Summary

A **nuclear reaction** occurs when two nuclei collide and two or more other nuclei (or particles) are produced. In this process, as in radioactivity, **transmutation** (change) of elements occurs.

The **reaction energy** or ***Q*-value** of a reaction $a + X \rightarrow Y + b$ is

$$Q = (M_a + M_X - M_b - M_Y)c^2 \qquad \textbf{(31–1)}$$

$$= K_b + K_Y - K_a - K_X. \qquad \textbf{(31–2)}$$

In **fission** a heavy nucleus such as uranium splits into two intermediate-sized nuclei after being struck by a neutron. ${}^{235}_{92}U$ is fissionable by slow neutrons, whereas some fissionable nuclei require fast neutrons. Much energy is released in fission because the binding energy per nucleon is lower for heavy nuclei than it is for intermediate-sized nuclei, so the mass of a heavy nucleus is greater than the total mass of its fission products. The fission process releases neutrons, so that a **chain reaction** is possible. The **critical mass** is the minimum mass of fuel needed to sustain a chain reaction. In a **nuclear reactor** or nuclear bomb, a **moderator** is needed to slow down the released neutrons.

The **fusion** process, in which small nuclei combine to form larger ones, also releases energy. The energy from our Sun is believed to originate in the fusion reactions known as the **proton–proton cycle** in which four protons fuse to form a ${}^{4}_{2}He$ nucleus producing over 25 MeV of energy. A useful fusion reactor for power generation has not yet proved possible because of the difficulty in containing the fuel (e.g., deuterium) long enough at the high temperature required.

Radiation can cause damage to materials, including biological tissue. Quantifying amounts of radiation is the subject of **dosimetry**. The **curie** (Ci) and the **becquerel** (Bq) are units that measure the **source activity** or rate of decay of a sample: $1\ \text{Ci} = 3.70 \times 10^{10}$ disintegrations per second, whereas $1\ \text{Bq} = 1$ disintegration/s. The **absorbed dose**, often specified in **rads**, measures the amount of energy deposited per unit mass of absorbing material: 1 rad is the amount of radiation that deposits energy at the rate of $10^{-2}\ \text{J/kg}$ of material. The SI unit of absorbed dose is the **gray**: $1\ \text{Gy} = 1\ \text{J/kg} = 100\ \text{rad}$. The **effective dose** is often specified by the **rem** = rad × QF, where QF is the "quality factor" of a given type of radiation; 1 rem of any type of radiation does approximately the same amount of biological damage. The average dose received per person per year in the United States is about 0.36 rem. The SI unit for effective dose is the **sievert**: $1\ \text{Sv} = 10^2$ rem.

[*Nuclear radiation is used in medicine as therapy and for imaging of biological processes, as well as several types of tomographic **imaging** of the human body: PET, SPET, and MRI; the latter makes use of **nuclear magnetic resonance** (NMR).]

Questions

(NOTE: Masses are found in Appendix B.)

1. Fill in the missing particles or nuclei:
(*a*) $n + {}^{137}_{56}Ba \rightarrow ? + \gamma$; (*b*) $n + {}^{137}_{56}Ba \rightarrow {}^{137}_{55}Cs + ?$;
(*c*) $d + {}^{2}_{1}H \rightarrow {}^{4}_{2}He + ?$; (*d*) $\alpha + {}^{197}_{79}Au \rightarrow ? + d$
where d stands for deuterium.

2. The isotope: ${}^{32}_{15}P$ is produced by the reaction: $n + ? \rightarrow {}^{32}_{15}P + p$. What must be the target nucleus?

3. When ${}^{22}_{11}Na$ is bombarded by deuterons (${}^{2}_{1}H$), an α particle is emitted. What is the resulting nuclide?

4. Why are neutrons such good projectiles for producing nuclear reactions?

5. A proton strikes a ${}^{20}_{10}Ne$ nucleus, and an α particle is observed to emerge. What is the residual nucleus? Write down the reaction equation.

6. Are fission fragments β^+ or β^- emitters? Explain.

7. If ${}^{235}_{92}U$ released only 1.5 neutrons per fission on the average, would a chain reaction be possible? If so, what would be different?

8. ${}^{238}_{92}U$ releases an average of 2.5 neutrons per fission compared to 2.9 for ${}^{239}_{94}Pu$. Pure samples of which of these two nuclei do you think would have the smaller critical mass? Explain.

9. The energy from nuclear fission appears in the form of thermal energy—but the thermal energy of what?

10. Why can't uranium be enriched by chemical means?

11. How can a neutron, with practically no kinetic energy, excite a nucleus to the extent shown in Fig. 31–2?

12. Why would a porous block of uranium be more likely to explode if kept under water rather than in air?

13. A reactor that uses highly enriched uranium can use ordinary water (instead of heavy water) as a moderator and still have a self-sustaining chain reaction. Explain.

14. Why must the fission process release neutrons if it is to be useful?

15. Discuss the relative merits and disadvantages, including pollution and safety, of power generation by fossil fuels, nuclear fission, and nuclear fusion.

16. What is the reason for the "secondary system" in a nuclear reactor, Fig. 31–7? That is, why is the water heated by the fuel in a nuclear reactor not used directly to drive the turbines?

17. Why are neutrons released in a fission reaction?

18. Why do gamma particles penetrate matter more easily than beta particles do?

19. A higher temperature is required for deuterium–deuterium ignition than for deuterium–tritium. Explain.

20. Light energy emitted by the Sun and stars comes from the fusion process. What conditions in the interior of stars make this possible?

21. How do stars, and our Sun, maintain confinement of the plasma for fusion?

22. What is the basic difference between fission and fusion?

23. People who work around metals that emit alpha particles are trained that there is little danger from proximity or even touching the material, but that they must take extreme precautions against ingesting it. Hence, there are strong rules against eating and drinking while working, and against machining the metal. Why?

24. Why is the recommended maximum radiation dose higher for women beyond the child-bearing age than for younger women?

25. Radiation is sometimes used to sterilize medical supplies and even food. Explain how it works.

26. What is the difference between absorbed dose and effective dose? What are the SI units for each?

* 27. How might radioactive tracers be used to find a leak in a pipe?

Problems

(NOTE: Masses are found in Appendix B.)

31–1 Nuclear Reactions, Transmutation

1. (I) Natural aluminum is all $^{27}_{13}\text{Al}$. If it absorbs a neutron, what does it become? Does it decay by β^+ or β^-? What will be the product nucleus?

2. (I) Determine whether the reaction $^2_1\text{H} + ^2_1\text{H} \rightarrow ^3_2\text{He} + \text{n}$ requires a threshold energy.

3. (I) Is the reaction $\text{n} + ^{238}_{92}\text{U} \rightarrow ^{239}_{92}\text{U} + \gamma$ possible with slow neutrons? Explain.

4. (II) Does the reaction $\text{p} + ^7_3\text{Li} \rightarrow ^4_2\text{He} + \alpha$ require energy, or does it release energy? How much energy?

5. (II) Calculate the energy released (or energy input required) for the reaction $\alpha + ^9_4\text{Be} \rightarrow ^{12}_6\text{C} + \text{n}$.

6. (II) (*a*) Can the reaction $\text{n} + ^{24}_{12}\text{Mg} \rightarrow ^{23}_{11}\text{Na} + \text{d}$ occur if the bombarding particles have 10.00 MeV of kinetic energy? (d stands for deuterium, ^2_1H.) (*b*) If so, how much energy is released?

7. (II) (*a*) Can the reaction $\text{p} + ^7_3\text{Li} \rightarrow ^4_2\text{He} + \alpha$ occur if the incident proton has kinetic energy = 2500 keV? (*b*) If so, what is the total kinetic energy of the products?

8. (II) In the reaction $\alpha + ^{14}_7\text{N} \rightarrow ^{17}_8\text{O} + \text{p}$, the incident α particles have 7.68 MeV of kinetic energy. (*a*) Can this reaction occur? (*b*) If so, what is the total kinetic energy of the products? The mass of $^{17}_8\text{O}$ is 16.999131 u.

9. (II) Calculate the *Q*-value for the "capture" reaction $\alpha + ^{16}_8\text{O} \rightarrow ^{20}_{10}\text{Ne} + \gamma$.

10. (II) Calculate the total kinetic energy of the products of the reaction $\text{d} + ^{13}_6\text{C} \rightarrow ^{14}_7\text{N} + \text{n}$ if the incoming deuteron (d) has KE = 36.3 MeV.

11. (II) Radioactive $^{14}_6\text{C}$ is produced in the atmosphere when a neutron is absorbed by $^{14}_7\text{N}$. Write the reaction and find its *Q*-value.

12. (II) An example of a "stripping" nuclear reaction is $\text{d} + ^6_3\text{Li} \rightarrow \text{X} + \text{p}$. (*a*) What is X, the resulting nucleus? (*b*) Why is it called a "stripping" reaction? (*c*) What is the *Q*-value of this reaction? Is the reaction endothermic or exothermic?

13. (II) An example of a "pick-up" nuclear reaction is $^3_2\text{He} + ^{12}_6\text{C} \rightarrow \text{X} + \alpha$. (*a*) Why is it called a "pickup" reaction? (*b*) What is the resulting nucleus? (*c*) What is the *Q*-value of this reaction? Is the reaction endothermic or exothermic?

14. (II) (*a*) Complete the following nuclear reaction, $\text{p} + ? \rightarrow ^{32}_{16}\text{S} + \gamma$. (*b*) What is the *Q*-value?

15. (II) The reaction $\text{p} + ^{18}_8\text{O} \rightarrow ^{18}_9\text{F} + \text{n}$ requires an input of energy equal to 2.453 MeV. What is the mass of $^{18}_9\text{F}$?

31–2 Nuclear Fission

16. (I) Calculate the energy released in the fission reaction $\text{n} + ^{235}_{92}\text{U} \rightarrow ^{88}_{38}\text{Sr} + ^{136}_{54}\text{Xe} + 12\text{n}$. Use Appendix B, and assume the initial kinetic energy of the neutron is very small.

17. (I) What is the energy released in the fission reaction of Eq. 31–4? (The masses of $^{141}_{56}\text{Ba}$ and $^{92}_{36}\text{Kr}$ are 140.914411 u and 91.926156 u, respectively.)

18. (I) How many fissions take place per second in a 200-MW reactor? Assume 200 MeV is released per fission.

19. (II) The energy produced by a fission reactor is about 200 MeV per fission. What fraction of the rest mass of a $^{235}_{92}\text{U}$ nucleus is this?

20. (II) Consider the fission reaction $^{235}_{92}\text{U} + \text{n} \rightarrow ^{133}_{51}\text{Sb} + ^{98}_{41}\text{Nb} + ?\text{n}$. (*a*) How many neutrons are produced in this reaction? (*b*) Calculate the energy release. The atomic masses for Sb and Nb isotopes are 132.915250 u and 97.910328 u, respectively.

21. (II) How much mass of $^{235}_{92}\text{U}$ is required to produce the same amount of energy as burning 1.0 kg of coal (about 3×10^7 J)?

22. (II) Suppose that the electric average power consumption, day and night, in a typical house is 950 W. What initial mass of $^{235}_{92}\text{U}$ would have to undergo fission to supply the electrical needs of such a house for a year? (Assume 200 MeV is released per fission, as well as 100% efficiency.)

23. (II) What initial mass of $^{235}_{92}\text{U}$ is required to operate a 650-MW reactor for 1 yr? Assume 40% efficiency.

24. (III) Assuming a fission of $^{236}_{92}\text{U}$ into two roughly equal fragments, estimate the electric potential energy just as the fragments separate from each other. Assume that the fragments are spherical (see Eq. 30–1) and compare your calculation to the nuclear fission energy released, about 200 MeV.

31–3 Nuclear Fusion

25. (I) What is the average kinetic energy of protons at the center of a star where the temperature is 10^7 K? [*Hint*: use Eq. 13–8.]

26. (II) Show that the energy released in the fusion reaction $^2_1\text{H} + ^3_1\text{H} \rightarrow ^4_2\text{He} + \text{n}$ is 17.59 MeV.

27. (II) Show that the energy released when two deuterium nuclei fuse to form ^3_2He with the release of a neutron is 3.27 MeV.

28. (II) Verify the *Q*-value stated for each of the reactions of Eqs. 31–6. [*Hint*: be careful with electrons.]

29. (II) Calculate the energy release per gram of fuel for the reactions of Eqs. 31–8a, b, and c. Compare to the energy release per gram of uranium in fission.

30. (II) How much energy is released when $^{238}_{92}\text{U}$ absorbs a slow neutron (KE ≈ 0) and becomes $^{239}_{92}\text{U}$?

31. (II) If a typical house requires 950 W of electric power on average, what minimum amount of deuterium fuel would have to be used in a year to supply these electrical needs? Assume the reaction of Eq. 31–8b.

32. (II) Show that the energies carried off by the $^{4}_{2}He$ nucleus and the neutron for the reaction of Eq. 31–8c are about 3.5 MeV and 14 MeV, respectively. Are these fixed values, independent of the plasma temperature?

33. (II) Suppose a fusion reactor ran on "d–d" reactions, Eqs. 31–8a and b. Estimate how much water, for fuel, would be needed per hour to run a 1000-MW reactor, assuming 30% efficiency.

34. (III) How much energy (J) is contained in 1.00 kg of water if its natural deuterium is used in the fusion reaction of Eq. 31–8a? Compare to the energy obtained from the burning of 1.0 kg of gasoline, about 5×10^7 J.

35. (III) The energy output of massive stars is believed to be due to the *carbon cycle* (see text). (*a*) Show that no carbon is consumed in this cycle and that the net effect is the same as for the proton–proton cycle. (*b*) What is the total energy release? (*c*) Determine the energy output for each reaction and decay. (*d*) Why does the carbon cycle require a higher temperature $(\approx 2 \times 10^7\,\mathrm{K})$ than the proton–proton cycle $(\approx 1.5 \times 10^7\,\mathrm{K})$?

36. (III) (*a*) Compare the energy needed for the first reaction of the carbon cycle to that for a deuterium–tritium reaction (Example 31–9). (*b*) If a deuterium–tritium reaction requires $T \approx 3 \times 10^8$ K, estimate the temperature needed for the first carbon-cycle reaction.

31–5 Dosimetry

37. (I) A dose of 4.0 Sv of γ rays in a short period would be lethal to about half the people subjected to it. How many grays is this?

38. (I) Fifty rads of α-particle radiation is equivalent to how many rads of X-rays in terms of biological damage?

39. (I) How many rads of slow neutrons will do as much biological damage as 75 rads of fast neutrons?

40. (I) How much energy is deposited in the body of a 65-kg adult exposed to a 2.0-Gy dose?

41. (II) A 0.025-μCi sample of $^{32}_{15}P$ is injected into an animal for tracer studies. If a Geiger counter intercepts 25% of the emitted β particles, what will be the counting rate, assumed 85% efficient?

42. (II) A cancer patient is undergoing radiation therapy in which protons with an energy of 1.2 MeV are incident on a 0.25-kg tumor. (*a*) If the patient receives an effective dose of 1.0 rem, what is the absorbed dose? (*b*) How many protons are absorbed by the tumor? Assume QF $\approx$ 1.

43. (II) A 1.0-mCi source of $^{32}_{15}P$ (in $NaHPO_4$), a β emitter, is implanted in a tumor where it is to administer 36 Gy. The half-life of $^{32}_{15}P$ is 14.3 days, and 1 mCi delivers about 10 mGy/min. Approximately how long should the source remain implanted?

44. (II) About 35 eV is required to produce one ion pair in air. Show that this is consistent with the two definitions of the roentgen given in the text.

45. (II) $^{57}_{27}Co$ emits 122-keV γ rays. If a 70-kg person swallowed 1.85 μCi of $^{57}_{27}Co$, what would be the dose rate (Gy/day) averaged over the whole body? Assume that 50% of the γ-ray energy is deposited in the body. [*Hint*: determine the rate of energy deposited in the body and use the definition of the gray.]

46. (II) What is the mass of a 1.00-μCi $^{14}_{6}C$ source?

47. (II) Huge amounts of radioactive $^{131}_{53}I$ were released in the accident at Chernobyl in 1986. Chemically, iodine goes to the human thyroid. (Doctors can use it for diagnosis and treatment of thyroid problems.) In a normal thyroid, $^{131}_{53}I$ absorption can cause damage to the thyroid. (*a*) Write down the reaction for the decay of $^{131}_{53}I$. (*b*) Its half-life is 8.0 d; how long would it take for ingested $^{131}_{53}I$ to become 10% of the initial value? (*c*) Absorbing 1 mCi of $^{131}_{53}I$ can be harmful; what mass of iodine is this?

48. (III) Assume a liter of milk typically has an activity of 2000 pCi due to $^{40}_{19}K$. If a person drinks two glasses (0.5 L) per day, estimate the total effective dose (in Sv and in rem) received in a year. As a crude model, assume the milk stays in the stomach 12 hr and is then released. Assume also that very roughly 10% of the 1.5 MeV released per decay is absorbed by the body. Compare your result to the normal allowed dose of 100 mrem per year. Make your estimate for (*a*) a 50-kg adult, and (*b*) a 5-kg baby.

49. (III) Radon gas, $^{222}_{86}Rn$, is considered a serious health hazard (see discussion in text). It decays by α-emission. (*a*) What is the daughter nucleus? (*b*) Is the daughter nucleus stable or radioactive? If the latter, how does it decay, and what is its half-life? (*c*) Is the daughter nucleus also a noble gas, or is it chemically reacting? (*d*) Suppose 1.0 ng of $^{222}_{86}Rn$ seeps into a basement. What will be its activity? If the basement is then sealed, what will be the activity 1 month later? [*Hint*: see Fig. 30–11.]

31–9 NMR

50. (II) Calculate the wavelength of photons needed to produce NMR transitions in free protons in a 1.000-T field. In what region of the spectrum does it lie?

General Problems

51. J. Chadwick discovered the neutron by bombarding $^{9}_{4}Be$ with the popular projectile of the day, alpha particles. (*a*) If one of the reaction products was the then unknown neutron, what was the other product? (*b*) What is the *Q*-value of this reaction?

52. Fusion temperatures are often given in keV. Determine the conversion factor from kelvins to keV using, as is common in this field, $\overline{\mathrm{KE}} = kT$ without the factor $\frac{3}{2}$.

53. One means of enriching uranium is by diffusion of the gas UF_6. Calculate the ratio of the speeds of molecules of this gas containing $^{235}_{92}U$ and $^{238}_{92}U$, on which this process depends.

54. (*a*) What mass of $^{235}_{92}U$ was actually fissioned in the first atomic bomb, whose energy was the equivalent of about 20 kilotons of TNT (1 kiloton of TNT releases 5×10^{12} J)? (*b*) What was the actual mass transformed to energy?

55. In a certain town the average yearly background radiation consists of 21 mrad of X-rays and γ rays plus 3.0 mrad of particles having a QF of 10. How many rem will a person receive per year on the average?

56. Deuterium makes up 0.0115% of natural hydrogen on average. Make a rough estimate of the total deuterium in the Earth's oceans and estimate the total energy released if all of it were used in fusion reactors.

57. A shielded γ-ray source yields a dose rate of 0.052 rad/h at a distance of 1.0 m for an average-sized person. If workers are allowed a maximum dose of 5.0 rem in 1 year, how close to the source may they operate, assuming a 40-h work week? Assume that the intensity of radiation falls off as the square of the distance. (It actually falls off more rapidly than $1/r^2$ because of absorption in the air, so your answer will give a better-than-permissible value.)

58. Radon gas, $^{222}_{86}\text{Rn}$, is formed by α decay. (*a*) Write the decay equation. (*b*) Ignoring the kinetic energy of the daughter nucleus (it's so massive), estimate the kinetic energy of the α particle produced. (*c*) Estimate the momentum of the alpha and of the daughter nucleus. (*d*) Estimate the kinetic energy of the daughter, and show that your approximation in (*b*) was valid.

59. Consider a system of nuclear power plants that produce 3400 MW. (*a*) What total mass of $^{235}_{92}\text{U}$ fuel would be required to operate these plants for 1 yr, assuming that 200 MeV is released per fission? (*b*) Typically 6% of the $^{235}_{92}\text{U}$ nuclei that fission produce $^{90}_{38}\text{Sr}$, a β^- emitter with a half-life of 29 yr. What is the total radioactivity of the $^{90}_{38}\text{Sr}$, in curies, produced in 1 yr? (Neglect the fact that some of it decays during the 1-yr period.)

60. In the net reaction, Eq. 31–7, for the proton–proton cycle in the Sun, the neutrinos escape from the Sun with energy of about 0.5 MeV. The remaining energy, 26.2 MeV, is available within the Sun. Use this value to calculate the "heat of combustion" per kilogram of hydrogen fuel and compare it to the heat of combustion of coal, about 3×10^7 J/kg.

61. Energy reaches Earth from the Sun at a rate of about 1400 W/m^2. Calculate (*a*) the total power output of the Sun, and (*b*) the number of protons consumed per second in the reaction of Eq. 31–7, assuming that this is the source of all the Sun's energy. (*c*) Assuming that the Sun's mass of 2.0×10^{30} kg was originally all protons and that all could be involved in nuclear reactions in the Sun's core, how long would you expect the Sun to "glow" at its present rate? See previous Problem.

62. Some stars, in a later stage of evolution, may begin to fuse two $^{12}_{6}\text{C}$ nuclei into one $^{24}_{12}\text{Mg}$ nucleus. (*a*) How much energy would be released in such a reaction? (*b*) What kinetic energy must two carbon nuclei each have when far apart, if they can then approach each other to within 6.0 fm, center-to-center? (*c*) Approximately what temperature would this require?

63. An average adult body contains about 0.10 μCi of $^{40}_{19}\text{K}$, which comes from food. (*a*) How many decays occur per second? (*b*) The potassium decays produce beta particles with energies of around 1.4 MeV. Calculate the dose per year in sieverts for a 50-kg adult. Is this a significant fraction of the 3.6 mSv/year background rate?

64. When the nuclear reactor accident occurred at Chernobyl in 1986, 2.0×10^7 Ci were released into the atmosphere. Assuming that this radiation was distributed uniformly over the surface of the Earth, what was the activity per square meter? (The actual activity was not uniform; even within Europe wet areas received more radioactivity from rainfall).

65. A star with a large helium abundance can burn helium in the reaction $^{4}_{2}\text{He} + ^{4}_{2}\text{He} + ^{4}_{2}\text{He} \rightarrow ^{12}_{6}\text{C}$. What is the Q-value for this reaction?

66. A 1.0-μCi $^{137}_{55}\text{Cs}$ source is used for 2.0 hours by a 75-kg student in a physics lab. Radioactive $^{137}_{55}\text{Cs}$ decays by β^- decay with a half-life of 30 years. The average energy of the emitted betas is about 190 keV per decay. The β decay is quickly followed by a γ with an energy of 660 keV. Assuming the student absorbs *all* emitted energy, what effective dose (in rem) is received during lab?

67. A large amount of $^{90}_{38}\text{Sr}$ was released during the Chernobyl nuclear reactor accident in 1986. The $^{90}_{38}\text{Sr}$ enters the body through the food chain. How long will it take for 90% of the $^{90}_{38}\text{Sr}$ released during the accident to decay? See Appendix B.

68. Three radioactive sources have the same activity, 25 mCi. Source A emits 1.0-MeV γ rays, source B emits 2.0-MeV γ rays, and source C emits 2.0-MeV alphas. What is the relative danger of these sources?

69. A 70-kg patient is to be given a medical test involving the ingestion of $^{99m}_{43}\text{Tc}$ (Section 31–7) which decays by emitting a 140-keV gamma. The half-life for this decay is 6 hours. Assuming that about half the gamma photons exit the body without interacting with anything, what must be the initial activity of the Tc sample if the whole-body dose cannot exceed 50 mrem? Make the rough approximation that biological elimination of Tc can be ignored.

Answers to Exercises

A: $^{138}_{56}\text{Ba}$.

B: 3 neutrons.

C: 2×10^{17}.