The energy levels in the hydrogen atom depend on n, whereas in other atoms they depend on n and l.

When an external magnetic field is applied, the spectral lines are split (the **Zeeman effect**), indicating that the energy depends also on m_l in this case.

Even in the absence of a magnetic field, precise measurements of spectral lines show a tiny splitting of the lines called **fine structure**, whose explanation is that the energy depends very slightly on m_l and m_s.

[*Transitions between states that obey the **selection rule** $\Delta l = \pm 1$ are far more probable than other so-called "forbidden" transitions.]

The arrangement of electrons in multi-electron atoms is governed by the **Pauli exclusion principle**, which states that no two electrons can occupy the same quantum state—that is, they cannot have the same set of quantum numbers n, l, m_l, and m_s.

As a result, electrons in multi-electron atoms are grouped into **shells** (according to the value of n) and **subshells** (according to l).

Electron configurations are specified using the numerical values of n, and using letters for l: s, p, d, f, etc., for $l = 0, 1, 2, 3,$ and so on, plus a superscript for the number of electrons in that subshell. Thus, the ground state of hydrogen is $1s^1$, whereas that for oxygen is $1s^2 2s^2 2p^4$.

The **periodic table** arranges the elements in horizontal rows according to increasing atomic number (number of electrons in the neutral atom). The shell structure gives rise to a periodicity in the properties of the elements, so that each vertical column can contain elements with similar chemical properties.

X-rays, which are a form of electromagnetic radiation of very short wavelength, are produced when high-speed electrons strike a target. The spectrum of X-rays so produced consists of two parts, a continuous spectrum produced when the electrons are decelerated by atoms of the target, and peaks representing photons emitted by atoms of the target after being excited by collision with the high-speed electrons. Measurement of these peaks allows determination of inner energy levels of atoms and determination of Z.

[***Fluorescence** occurs when absorbed UV photons are followed by emission of visible light, due to the special arrangement of energy levels of the material. **Phosphorescent** materials have **metastable** states (long-lived) that emit light seconds or minutes after absorption of light.]

[***Lasers** produce a narrow beam of monochromatic coherent light (light waves *in phase*). **Holograms** are images with a 3-dimensional quality, formed by interference of laser light.]

Questions

1. Compare a matter wave Ψ to (*a*) a wave on a string, (*b*) an EM wave. Discuss similarities and differences.
2. Explain why Bohr's theory of the atom is not compatible with quantum mechanics, particularly the uncertainty principle.
3. Explain why it is that the more massive an object is, the easier it becomes to predict its future position.
4. In view of the uncertainty principle, why does a baseball seem to have a well-defined position and speed whereas an electron does not?
5. Would it ever be possible to balance a very sharp needle precisely on its point? Explain.
6. A cold thermometer is placed in a hot bowl of soup. Will the temperature reading of the thermometer be the same as the temperature of the hot soup before the measurement was made? Explain.
7. Does the uncertainty principle set a limit to how well you can make any single measurement of position?
8. If you knew the position of an object precisely, with no uncertainty, how well would you know its momentum?
9. When you check the pressure in a tire, doesn't some air inevitably escape? Is it possible to avoid this escape of air altogether? What is the relation to the uncertainty principle?
10. It has been said that the ground-state energy in the hydrogen atom can be precisely known but the excited states have some uncertainty in their values (an "energy width"). Is this consistent with the uncertainty principle in its energy form? Explain.
11. Which model of the hydrogen atom, the Bohr model or the quantum-mechanical model, predicts that the electron spends more time near the nucleus?
12. The size of atoms varies by only a factor of three or so, from largest to smallest, yet the number of electrons varies from one to over 100. Explain.
13. Excited hydrogen and excited helium atoms both radiate light as they jump down to the $n = 1$, $l = 0$, $m_l = 0$ state. Yet the two elements have very different emission spectra. Why?
14. How would the periodic table look if there were no electron spin but otherwise quantum mechanics were valid? Consider the first 20 elements or so.
15. Which of the following electron configurations are not allowed: (*a*) $1s^2 2s^2 2p^4 3s^2 4p^2$; (*b*) $1s^2 2s^2 2p^8 3s^1$; (*c*) $1s^2 2s^2 2p^6 3s^2 3p^5 4s^2 4d^5 4f^1$? If not allowed, explain why.
16. Give the complete electron configuration for a uranium atom (careful scrutiny across the periodic table on the inside back cover will provide useful hints).
17. In what column of the periodic table would you expect to find the atom with each of the following configurations: (*a*) $1s^2 2s^2 2p^6 3s^2$; (*b*) $1s^2 2s^2 2p^6 3s^2 3p^6$; (*c*) $1s^2 2s^2 2p^6 3s^2 3p^6 4s^1$; (*d*) $1s^2 2s^2 2p^5$?
18. Why do chlorine and iodine exhibit similar properties?
19. Explain why potassium and sodium exhibit similar properties.
20. The ionization energy for neon ($Z = 10$) is 21.6 eV, and that for sodium ($Z = 11$) is 5.1 eV. Explain the large difference.

* 21. Why does the cutoff wavelength in Fig. 28–11 imply a photon nature for light?

* 22. Why do we not expect perfect agreement between measured values of X-ray line wavelengths and those calculated using Bohr theory, as in Example 28–6?

* 23. How would you figure out which lines in an X-ray spectrum correspond to K_α, K_β, L etc., transitions?

24. Why do we expect electron transitions deep within an atom to produce shorter wavelengths than transitions by outer electrons?

* 25. Compare spontaneous emission to stimulated emission.

* 26. How does laser light differ from ordinary light? How is it the same?

* 27. Explain how a 0.0005-W laser beam, photographed at a distance, can seem much stronger than a 1000-W street lamp at the same distance.

* 28. Does the intensity of light from a laser fall off as the inverse square of the distance? Explain.

Problems

28–2 Wave Function, Double-Slit

1. (II) The neutrons in a parallel beam, each having kinetic energy $\frac{1}{40}$ eV, are directed through two slits 0.50 mm apart. How far apart will the interference peaks be on a screen 1.0 m away? [*Hint*: first find the wavelength of the neutron.]

2. (II) Bullets of mass 3.0 g are fired in parallel paths with speeds of 220 m/s through a hole 3.0 mm in diameter. How far from the hole must you be to detect a 1.0-cm-diameter spread in the beam of bullets?

28–3 Uncertainty Principle

3. (I) A proton is traveling with a speed of $(6.560 \pm 0.012) \times 10^5$ m/s. With what maximum accuracy can its position be ascertained? [*Hint*: $\Delta p = m\,\Delta v$.]

4. (I) If an electron's position can be measured to an accuracy of 2.0×10^{-8} m, how accurately can its speed be known?

5. (I) An electron remains in an excited state of an atom for typically 10^{-8} s. What is the minimum uncertainty in the energy of the state (in eV)?

6. (I) The Z^0 boson, discovered in 1985, is the mediator of the weak nuclear force, and it typically decays very quickly. Its average rest energy is 91.19 GeV, but its short lifetime shows up as an intrinsic width of 2.5 GeV (rest energy uncertainty). What is the lifetime of this particle?

7. (II) What is the uncertainty in the mass of a muon $(m = 105.7\ \text{MeV}/c^2)$, specified in eV/$c^2$, given its lifetime of 2.20 μs? [*Hint*: $\Delta E \approx \hbar/\Delta t$.]

8. (II) A free neutron $(m = 1.67 \times 10^{-27}\ \text{kg})$ has a mean life of 900 s. What is the uncertainty in its mass (in kg)?

9. (II) An electron and a 140-g baseball are each traveling 150 m/s measured to an accuracy of 0.055%. Calculate and compare the uncertainty in position of each.

10. (III) Estimate the lowest possible energy of a neutron contained in a typical nucleus of radius 1.0×10^{-15} m. [*Hint*: a particle can have an energy at least as large as its uncertainty.]

11. (III) Use the uncertainty principle to show that if an electron were present in the nucleus $(r \approx 10^{-15} m)$, its kinetic energy (use relativity) would be hundreds of MeV. (Since such electron energies are not observed, we conclude that electrons are not present in the nucleus.) [*Hint*: a particle can have an energy at least as large as its uncertainty.]

12. (III) How accurately can the position of a 3.00-keV electron be measured assuming its energy is known to 1.00%?

28–6 to 28–8 Quantum Numbers, Exclusion Principle

13. (I) For $n = 6$, what values can l have?

14. (I) For $n = 5$, $l = 3$, what are the possible values of m_l and m_s?

15. (I) How many electrons can be in the $n = 6$, $l = 3$ subshell?

16. (I) How many different states are possible for an electron whose principal quantum number is $n = 4$? Write down the quantum numbers for each state.

17. (I) List the quantum numbers for each electron in the ground state of (*a*) carbon $(Z = 6)$, (*b*) magnesium $(Z = 12)$.

18. (I) List the quantum numbers for each electron in the ground state of nitrogen $(Z = 7)$.

19. (I) Suppose a certain hydrogen atom has $l = 4$. What are the possible values for n, m_l, and m_s?

20. (I) Calculate the magnitude of the angular momentum of an electron in the $n = 4$, $l = 3$ state of hydrogen.

21. (II) If a hydrogen atom has $m_l = -3$, what are the possible values of n, l, and m_s?

22. (II) Show that there can be 18 electrons in a "g" subshell.

23. (II) What is the full electron configuration in the ground state for elements with Z equal to (*a*) 27, (*b*) 36, (*c*) 38? [*Hint*: see the periodic table inside the back cover.]

24. (II) What is the full electron configuration for (*a*) selenium (Se), (*b*) gold (Au), (*c*) radium (Ra)? [*Hint*: see the periodic table inside the back cover.]

25. (II) A hydrogen atom is in the 6s state. Determine (*a*) the principal quantum number, (*b*) the energy of the state, (*c*) the orbital angular momentum and its quantum number l, and (*d*) the possible values for the magnetic quantum number.

26. (II) Estimate the binding energy of the third electron in lithium using the Bohr theory. [*Hint*: this electron has $n = 2$ and "sees" a net charge of approximately $+1e$.] The measured value is 5.36 eV.

27. (II) Show that the total angular momentum is zero for a filled subshell.

* 28. (II) For each of the following atomic transitions, state whether the transition is *allowed* or *forbidden*, and if forbidden, what rule is being violated: (*a*) $4p \rightarrow 3p$; (*b*) $2p \rightarrow 1s$; (*c*) $3d \rightarrow 2d$; (*d*) $4d \rightarrow 3s$; (*e*) $4s \rightarrow 2p$.

* **29.** (II) An excited H atom is in a 6*d* state. (*a*) Name all the states (n, l) to which the atom is "allowed" to jump with the emission of a photon. (*b*) How many different wavelengths are there (ignoring fine structure)?

* 28–9 X-Rays

* **30.** (I) What are the shortest-wavelength X-rays emitted by electrons striking the face of a 33.5-kV TV picture tube? What are the longest wavelengths?

* **31.** (I) If the shortest-wavelength bremsstrahlung X-rays emitted from an X-ray tube have $\lambda = 0.030\,\text{nm}$, what is the voltage across the tube?

* **32.** (I) Show that the cutoff wavelength λ_0 is given by

$$\lambda_0 = \frac{1240\,\text{nm}}{V},$$

where V is the X-ray tube voltage in volts.

* **33.** (II) Use the result of Example 28–6 to estimate the X-ray wavelength emitted when a Co $(Z = 27)$ atom jumps from $n = 2$ to $n = 1$.

* **34.** (II) Estimate the wavelength for an $n = 2$ to $n = 1$ transition in iron $(Z = 26)$.

* **35.** (II) Use the Bohr theory to estimate the wavelength for an $n = 3$ to $n = 1$ transition in molybdenum $(Z = 42)$. The measured value is 0.063 nm. Why do we not expect perfect agreement?

* **36.** (II) A mixture of iron and an unknown material is bombarded with electrons. The wavelength of the K_α lines are 194 pm for iron and 229 pm for the unknown. What is the unknown material?

* 28–11 Lasers

* **37.** (II) A laser used to weld detached retinas puts out 28-ms-long pulses of 640-nm light which average 0.68-W output during a pulse. How much energy can be deposited per pulse and how many photons does each pulse contain? [*Hint*: see Example 27–5.]

* **38.** (II) A low-power laser used in a physics lab might have a power of 0.50 mW and a beam diameter of 3.0 mm. Calculate (*a*) the average light intensity of the laser beam, and (*b*) compare it to the intensity of a lightbulb producing 40-W light viewed from 2.0 m.

* **39.** (II) Estimate the angular spread of a laser beam due to diffraction if the beam emerges through a 3.0-mm-diameter mirror. Assume that $\lambda = 694\,\text{nm}$. What would be the diameter of this beam if it struck (*a*) a satellite 300 km above the Earth, or (*b*) the Moon? [*Hint*: see Section 25–7.]

* **40.** (II) What is the wavelength of the He–Ne laser?

General Problems

41. Use the uncertainty principle to estimate the position uncertainty for the electron in the ground state of the hydrogen atom. [*Hint*: determine the momentum using the Bohr model of Section 27–12 and assume the momentum can be anywhere between this value and zero.] How does this result compare to the Bohr radius?

42. An electron in the $n = 2$ state of hydrogen remains there on average about 10^{-8} s before jumping to the $n = 1$ state. (*a*) Estimate the uncertainty in the energy of the $n = 2$ state. (*b*) What fraction of the transition energy is this? (*c*) What is the wavelength, and width (in nm), of this line in the spectrum of hydrogen?

43. What are the largest and smallest possible values for the angular momentum L of an electron in the $n = 5$ shell?

44. Estimate (*a*) the quantum number l for the orbital angular momentum of the Earth about the Sun, and (*b*) the number of possible orientations for the plane of Earth's orbit.

45. A 12-g bullet leaves a rifle at a speed of 180 m/s. (*a*) What is the wavelength of this bullet? (*b*) If the position of the bullet is known to an accuracy of 0.60 cm (radius of the barrel), what is the minimum uncertainty in its momentum?

46. Using the Bohr formula for the radius of an electron orbit, estimate the average distance from the nucleus for an electron in the innermost $(n = 1)$ orbit of a uranium atom $(Z = 92)$. Approximately how much energy would be required to remove this innermost electron?

47. An X-ray tube operates at 95 kV with a current of 25 mA and nearly all the electron energy goes into heat. If the specific heat of the 0.085-kg plate is 0.11 kcal/kg·C°, what will be the temperature rise per minute if no cooling water is used?

48. The ionization (binding) energy of the outermost electron in boron is 8.26 eV. (*a*) Use the Bohr model to estimate the "effective charge," Z_{eff}, seen by this electron. (*b*) Estimate the average orbital radius.

49. Use the Bohr theory (especially Eq. 27–16) to show that the Moseley plot (Fig. 28–12) can be written

$$\sqrt{\frac{1}{\lambda}} = a(Z - b),$$

where $b \approx 1$, and evaluate a.

50. (*a*) Show that the number of different states for a given value of l is equal to $2(2l + 1)$. (*b*) What is this number for $l = 0, 1, 2, 3, 4, 5$, and 6?

51. Show that the number of different electron states possible for a given value of n is $2n^2$. (See Problem 50.)

52. A beam of electrons with kinetic energy 45 keV is shot through two narrow slits in a barrier. The slits are a distance 2.0×10^{-6} m apart. If a screen is placed 35.0 cm behind the barrier, calculate the spacing between the "bright" fringes of the interference pattern produced on the screen.

53. The angular momentum in the hydrogen atom is given both by the Bohr model and by quantum mechanics. Compare the results for $n = 2$.

54. An 1100-kg car is traveling with a speed of (22 ± 0.22) m/s. With what maximum accuracy can its position be determined?

55. An atomic spectrum contains a line with a wavelength centered at 488 nm. Careful measurements show the line is really spread out between 487 and 489 nm. Estimate the lifetime of the excited state that produced this line.

56. Protons are accelerated from rest across 550 V. They are then directed at two slits 0.70 mm apart. How far apart will the interference peaks be on a screen 28 m away?

57. An electron and a proton, each initially at rest, are accelerated across the same voltage. Assuming that the uncertainty in their position is given by their de Broglie wavelength, find the ratio of the uncertainty in their momentum.

58. If the principal quantum number n were limited to the range from 1 to 6, how many elements would we find in nature?

59. If your de Broglie wavelength were 0.50 m, how fast would you be moving if your mass is 75.0 kg? Would you notice diffraction effects as you walk through a doorway? Approximately how long would it take you to walk through the doorway?

60. Suppose that the spectrum of an unknown element shows a series of lines with one out of every four matching a line from the Lyman series of hydrogen. Assuming that the unknown element is an ion with Z protons and one electron, determine Z and the element in question.

* 61. Photons of wavelength 0.154 nm are emitted from the surface of a certain metal when it is bombarded with high energy radiation. If this photon wavelength corresponds to the K_α line, what is the element?

* 62. Show that the diffractive spread of a laser beam, $\approx \lambda/D$ as described in Section 28–11, is precisely what you might expect from the uncertainty principle. [*Hint*: since the beam's width is constrained by the dimension of the aperture D, the component of the light's momentum perpendicular to the laser axis is uncertain.]

Answers to Exercises

A: 2.1×10^{-24} kg·m/s, 2.3×10^6 m/s.

B: 2, 1, 0, −1, −2.

C: −0.38 eV, $\sqrt{20}\,\hbar$.

D: $1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^1$.