At speeds much less than the speed of light, the relativistic formulas reduce to the old classical ones, as we have discussed. We would, of course, hope—or rather, insist—that this be true since Newtonian mechanics works so well for objects moving with speeds $v \ll c$. This insistence that a more general theory (such as relativity) give the same results as a more restricted theory (such as classical mechanics which works for $v \ll c$) is called the **correspondence principle**. The two theories must correspond where their realms of validity overlap. Relativity thus does not contradict classical mechanics. Rather, it is a more general theory, of which classical mechanics is now considered to be a limiting case.

Correspondence principle

The importance of relativity is not simply that it gives more accurate results, especially at very high speeds. Much more than that, it has changed the way we view the world. The concepts of space and time are now seen to be relative, and intertwined with one another, whereas before they were considered absolute and separate. Even our concepts of matter and energy have changed: either can be converted to the other. The impact of relativity extends far beyond physics. It has influenced the other sciences, and even the world of art and literature; it has, indeed, entered the general culture.

From a practical point of view, we do not have much opportunity in our daily lives to use the mathematics of relativity. For example, the γ factor $1/\sqrt{1 - v^2/c^2}$, which appears in relativistic formulas, has a value of only 1.005 even for a speed as high as $0.10c = 3.0 \times 10^7\ \mathrm{m/s}$, giving a correction of less than 1%. For speeds less than $0.10c$, or unless mass and energy are interchanged, we don't usually need to use the more complicated relativistic formulas, and can use the simpler classical formulas.

The special theory of relativity we have studied in this Chapter deals with inertial (nonaccelerating) reference frames. In Chapter 33 we will discuss briefly the more complicated "general theory of relativity" which can deal with noninertial reference frames.

Summary

An **inertial reference frame** is one in which Newton's law of inertia holds. Inertial reference frames can move at constant velocity relative to one another; accelerating reference frames are **noninertial**.

The **special theory of relativity** is based on two principles: the **relativity principle**, which states that the laws of physics are the same in all inertial reference frames, and the principle of the **constancy of the speed of light**, which states that the speed of light in empty space has the same value in all inertial reference frames.

One consequence of relativity theory is that two events that are simultaneous in one reference frame may not be simultaneous in another. Other effects are **time dilation**: moving clocks are measured to run slow; and **length contraction**: the length of a moving object is measured to be shorter (in its direction of motion) than when it is at rest. Quantitatively,

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}} = \gamma\, \Delta t_0 \qquad \textbf{(26–1)}$$

$$L = L_0\sqrt{1 - v^2/c^2} = \frac{L_0}{\gamma} \qquad \textbf{(26–3)}$$

where L and Δt are the length and time interval of objects (or events) observed as they move by at the speed v; L_0 and Δt_0 are the **proper length** and **proper time**—that is, the same quantities as measured in the rest frame of the objects or events. The quantity γ is shorthand for

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}. \qquad \textbf{(26–2)}$$

The theory of relativity has changed our notions of space and time, and of momentum, energy, and mass. Space and time are seen to be intimately connected, with time being the fourth dimension in addition to the three dimensions of space.

The **momentum** of an object is given by

$$p = \gamma m_0 v = \frac{m_0 v}{\sqrt{1 - v^2/c^2}}. \qquad \textbf{(26–4)}$$

This formula can be interpreted as a **mass increase**, where the relativistic mass is

$$m_{\mathrm{rel}} = \gamma m_0 = \frac{m_0}{\sqrt{1 - v^2/c^2}}, \qquad \textbf{(26–5)}$$

and m_0 is the **rest mass** of the object $(v = 0)$.

Mass and energy are interconvertible. The equation

$$E_0 = m_0c^2 \tag{26–8}$$

tells how much energy E_0 is needed to create a mass m_0, or vice versa. Said another way, $E_0 = m_0c^2$ is the amount of energy an object has because of its mass m_0. The law of conservation of energy must include mass as a form of energy.

The kinetic energy KE of an object moving at speed v is given by

$$\text{KE} = \frac{m_0c^2}{\sqrt{1 - v^2/c^2}} - m_0c^2 = (\gamma - 1)m_0c^2 \tag{26–6}$$

where m_0 is the rest mass of the object. The total energy E, if there is no potential energy, is

$$E = \text{KE} + m_0c^2$$
$$= \gamma m_0c^2. \tag{26–7}$$

The momentum p of an object is related to its total energy E (assuming no potential energy) by

$$E^2 = p^2c^2 + m_0^2c^4. \tag{26–10}$$

Velocity addition also must be done in a special way. All these relativistic effects are significant only at high speeds, close to the speed of light, which itself is the ultimate speed in the universe.

Questions

1. You are in a windowless car in an exceptionally smooth train moving at constant velocity. Is there any physical experiment you can do in the train car to determine whether you are moving? Explain.
2. You might have had the experience of being at a red light when, out of the corner of your eye, you see the car beside you creep forward. Instinctively you stomp on the brake pedal, thinking that you are rolling backward. What does this say about absolute and relative motion?
3. A worker stands on top of a moving railroad car, and throws a heavy ball straight up (from his point of view). Ignoring air resistance, will the ball land on the car or behind it?
4. Does the Earth really go around the Sun? Or is it also valid to say that the Sun goes around the Earth? Discuss in view of the first principle of relativity (that there is no best reference frame). Explain.
5. If you were on a spaceship traveling at $0.5c$ away from a star, at what speed would the starlight pass you?
6. The time dilation effect is sometimes expressed as "moving clocks run slowly." Actually, this effect has nothing to do with motion affecting the functioning of clocks. What then does it deal with?
7. Does time dilation mean that time actually passes more slowly in moving reference frames or that it only *seems* to pass more slowly?
8. A young-looking woman astronaut has just arrived home from a long trip. She rushes up to an old gray-haired man and in the ensuing conversation refers to him as her son. How might this be possible?
9. If you were traveling away from Earth at speed $0.5c$, would you notice a change in your heartbeat? Would your mass, height, or waistline change? What would observers on Earth using telescopes say about you?
10. Do time dilation and length contraction occur at ordinary speeds, say 90 km/h?
11. Suppose the speed of light were infinite. What would happen to the relativistic predictions of length contraction and time dilation?
12. Discuss how our everyday lives would be different if the speed of light were only 25 m/s.
13. Explain how the length contraction and time dilation formulas might be used to indicate that c is the limiting speed in the universe.
14. The drawing at the start of this Chapter shows the street as seen by Mr Tompkins, for whom the speed of light is $c = 20$ mi/h. What does Mr Tompkins look like to the people standing on the street (Fig. 26–11)? Explain.

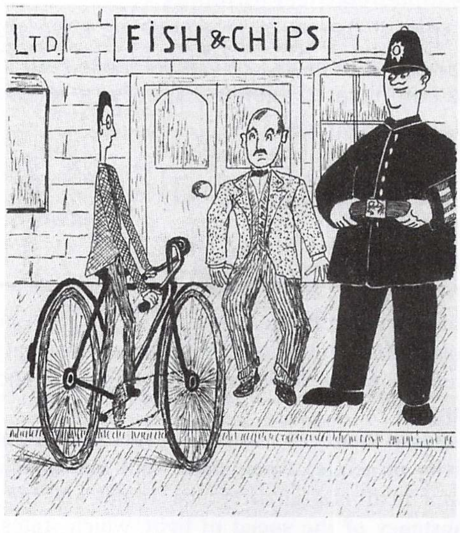

FIGURE 26–11 Question 14. Mr Tompkins as seen by people on the sidewalk. See also Chapter-opening photograph.

15. An electron is limited to travel at speeds less than c. Does this put an upper limit on the momentum of an electron? If so, what is this upper limit? If not, explain.
16. Can a particle of nonzero rest mass attain the speed of light?
17. Does the equation $E = mc^2$ conflict with the conservation of energy principle? Explain.
18. If mass is a form of energy, does this mean that a spring has more mass when compressed than when relaxed?
19. It is not correct to say that "matter can neither be created nor destroyed." What must we say instead?
20. Is our intuitive notion that velocities simply add, as we did in Section 3–8, completely wrong?

Problems

26–4 and 26–5 Time Dilation, Length Contraction

1. (I) A spaceship passes you at a speed of $0.750c$. You measure its length to be 28.2 m. How long would it be when at rest?

2. (I) A certain type of elementary particle travels at a speed of 2.70×10^8 m/s. At this speed, the average lifetime is measured to be 4.76×10^{-6} s. What is the particle's lifetime at rest?

3. (I) Lengths and time intervals depend on the factor
$$\sqrt{1 - v^2/c^2}$$
according to the theory of relativity (Eqs. 26–1 and 26–3). Evaluate this factor for speeds of: (*a*) $v = 20{,}000$ m/s (typical speed of a satellite); (*b*) $v = 0.020c$; (*c*) $v = 0.200c$; (*d*) $v = 0.95c$; (*e*) $v = 0.98c$; (*f*) $v = 0.999c$.

4. (II) If you were to travel to a star 125 light-years from Earth at a speed of 2.50×10^8 m/s, what would you measure this distance to be?

5. (II) What is the speed of a pion if its average lifetime is measured to be 4.10×10^{-8} s? At rest, its average lifetime is 2.60×10^{-8} s.

6. (II) In an Earth reference frame, a star is 82 light-years away. How fast would you have to travel so that to you the distance would be only 35 light-years?

7. (II) Suppose you decide to travel to a star 85 light-years away at a speed that tells you the distance is only 25 light-years. How many years would it take you to make the trip?

8. (II) At what speed v will the length of a 1.00-m stick look 10.0% shorter (90.0 cm)?

9. (II) Escape velocity from the Earth is 40,000 km/h. What would be the percent decrease in length of a 95.2-m-long spacecraft traveling at that speed?

10. (II) At what speed do the relativistic formulas for (*a*) length and (*b*) time intervals differ from classical values by 1.00%? (This is a reasonable way to estimate when to do relativistic calculations rather than classical.)

11. (II) Suppose a news report stated that starship *Enterprise* had just returned from a 5-year voyage while traveling at $0.84c$. (*a*) If the report meant 5.0 years of *Earth time*, how much time elapsed on the ship? (*b*) If the report meant 5.0 years of *ship time*, how much time passed on Earth?

12. (II) A certain star is 10.6 light-years away. How long would it take a spacecraft traveling $0.960c$ to reach that star from Earth, as measured by observers: (*a*) on Earth, (*b*) on the spacecraft? (*c*) What is the distance traveled according to observers on the spacecraft? (*d*) What will the spacecraft occupants compute their speed to be from the results of (*b*) and (*c*)?

13. (II) A friend speeds by you in her "Ferrari" spacecraft at a speed of $0.660c$. It is measured in your frame to be 4.80 m long and 1.25 m high. (*a*) What will be its length and height at rest? (*b*) How many seconds would you say elapsed on your friend's watch when 20.0 s passed on yours? (*c*) How fast did you appear to be traveling according to your friend? (*d*) How many seconds would she say elapsed on your watch when she saw 20.0 s pass on hers?

14. (III) How fast must an average pion be moving to travel 15 m before it decays? The average lifetime, at rest, is 2.6×10^{-8} s.

26–7 Relativistic Momentum

15. (I) What is the momentum of a proton traveling at $v = 0.85c$?

16. (I) At what speed will an object's relativistic mass be twice its rest mass?

17. (II) A particle of rest mass m_0 travels at a speed $v = 0.20c$. At what speed will its momentum be doubled?

18. (II) (*a*) A particle travels at $v = 0.10c$. By what percentage will a calculation of its momentum be wrong if you use the classical formula? (*b*) Repeat for $v = 0.50c$.

19. (II) What is the percent change in momentum of a proton that accelerates (*a*) from $0.45c$ to $0.90c$, (*b*) from $0.90c$ to $0.98c$?

26–9 $E = mc^2$

20. (I) A certain chemical reaction requires 4.82×10^4 J of energy input for it to go. What is the increase in mass of the products over the reactants?

21. (I) When a uranium nucleus at rest breaks apart in the process known as fission in a nuclear reactor, the resulting fragments have a total kinetic energy of about 200 MeV. How much mass was lost in the process?

22. (I) Calculate the rest energy of an electron in joules and in MeV ($1 \text{ MeV} = 1.60 \times 10^{-13}$ J).

23. (I) Calculate the rest mass of a proton in MeV/c^2.

24. (I) The total annual energy consumption in the United States is about 8×10^{19} J. How much mass would have to be converted to energy to fuel this need?

25. (II) How much energy can be obtained from conversion of 1.0 gram of mass? How much mass could this energy raise to a height of 0.25 km above the Earth's surface?

26. (II) What is the speed of a particle when its kinetic energy equals its rest energy?

27. (II) At what speed will an object's kinetic energy be 25% of its rest energy?

28. (II) (*a*) How much work is required to accelerate a proton from rest up to a speed of $0.997c$? (*b*) What would be the momentum of this proton?

29. (II) Calculate the kinetic energy and momentum of a proton traveling 2.60×10^8 m/s.

30. (II) What is the momentum of a 750-MeV proton (that is, its kinetic energy is 750 MeV)?

31. (II) What is the speed of a proton accelerated by a potential difference of 105 MV?

32. (II) What is the speed of an electron whose kinetic energy is 1.00 MeV?

33. (II) What is the speed of an electron just before it hits a television screen after being accelerated from rest by the 25,000 V of the picture tube?

34. (II) Two identical particles of rest mass m_0 approach each other at equal and opposite speeds, v. The collision is completely inelastic and results in a single particle at rest. What is the rest mass of the new particle? How much energy was lost in the collision? How much kinetic energy is lost in this collision?

35. (II) Calculate the speed of a proton $(m_0 = 1.67 \times 10^{-27}\,\text{kg})$ whose kinetic energy is exactly half (*a*) its total energy, (*b*) its rest energy.

36. (II) What is the speed and the momentum of an electron $(m = 9.11 \times 10^{-31}\,\text{kg})$ whose kinetic energy equals its rest energy?

37. (II) Suppose a spacecraft of mass 27,000 kg is accelerated to $0.21c$. (*a*) How much kinetic energy would it have? (*b*) If you used the classical formula for kinetic energy, by what percentage would you be in error?

38. (II) Calculate the kinetic energy and momentum of a proton $(m_0 = 1.67 \times 10^{-27}\,\text{kg})$ traveling $7.35 \times 10^7\,\text{m/s}$. By what percentages would your calculations have been in error if you had used classical formulas?

39. (II) The americium nucleus, ${}^{241}_{95}\text{Am}$, decays to a neptunium nucleus, ${}^{237}_{93}\text{Np}$, by emitting an alpha particle of mass 4.00260 u and kinetic energy 5.5 MeV. Estimate the mass of the neptunium nucleus, ignoring its recoil, given that the americium mass is 241.05682 u.

40. (II) An electron $(m_0 = 9.11 \times 10^{-31}\,\text{kg})$ is accelerated from rest to speed v by a conservative force. In this process, its potential energy decreases by $6.60 \times 10^{-14}\,\text{J}$. Determine the electron's speed, v.

41. (II) Make a graph of the kinetic energy versus momentum for (*a*) a particle of nonzero rest mass, and (*b*) a particle with zero rest mass.

42. (II) What magnetic field intensity is needed to keep 998-GeV protons revolving in a circle of radius 1.0 km (at, say, the Fermilab synchrotron)? Use the relativistic mass. The proton's rest mass is $0.938\,\text{GeV}/c^2$. ($1\,\text{GeV} = 10^9\,\text{eV}$.) [*Hint*: in relativity, it is still true that $mv^2/r = qvB$ in a magnetic field.]

26–10 Relativistic Addition of Velocities

43. (I) A person on a rocket traveling at $0.50c$ (with respect to the Earth) observes a meteor come from behind and pass her at a speed she measures as $0.50c$. How fast is the meteor moving with respect to the Earth?

44. (II) Two spaceships leave Earth in opposite directions, each with a speed of $0.50c$ with respect to Earth. (*a*) What is the velocity of spaceship 1 relative to spaceship 2? (*b*) What is the velocity of spaceship 2 relative to spaceship 1?

45. (II) A spaceship leaves Earth traveling at $0.71c$. A second spaceship leaves the first at a speed of $0.87c$ with respect to the first. Calculate the speed of the second ship with respect to Earth if it is fired (*a*) in the same direction the first spaceship is already moving, (*b*) directly backward toward Earth.

46. (II) An observer on Earth sees an alien vessel approach at a speed of $0.60c$. The *Enterprise* comes to the rescue (Fig. 26–12), overtaking the aliens while moving directly toward Earth at a speed of $0.90c$ relative to Earth. What is the relative speed of one vessel as seen by the other?

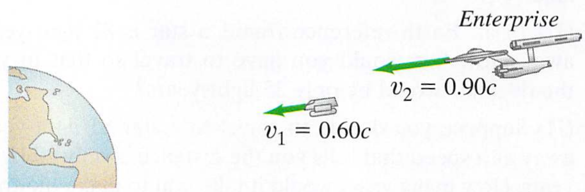

FIGURE 26–12 Problem 46.

47. (II) A spaceship in distress sends out two escape pods in opposite directions. One travels at a speed $v_1 = -0.60c$ in one direction, and the other travels at a speed $v_2 = +0.70c$ in the other direction, as observed from the spaceship. What speed does the first escape pod measure for the second escape pod?

48. (II) Rocket A passes Earth at a speed of $0.75c$. At the same time, rocket B passes Earth moving $0.95c$ relative to Earth in the same direction. How fast is B moving relative to A when it passes A?

General Problems

49. The nearest star to Earth is Proxima Centauri, 4.3 light-years away. (*a*) At what constant velocity must a spacecraft travel from Earth if it is to reach the star in 4.0 years, as measured by travelers on the spacecraft? (*b*) How long does the trip take according to Earth observers?

50. As a rule of thumb, anything traveling faster than about $0.1c$ is called *relativistic*—i.e., for which the correction using special relativity is a significant effect. Determine the speed of an electron in a hydrogen atom (radius $0.5 \times 10^{-10}\,\text{m}$) and state if it is relativistic. (Treat the electron as though it were in a circular orbit around the proton.)

51. (*a*) What is the speed v of an electron whose kinetic energy is 14,000 times its rest energy? You can state the difference, $c - v$. Such speeds are reached in the Stanford Linear Accelerator, SLAC. (*b*) If the electrons travel in the lab through a tube 3.0 km long (as at SLAC), how long is this tube in the electrons' reference frame? [*Hint*: use the binomial expansion.]

52. How many grams of matter would have to be totally destroyed to run a 100-W lightbulb for 1 year?

53. What minimum amount of electromagnetic energy is needed to produce an electron and a positron together? A positron is a particle with the same rest mass as an electron, but has the opposite charge. (Note that electric charge is conserved in this process. See Section 27–6.)

54. An electron $(m = 9.11 \times 10^{-31}\,\text{kg})$ enters a uniform magnetic field $B = 1.8\,\text{T}$, and moves perpendicular to the field lines with a speed $v = 0.92c$. What is the radius of curvature of its path? See hint for Problem 42.

55. A negative muon traveling at 33% the speed of light collides head on with a positive muon traveling at 50% the speed of light. The two muons (each of rest mass $105.7\,\text{MeV}/c^2$) annihilate, and produce how much electromagnetic energy?

56. A free neutron can decay into a proton, an electron, and a neutrino. Assume the neutrino's rest mass is zero, and the other masses can be found in the Table inside the front cover. Determine the total kinetic energy shared among the three particles when a neutron decays at rest.

57. The Sun radiates energy at a rate of about $4 \times 10^{26}\,\text{W}$. (*a*) At what rate is the Sun's mass decreasing? (*b*) How long does it take for the Sun to lose a mass equal to that of Earth? (*c*) Estimate how long the Sun could last if it radiated constantly at this rate.

58. An unknown particle is measured to have a negative charge and a speed of $2.24 \times 10^8\,\text{m/s}$. Its momentum is determined to be $3.07 \times 10^{-22}\,\text{kg}\cdot\text{m/s}$. Identify the particle by finding its rest mass.

59. How much energy would be required to break a helium nucleus into its constituents, two protons and two neutrons? The rest masses of a proton (including an electron), a neutron, and helium are, respectively, 1.00783 u, 1.00867 u, and 4.00260 u. (This energy difference is called the *total binding energy* of the ^4_2He nucleus.)

60. What is the percentage increase in the (relativistic) mass of a car traveling 110 km/h as compared to at rest? [*Hint*: use the binomial expansion.]

61. Two protons, each having a speed of $0.935c$ in the laboratory, are moving toward each other. Determine (*a*) the momentum of each proton in the laboratory, (*b*) the total momentum of the two protons in the laboratory, and (*c*) the momentum of one proton as seen by the other proton.

62. Show analytically that a particle with momentum p and energy E has a speed given by

$$v = \frac{pc^2}{E} = \frac{pc}{\sqrt{m_0^2c^2 + p^2}}.$$

63. The fictional starship *Enterprise* obtains its power by combining matter and antimatter, achieving complete conversion of mass into energy. If the mass of the *Enterprise* is approximately $5 \times 10^9\,\text{kg}$, how much mass must be converted into kinetic energy to accelerate it from rest to one-tenth the speed of light?

64. An electron is accelerated so that its kinetic energy is greater than its rest energy m_0c^2 by a factor of (*a*) 5.00, (*b*) 999. What is the speed of the electron in each case?

65. A farm boy studying physics believes that he can fit a 15.0-m-long pole into a 12.0-m-long barn if he runs fast enough (carrying the pole). Can he do it? Explain in detail. How does this fit with the idea that when he is running the barn looks even shorter to him than 12.0 m?

66. When two moles of hydrogen and one mole of oxygen react to form two moles of water, the energy released is 484 kJ. How much does the mass of the elements decrease in this reaction? What % of the total original mass of the system does this mass change represent?

67. In a nuclear reaction two identical particles are created, traveling in opposite directions. If the speed of each particle is $0.75c$, relative to the laboratory frame of reference, what is one particle's speed relative to the other particle?

68. An astronaut on a spaceship traveling at $0.75c$ relative to Earth measures his ship to be 25 m long. On the ship, he eats his lunch in 23 min. (*a*) What length is the spaceship according to observers on Earth? (*b*) How long does the astronaut's lunch take to eat according to observers on Earth?

69. You are traveling in a spaceship at a speed of $0.85c$ away from Earth. You send a laser beam toward the Earth traveling at velocity c relative to you. What do observers on the Earth measure for the speed of the laser beam?

70. A spaceship and its occupants have a total mass of 150,000 kg. The occupants would like to travel to a star that is 25 light-years away at a speed of $0.60c$. To accelerate, the engine of the spaceship changes mass directly to energy. How much mass will be converted to energy to accelerate the spaceship to this speed? Assume the acceleration is rapid, so the speed for the entire trip can be taken to be $0.60c$, and ignore decrease in total mass for the calculation. How long will the trip take according to the astronauts on board?

71. Suppose a 12,500-kg spaceship left Earth at a speed of $0.99c$. What is the spaceship's kinetic energy? Compare with the total U.S. annual energy consumption (about $10^{20}\,\text{J}$).

72. A 42,000-kg spaceship is to travel to the vicinity of a star 6.0 light-years from Earth. Passengers on the ship want the (one-way) trip to take no more than 1.0 year. How much work must be done on the spaceship to bring it to the speed necessary for this trip?

73. A 1.68-kg mass oscillates on the end of a spring whose spring stiffness constant is $k = 48.7\,\text{N/m}$. If this system is in a spaceship moving past Earth at $0.900c$, what is its period of oscillation according to (*a*) observers on the ship, and (*b*) observers on Earth?

74. A pi meson of rest mass m_π decays at rest into a muon (rest mass m_μ) and a neutrino of negligible or zero rest mass. Show that the kinetic energy of the muon is $\text{KE}_\mu = (m_\pi - m_\mu)^2c^2/2m_\pi$.

Answers to Exercises

A: Yes.
B: (*a*) 2.21 μs; (*b*) 5.0 μs.
C: (*a*) No; (*b*) yes.
D: $0.36c$.
E: No.
F: $0.030c$, same as classical, to an accuracy of better than 0.1%.