicinity of the point charge. The tension in the string is measured to be 5.67 N. Calculate the magnitude and direction of the electric field due to the sheet of charge (Fig. 16–67).

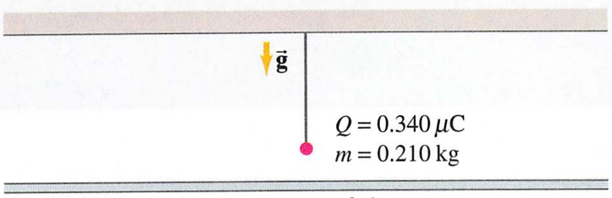

FIGURE 16–67 Problem 68.

69. What is the total charge of all the electrons in a 15-kg bar of aluminum? What is the net charge of the bar? (Aluminum has 13 electrons per atom and an atomic mass of 27 u.)

70. Two small, identical conducting spheres A and B are a distance R apart; each carries the same charge Q. (*a*) What is the force sphere B exerts on sphere A? (*b*) An identical sphere with zero charge, sphere C, makes contact with sphere B and is then moved very far away. What is the net force now acting on sphere A? (*c*) Sphere C next makes contact with sphere A and is then moved far away. What is the force on sphere A in this third case?

71. Given the two charges shown in Fig. 16–68, at what position(s) x is the electric field zero? Is the field zero at any other points, not on the x axis?

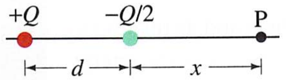

FIGURE 16–68 Problem 71.

72. Two point charges, $+Q$ and $-Q$ of mass m, are placed on the ends of a massless rod of length L, which is fixed to a table by a pin through its center. If the apparatus is then subjected to a uniform electric field E parallel to the table and perpendicular to the rod, find the net torque on the system of rod plus charges.

73. Four equal positive point charges, each of charge $8.0\,\mu C$, are at the corners of a square of side 9.2 cm. What charge should be placed at the center of the square so that all charges are at equilibrium? Is this a stable or unstable equilibrium (Section 9–4) in the plane?

Answers to Exercises

A: Opposite.
B: 5 N.
C: 1.2 N, to the right.
D: 0.32 m.
E: (*a*) No; (*b*) yes, midway between them.
F: 9.0×10^4 N/C, vertically down.
G: (*a*) $\vec{E}_1$ right; $\vec{E}_2$ left; $\vec{E}$ right; (*b*) $\vec{E}_1$ left; $\vec{E}_2$ right; $\vec{E}$ right.
H: $\sum E_{\perp}\,\Delta A = E\sum \Delta A = E(2\pi r L) = \dfrac{Q_{\text{encl}}}{\epsilon_0}$, so $E = \dfrac{1}{2\pi\epsilon_0 r}\dfrac{Q}{L}$.