An α particle is a ^4_2He nucleus; a β particle is an electron or positron; and a γ ray is a high-energy photon. In β decay, a **neutrino** is also emitted. The transformation of the parent into the daughter nucleus is called **transmutation** of the elements. Radioactive decay occurs spontaneously only when the rest mass of the products is less than the mass of the parent nucleus. The loss in mass appears as kinetic energy of the products.

Nuclei are held together by the **strong nuclear force**. The **weak nuclear force** makes itself apparent in β decay. These two forces, plus the gravitational and electromagnetic forces, are the four known types of force.

Electric charge, linear and angular momentum, mass–energy, and **nucleon number** are **conserved** in all decays.

Radioactive decay is a statistical process. For a given type of radioactive nucleus, the number of nuclei that decay (ΔN) in a time Δt is proportional to the number N of parent nuclei present:

$$\Delta N = -\lambda N \, \Delta t; \qquad \textbf{(30–3a)}$$

the minus sign means N *decreases* in time.

The proportionality constant λ is called the **decay constant** and is characteristic of the given nucleus. The number N of nuclei remaining after a time t decreases exponentially

$$N = N_0 e^{-\lambda t}, \qquad \textbf{(30–4)}$$

as does the **activity**, $\Delta N/\Delta t$:

$$\frac{\Delta N}{\Delta t} = \left(\frac{\Delta N}{\Delta t}\right)_0 e^{-\lambda t}. \qquad \textbf{(30–5)}$$

The **half-life**, $T_{\frac{1}{2}}$, is the time required for half the nuclei of a radioactive sample to decay. It is related to the decay constant by

$$T_{\frac{1}{2}} = \frac{0.693}{\lambda}. \qquad \textbf{(30–6)}$$

Radioactive decay can be used to determine the age of certain objects.

[*Alpha decay occurs via a purely quantum mechanical process called **tunneling** through a barrier.]

[*Particle **detectors** include **Geiger counters**, **scintillators** with attached **photomultiplier tubes**, and **semiconductor detectors**. Detectors that can image particle tracks include photographic **emulsions**, **bubble chambers**, and today **wire drift chambers**.]

Questions

1. What do different isotopes of a given element have in common? How are they different?
2. What are the elements represented by the X in the following: (*a*) $^{232}_{92}X$; (*b*) $^{18}_{7}X$; (*c*) $^{1}_{1}X$; (*d*) $^{82}_{38}X$; (*e*) $^{247}_{97}X$?
3. How many protons and how many neutrons do each of the isotopes in Question 2 have?
4. Identify the element that has 88 nucleons and 50 neutrons.
5. Why are the atomic masses of many elements (see the periodic table) not close to whole numbers?
6. How do we know there is such a thing as the strong nuclear force?
7. What are the similarities and the differences between the strong nuclear force and the electric force?
8. What is the experimental evidence in favor of radioactivity being a nuclear process?
9. The isotope $^{64}_{29}Cu$ is unusual in that it can decay by γ, β^-, and β^+ emission. What is the resulting nuclide for each case?
10. A $^{238}_{92}U$ nucleus decays to a nucleus containing how many neutrons?
11. Describe, in as many ways as you can, the difference between α, β, and γ rays.
12. What element is formed by the radioactive decay of (*a*) $^{24}_{11}Na$ (β^-); (*b*) $^{22}_{11}Na$ (β^+); (*c*) $^{210}_{84}Po$ (α)? [*Hint*: see Appendix B.]
13. What element is formed by the decay of (*a*) $^{32}_{15}P$ (β^-); (*b*) $^{35}_{16}S$ (β^-); (*c*) $^{211}_{83}Bi$ (α)? [*Hint*: see Appendix B.]
14. Fill in the missing particle or nucleus:
    (*a*) $^{45}_{20}Ca \rightarrow ? + e^- + \bar{\nu}$
    (*b*) $^{58}_{29}Cu \rightarrow ? + \gamma$
    (*c*) $^{46}_{24}Cr \rightarrow {}^{46}_{23}V + ?$
    (*d*) $^{234}_{94}Pu \rightarrow ? + \alpha$
    (*e*) $^{239}_{93}Np \rightarrow {}^{239}_{94}Pu + ?$
15. Immediately after a $^{238}_{92}U$ nucleus decays to $^{234}_{90}Th + {}^4_2He$, the daughter thorium nucleus still has 92 electrons circling it. Since thorium normally holds only 90 electrons, what do you suppose happens to the two extra ones?
16. When a nucleus undergoes either β^- or β^+ decay, what happens to the energy levels of the atomic electrons? What is likely to happen to these electrons following the decay?
17. The alpha particles from a given alpha-emitting nuclide are generally monoenergetic; that is, they all have the same kinetic energy. But the beta particles from a beta-emitting nuclide have a spectrum of energies. Explain the difference between these two cases.
18. Do isotopes that undergo electron capture generally lie above or below the line of stability in Fig. 30–2?
19. Can hydrogen or deuterium emit an α particle? Explain.
20. Why are many artificially produced radioactive isotopes rare in nature?
21. An isotope has a half-life of one month. After two months, will a given sample of this isotope have completely decayed? If not, how much remains?
22. Why are none of the elements with $Z > 92$ stable?
23. A proton strikes a ^6_3Li nucleus. As a result, an α particle and another particle are released. What is the other particle?
24. Can $^{14}_{6}C$ dating be used to measure the age of stone walls and tablets of ancient civilizations? Explain.
25. In both internal conversion and β decay, an electron is emitted. How could you determine which decay process occurred?

Problems

30–1 Nuclear Properties

1. (I) A pi meson has a mass of $139\,\mathrm{MeV}/c^2$. What is this in atomic mass units?

2. (I) What is the approximate radius of an alpha particle ($^4_2\mathrm{He}$)?

3. (II) What is the rest mass of a bare α particle in MeV/c^2?

4. (II) (*a*) What is the approximate radius of a $^{64}_{29}\mathrm{Cu}$ nucleus? (*b*) Approximately what is the value of A for a nucleus whose radius is $3.9 \times 10^{-15}\,\mathrm{m}$?

5. (II) (*a*) Show that the density of nuclear matter is essentially the same for all nuclei. (*b*) What would be the radius of the Earth if it had its actual mass but had the density of nuclei? (*c*) What would be the radius of a $^{238}_{92}\mathrm{U}$ nucleus if it had the density of the Earth?

6. (II) (*a*) What is the fraction of the hydrogen atom's mass that is in the nucleus? (*b*) What is the fraction of the hydrogen atom's volume that is occupied by the nucleus?

7. (II) Approximately how many nucleons are there in a 1.0-kg object? Does it matter what the object is made of? Why or why not?

8. (III) How much energy must an α particle have to just "touch" the surface of a $^{238}_{92}\mathrm{U}$ nucleus?

30–2 Binding Energy

9. (I) Estimate the total binding energy for $^{40}_{20}\mathrm{Ca}$, using Fig. 30–1.

10. (I) Use Fig. 30–1 to estimate the total binding energy of (*a*) $^{238}_{92}\mathrm{U}$, and (*b*) $^{84}_{36}\mathrm{Kr}$.

11. (II) Use Appendix B to calculate the binding energy of $^2_1\mathrm{H}$ (deuterium).

12. (II) Calculate the binding energy per nucleon for a $^{14}_{7}\mathrm{N}$ nucleus.

13. (II) Determine the binding energy of the last neutron in a $^{40}_{19}\mathrm{K}$ nucleus.

14. (II) Calculate the total binding energy, and the binding energy per nucleon, for (*a*) $^6_3\mathrm{Li}$, (*b*) $^{208}_{82}\mathrm{Pb}$. Use Appendix B.

15. (II) Compare the average binding energy of a nucleon in $^{23}_{11}\mathrm{Na}$ to that in $^{24}_{11}\mathrm{Na}$.

16. (III) How much energy is required to remove (*a*) a proton, (*b*) a neutron, from $^{16}_{8}\mathrm{O}$? Explain the difference in your answers.

17. (III) (*a*) Show that the nucleus $^8_4\mathrm{Be}$ (mass $= 8.005305\,\mathrm{u}$) is unstable and will decay into two α particles. (*b*) Is $^{12}_{6}\mathrm{C}$ stable against decay into three α particles? Show why or why not.

30–3 to 30–7 Radioactive Decay

18. (I) How much energy is released when tritium, $^3_1\mathrm{H}$, decays by β^- emission?

19. (I) What is the maximum kinetic energy of an electron emitted in the β decay of a free neutron?

20. (I) Show that the decay $^{11}_{6}\mathrm{C} \rightarrow {}^{10}_{5}\mathrm{B} + \mathrm{p}$ is not possible because energy would not be conserved.

21. (II) $^{22}_{11}\mathrm{Na}$ is radioactive. (*a*) Is it a β^- or β^+ emitter? (*b*) Write down the decay reaction, and estimate the maximum kinetic energy of the emitted β.

22. (II) Give the result of a calculation that shows whether or not the following decays are possible:

    (*a*) $^{236}_{92}\mathrm{U} \rightarrow {}^{235}_{92}\mathrm{U} + \mathrm{n}$;

    (*b*) $^{16}_{8}\mathrm{O} \rightarrow {}^{15}_{8}\mathrm{O} + \mathrm{n}$;

    (*c*) $^{23}_{11}\mathrm{Na} \rightarrow {}^{22}_{11}\mathrm{Na} + \mathrm{n}$.

23. (II) A $^{238}_{92}\mathrm{U}$ nucleus emits an α particle with kinetic energy $= 4.20\,\mathrm{MeV}$. (*a*) What is the daughter nucleus, and (*b*) what is the approximate atomic mass (in u) of the daughter atom? Ignore recoil of the daughter nucleus.

24. (II) When $^{23}_{10}\mathrm{Ne}$ (mass $= 22.9945\,\mathrm{u}$) decays to $^{23}_{11}\mathrm{Na}$ (mass $= 22.9898\,\mathrm{u}$), what is the maximum kinetic energy of the emitted electron? What is its minimum energy? What is the energy of the neutrino in each case? Ignore recoil of the daughter nucleus.

25. (II) A nucleus of mass 238 u, initially at rest, emits an α particle with a KE of 5.0 MeV. What is the KE of the recoiling daughter nucleus?

26. (II) What is the maximum KE of the emitted β particle during the decay of $^{60}_{27}\mathrm{Co}$?

27. (II) The nuclide $^{32}_{15}\mathrm{P}$ decays by emitting an electron whose maximum kinetic energy can be 1.71 MeV. (*a*) What is the daughter nucleus? (*b*) Calculate the daughter's atomic mass (in u).

28. (II) The isotope $^{218}_{84}\mathrm{Po}$ can decay by either α or β^- emission. What is the energy release in each case? The mass of $^{218}_{84}\mathrm{Po}$ is 218.008965 u.

29. (II) How much energy is released in electron capture by beryllium: $^7_4\mathrm{Be} + {}^{0}_{-1}\mathrm{e} \rightarrow {}^7_3\mathrm{Li} + \nu$?

30. (II) A photon with a wavelength of $1.00 \times 10^{-13}\,\mathrm{m}$ is ejected from an atom. Calculate its energy and explain why it is a γ ray from the nucleus or a photon from the atom.

31. (II) Determine the maximum kinetic energy of β^+ particles released when $^{11}_{6}\mathrm{C}$ decays to $^{11}_{5}\mathrm{B}$. What is the maximum energy the neutrino can have? What is its minimum energy?

32. (II) How much recoil energy does a $^{40}_{19}\mathrm{K}$ nucleus get when it emits a 1.46-MeV gamma ray?

33. (III) What is the energy of the α particle emitted in the decay $^{210}_{84}\mathrm{Po} \rightarrow {}^{206}_{82}\mathrm{Pb} + \alpha$? Take into account the recoil of the daughter nucleus.

34. (III) The α particle emitted when $^{238}_{92}\mathrm{U}$ decays has 4.20 MeV of kinetic energy. Calculate the recoil kinetic energy of the daughter nucleus and the Q-value of the decay.

35. (III) Show that when a nucleus decays by β^+ decay, the total energy released is equal to

$$(M_\mathrm{P} - M_\mathrm{D} - 2m_\mathrm{e})c^2,$$

where M_P and M_D are the masses of the parent and daughter atoms (neutral), and m_e is the mass of an electron or positron.

30–8 to 30–11 Half-Life, Decay Rates, Decay Series, Dating

36. (I) A radioactive material produces 1280 decays per minute at one time, and 4.6 h later produces 320 decays per minute. What is its half-life?

37. (I) (*a*) What is the decay constant of $^{238}_{92}U$ whose half-life is 4.5×10^9 yr? (*b*) The decay constant of a given nucleus is $8.2 \times 10^{-5}\,s^{-1}$. What is its half-life?

38. (I) What is the activity of a sample of $^{14}_{6}C$ that contains 3.1×10^{20} nuclei?

39. (I) What fraction of a sample of $^{68}_{32}Ge$, whose half-life is about 9 months, will remain after 3.0 yr?

40. (I) What fraction of a sample is left after exactly 6 half-lives?

41. (II) How many nuclei of $^{238}_{92}U$ remain in a rock if the activity registers 640 decays per second?

42. (II) In a series of decays, the nuclide $^{235}_{92}U$ becomes $^{207}_{82}Pb$. How many α and β^- particles are emitted in this series?

43. (II) The iodine isotope $^{131}_{53}I$ is used in hospitals for diagnosis of thyroid function. If 682 μg are ingested by a patient, determine the activity (*a*) immediately, (*b*) 1.0 h later when the thyroid is being tested, and (*c*) 6 months later. Use Appendix B.

44. (II) $^{124}_{55}Cs$ has a half-life of 30.8 s. (*a*) If we have 8.8 μg initially, how many Cs nuclei are present? (*b*) How many are present 2.0 min later? (*c*) What is the activity at this time? (*d*) After how much time will the activity drop to less than about 1 per second?

45. (II) Calculate the mass of a sample of pure $^{40}_{19}K$ with an initial decay rate of $2.0 \times 10^5\,s^{-1}$. The half-life of $^{40}_{19}K$ is 1.28×10^9 yr.

46. (II) Calculate the activity of a pure 9.7-μg sample of $^{32}_{15}P$ $\left(T_{\frac{1}{2}} = 1.23 \times 10^6\,s\right)$.

47. (II) The activity of a sample of $^{35}_{16}S$ $\left(T_{\frac{1}{2}} = 7.55 \times 10^6\,s\right)$ is 2.65×10^5 decays per second. What is the mass of the sample?

48. (II) A sample of $^{233}_{92}U$ $\left(T_{\frac{1}{2}} = 1.59 \times 10^5\,yr\right)$ contains 7.50×10^{19} nuclei. (*a*) What is the decay constant? (*b*) Approximately how many disintegrations will occur per minute?

49. (II) The activity of a sample drops by a factor of 10 in 8.6 minutes. What is its half-life?

50. (II) A 285-g sample of pure carbon contains 1.3 parts in 10^{12} (atoms) of $^{14}_{6}C$. How many disintegrations occur per second?

51. (II) A sample of $^{40}_{19}K$ is decaying at a rate of 6.70×10^2 decays/s. What is the mass of the sample?

52. (II) The rubidium isotope $^{87}_{37}Rb$, a β emitter with a half-life of 4.75×10^{10} yr, is used to determine the age of rocks and fossils. Rocks containing fossils of ancient animals contain a ratio of $^{87}_{38}Sr$ to $^{87}_{37}Rb$ of 0.0160. Assuming that there was no $^{87}_{38}Sr$ present when the rocks were formed, estimate the age of these fossils. [*Hint*: use Eq. 30–3.]

53. (II) Use Fig. 30–11 and calculate the relative decay rates for α decay of $^{218}_{84}Po$ and $^{214}_{84}Po$.

54. (II) $^{7}_{4}Be$ decays with a half-life of about 53 d. It is produced in the upper atmosphere, and filters down onto the Earth's surface. If a plant leaf is detected to have 450 decays/s of $^{7}_{4}Be$, (*a*) how long do we have to wait for the decay rate to drop to 15 per second? (*b*) Estimate the initial mass of $^{7}_{4}Be$ on the leaf.

55. (II) Two of the naturally occurring radioactive decay sequences start with $^{232}_{90}Th$, and $^{235}_{92}U$. The first five decays of these two sequences are:

$$\alpha, \beta, \beta, \alpha, \alpha$$

and

$$\alpha, \beta, \alpha, \beta, \alpha.$$

Determine the resulting intermediate daughter nuclei in each case.

56. (II) An ancient wooden club is found that contains 290 g of carbon and has an activity of 8.0 decays per second. Determine its age assuming that in living trees the ratio of $^{14}C/^{12}C$ atoms is about 1.3×10^{-12}.

57. (III) At $t = 0$, a pure sample of radioactive nuclei contains N_0 nuclei whose decay constant is λ. Determine a formula for the number of daughter nuclei, N_D, as a function of time; assume the daughter is stable and that $N_D = 0$ at $t = 0$.

General Problems

58. Which radioactive isotope of lead is being produced in a reaction where the measured activity of a sample drops to 1.050% of its original activity in 4.00 h?

59. An old wooden tool is found to contain only 6.0% of $^{14}_{6}C$ that a sample of fresh wood would. How old is the tool?

60. A neutron star consists of neutrons at approximately nuclear density. Estimate, for a 10-km-diameter neutron star, (*a*) its mass number, (*b*) its mass (kg), and (*c*) the acceleration of gravity at its surface.

61. The $^{3}_{1}H$ isotope of hydrogen, which is called *tritium* (because it contains three nucleons), has a half-life of 12.33 yr. It can be used to measure the age of objects up to about 100 yr. It is produced in the upper atmosphere by cosmic rays and brought to Earth by rain. As an application, determine approximately the age of a bottle of wine whose $^{3}_{1}H$ radiation is about $\frac{1}{10}$ that present in new wine.

62. Some elementary particle theories (Section 32–11) suggest that the proton may be unstable, with a half-life $\geq 10^{32}$ yr. How long would you expect to wait for one proton in your body to decay (consider that your body is all water)?

63. How long must you wait (in half-lives) for a radioactive sample to drop to 1.00% of its original activity?

64. If the potassium isotope $^{40}_{19}K$ gives 60 decays/s in a liter of milk, estimate how much $^{40}_{19}K$ and regular $^{39}_{19}K$ are in a liter of milk. Use Appendix B.

65. (*a*) In α decay of, say, a $^{226}_{88}Ra$ nucleus, show that the nucleus carries away a fraction $1/(1 + \frac{1}{4}A_D)$ of the total energy available, where A_D is the mass number of the daughter nucleus. [*Hint*: use conservation of momentum as well as conservation of energy.] (*b*) Approximately what percentage of the energy available is thus carried off by the α particle in the case cited?

66. Strontium-90 is produced as a nuclear fission product of uranium in both reactors and atomic bombs. Look at its location in the periodic table to see what other elements it might be similar to chemically, and tell why you think it might be dangerous to ingest. It has too many neutrons, and it decays with a half-life of about 29 yr. How long will we have to wait for the amount of $^{90}_{38}Sr$ on the Earth's surface to reach 1% of its current level, assuming no new material is scattered about? Write down the decay reaction, including the daughter nucleus. The daughter is radioactive: write down its decay.

67. The nuclide $^{191}_{76}Os$ decays with β^- energy of 0.14 MeV accompanied by γ rays of energy 0.042 MeV and 0.129 MeV. (*a*) What is the daughter nucleus? (*b*) Draw an energy-level diagram showing the ground states of the parent and daughter and excited states of the daughter. To which of the daughter states does β^- decay of $^{191}_{76}Os$ occur?

68. Determine the activities of (*a*) 1.0 g of $^{131}_{53}I$ ($T_{\frac{1}{2}} = 8.02$ days) and (*b*) 1.0 g of $^{238}_{92}U$ ($T_{\frac{1}{2}} = 4.47 \times 10^9$ yr).

69. Estimate the total binding energy for copper and then estimate the energy, in joules, needed to break a 3.0-g copper penny into its constituent nucleons. [*Hint*: use Fig. 30–1.]

70. Instead of giving atomic masses for nuclides as in Appendix B, some Tables give the *mass excess*, Δ, defined as $\Delta = M - A$, where A is the atomic number and M is the mass in u. Determine the mass excess, in u and in MeV/c^2, for: (*a*) ^4_2He; (*b*) $^{12}_6C$; (*c*) $^{107}_{47}Ag$; (*d*) $^{235}_{92}U$. (*e*) From a glance at Appendix B, can you make a generalization about the sign of Δ as a function of Z or A?

71. (*a*) A 92-gram sample of natural carbon contains the usual fraction of $^{14}_6C$. Estimate how long it will take before there is only one $^{14}_6C$ nucleus left. (*b*) How does the answer in (*a*) change if the sample is 280 grams? What does this tell you about the limits of carbon dating?

72. If the mass of the proton were just a little closer to the mass of the neutron, the following reaction would be possible even at low collision energies:

$$e^- + p \rightarrow n + \nu.$$

Why would this situation be catastrophic? By what percentage would the proton's mass have to be increased to make this reaction possible?

73. What is the ratio of the kinetic energies for an alpha particle and a beta particle if both make tracks with the same radius of curvature in a magnetic field, oriented perpendicular to the paths of the particles?

74. A 1.00-g sample of natural samarium emits α particles at a rate of 120 s^{-1} due to the presence of $^{147}_{62}Sm$. The natural abundance of $^{147}_{62}Sm$ is 15%. Calculate the half-life for this decay process.

75. Almost all of naturally occurring uranium is $^{238}_{92}U$ with a half-life of 4.468×10^9 yr. Most of the rest of natural uranium is $^{235}_{92}U$ with a half-life of 7.038×10^8 yr. Today a sample contains 0.72% $^{235}_{92}U$. (*a*) What was this percentage 1.0 billion years ago? (*b*) What percentage of the sample would be $^{235}_{92}U$ in 100 million years?

76. A typical banana contains 400 mg of potassium, of which a small fraction is the radioactive isotope $^{40}_{19}K$ (see Appendix B). Estimate the activity of an average banana due to $^{40}_{19}K$.

77. The practical limit for carbon-14 dating is about 60,000 years. If a bone contains 1.0 kg of carbon, and the animal died 60,000 years ago, what is the activity today?

78. Decay series, such as that shown in Fig. 30–11, can be classified into four families, depending on whether the mass numbers have the form $4n$, $4n + 1$, $4n + 2$, or $4n + 3$, where n is an integer. Justify this statement and show that for a nuclide in any family, all its daughters will be in the same family.

Answers to Exercises

A: 0.0421 u.
B: 7.98 MeV/nucleon.
C: $1.37 \times 10^{-11}\ s^{-1}$.
D: No: $(\frac{1}{2})(\frac{1}{2}) = \frac{1}{4}$ will be left.