32–12 Strings and Supersymmetry

Even more ambitious than grand unified theories are attempts to also incorporate gravity, and thus unify all four forces in nature into a single theory. (Such theories are sometimes referred to misleadingly as **theories of everything**.) There are consistent theories that attempt to unify all four forces called **string theories**, in which the elementary particles (Table 32–5) are imagined not as points but as one-dimensional strings perhaps 10^{-35} m long.

String theory

Supersymmetry

A related idea is **supersymmetry**, which applied to strings is known as **superstring theory**. Supersymmetry predicts that interactions exist that would change fermions into bosons and vice versa, and that all known fermions have supersymmetric boson partners. Thus, for each quark we know (a fermion), there would be a *squark* (a *boson*) or "supersymmetric" quark. For every lepton there would be a *slepton*. Likewise, for every known boson (photons and gluons, for example), there would be a supersymmetric fermion (*photinos* and *gluinos*). Supersymmetry predicts also that a *graviton*, which transmits the gravity force, has a partner, the gravitino. Supersymmetric particles are sometimes called "SUSYs" for short, and may be a candidate for the "dark matter" of the universe (discussed in Chapter 33). But why hasn't this "missing part" of the universe ever been detected? The best guess is that supersymmetric particles might be heavier than their conventional counterparts, perhaps too heavy to have been produced in today's accelerators. Until a supersymmetric particle is found, and it may be possible at CERN's new LHC, supersymmetry is just an elegant guess.

The world of elementary particles is opening new vistas. What happens in the future is bound to be exciting.

Summary

Particle accelerators are used to accelerate charged particles, such as electrons and protons, to very high energy. High-energy particles have short wavelength and so can be used to probe the structure of matter at very small distances in great detail. High kinetic energy also allows the creation of new particles through collision (via $E = mc^2$).

Cyclotrons and **synchrotrons** use a magnetic field to keep the particles in a circular path and accelerate them at intervals by high voltage. **Linear accelerators** accelerate particles along a line. **Colliding beams** allow higher interaction energy.

An **antiparticle** has the same mass as a particle but opposite charge. Certain other properties may also be opposite: for example, the antiproton has **baryon number** (nucleon number) opposite to that for the proton.

In all nuclear and particle reactions, the following conservation laws hold: momentum, angular momentum, mass–energy, electric charge, baryon number, and **lepton numbers**.

Certain particles have a property called **strangeness**, which is conserved by the strong force but not by the weak force. The properties **charm**, **bottomness**, and **topness** also are conserved by the strong force but not by the weak.

Just as the electromagnetic force can be said to be due to an exchange of photons, the strong nuclear force was first thought to be carried by *mesons* that have rest mass, but recent theory says the force is carried by massless **gluons**. The W and Z particles carry the weak force. These fundamental force carriers (photon, W and Z, gluons) are called **gauge bosons**.

Other particles can be classified as either *leptons* or *hadrons*. **Leptons** participate in the weak and electrically charged electromagnetic interactions. **Hadrons**, which today are considered to be made up of **quarks**, participate in the strong interaction as well. The hadrons can be classified as **mesons**, with baryon number zero, and **baryons**, with nonzero baryon number.

All particles, except for the photon, electron, neutrinos, and proton, decay with measurable half-lives varying from 10^{-25} s to 10^{3} s. The half-life depends on which force is predominant. Weak decays usually have half-lives greater than about 10^{-13} s. Electromagnetic decays have half-lives on the order of 10^{-16} to 10^{-19} s. The shortest lived particles, called **resonances**, decay via the strong interaction and live typically for only about 10^{-23} s.

Today's standard model of elementary particles considers **quarks** as the basic building blocks of the hadrons. The six quark "flavors" are called **up**, **down**, **strange**, **charmed**, **bottom**, and **top**. It is expected that there are the same number of quarks as leptons (six of each), and that quarks and leptons are the truly elementary particles along with the gauge bosons (γ, W, Z, gluons). Quarks are said to have **color**, and, according to **quantum chromodynamics** (QCD), the strong color force acts between their color charges and is transmitted by **gluons**. **Electroweak theory** views the weak and electromagnetic forces as two aspects of a single underlying interaction. QCD plus the electroweak theory are referred to as the **Standard Model**.

Grand unified theories of forces suggest that at very short distances (10^{-32} m) and very high energy, the weak, electromagnetic, and strong forces appear as a single force, and the fundamental difference between quarks and leptons disappears.

Questions

1. Give a reaction between two nucleons, similar to Eq. 32–4, that could produce a π^-.
2. If a proton is moving at very high speed, so that its kinetic energy is much greater than its rest energy (m_0c^2), can it then decay via $p \rightarrow n + \pi^+$?
3. What would an "antiatom," made up of the antiparticles to the constituents of normal atoms, consist of? What might happen if *antimatter*, made of such antiatoms, came in contact with our normal world of matter?
4. What particle in a decay signals the electromagnetic interaction?
5. Does the presence of a neutrino among the decay products of a particle necessarily mean that the decay occurs via the weak interaction? Do all decays via the weak interaction produce a neutrino? Explain.
6. Why is it that a neutron decays via the weak interaction even though the neutron and one of its decay products (proton) are strongly interacting?
7. Which of the four interactions (strong, electromagnetic, weak, gravitational) does an electron take part in? A neutrino? A proton?
8. Check that charge and baryon number are conserved in each of the decays in Table 32–2.
9. Which of the particle decays in Table 32–2 occur via the electromagnetic interaction?
10. Which of the particle decays in Table 32–2 occur by the weak interaction?
11. By what interaction, and why, does $\Sigma^{\pm}$ decay to Λ^0? What about Σ^0 decaying to Λ^0?
12. The Δ baryon has spin $\frac{3}{2}$, baryon number 1, and charge $Q = +2, \ +1, \ 0,$ or -1. Why is there no charge state $Q = -2$?
13. Which of the particle decays in Table 32–4 occur via the electromagnetic interaction?
14. Which of the particle decays in Table 32–4 occur by the weak interaction?
15. Quarks have spin $\frac{1}{2}$. How do you account for the fact that baryons have spin $\frac{1}{2}$ or $\frac{3}{2}$, and mesons have spin 0 or 1?
16. Suppose there were a kind of "neutrinolet" that was massless, had no color charge or electrical charge, and did not feel the weak force. Could you say that this particle even exists?
17. Is it possible for a particle to be both (*a*) a lepton and a baryon? (*b*) a baryon and a hadron? (*c*) a meson and a quark? (*d*) a hadron and a lepton? Explain.
18. Using the ideas of quantum chromodynamics, would it be possible to find particles made up of two quarks and no antiquarks? What about two quarks and two antiquarks?
19. Why do neutrons decay when they are free but not when they are inside the nucleus?
20. Is the reaction $e^- + p \rightarrow n + \bar{\nu}_e$ possible? Explain.
21. Occasionally, the Λ will decay by the following reaction: $\Lambda^0 \rightarrow p^+ + e^- + \bar{\nu}_e$. Which of the four forces in nature is responsible for this decay? How do you know?

Problems

32–1 Particles and Accelerators

1. (I) What is the total energy of a proton whose kinetic energy is 6.35 GeV?
2. (I) Calculate the wavelength of 35-GeV electrons.
3. (I) What strength of magnetic field is used in a cyclotron in which protons make 2.8×10^7 revolutions per second?
4. (I) What is the time for one complete revolution for a very high-energy proton in the 1.0-km-radius Fermilab accelerator?
5. (I) If α particles are accelerated by the cyclotron of Example 32–2, what must be the frequency of the voltage applied to the dees?
6. (II) (*a*) If the cyclotron of Example 32–2 accelerated α particles, what maximum energy could they attain? What would their speed be? (*b*) Repeat for deuterons ($^2_1$H). (*c*) In each case, what frequency of voltage is required?
7. (II) Which is better for picking out details of the nucleus: 30-MeV alpha particles or 30-MeV protons? Compare each of their wavelengths with the size of a nucleon in a nucleus.
8. (II) The voltage across the dees of a cyclotron is 55 kV. How many revolutions do protons make to reach a kinetic energy of 25 MeV?
9. (II) What is the wavelength (= maximum resolvable distance) of 7.0-TeV protons?
10. (II) A cyclotron with a radius of 1.0 m is to accelerate deuterons ($^2_1$H) to an energy of 12 MeV. (*a*) What is the required magnetic field? (*b*) What frequency is needed for the voltage between the dees? (*c*) If the potential difference between the dees averages 22 kV, how many revolutions will the particles make before exiting? (*d*) How much time does it take for one deuteron to go from start to exit. (*e*) Estimate how far it travels during this time.
11. (II) The 4.25-km-radius tunnel that will be used to house the magnets for the Large Hadron Collider (LHC) calls for proton beams of energy 7.0 TeV. What magnetic field will be required?
12. (II) The 1.0-km radius Fermilab Tevatron takes about 20 seconds to bring the energies of the stored protons from 150 GeV to 1.0 TeV. The acceleration is done once per turn. Estimate the energy given to the protons on each turn. (You can assume that the speed of the protons is essentially *c* the whole time.)
13. (III) Show that the energy of a particle (charge *e*) in a synchrotron, in the relativistic limit ($v \approx c$), is given by $E\,(\text{in eV}) = Brc$, where B is magnetic field strength and r the radius of the orbit (SI units).
14. (III) What magnetic field intensity is needed at the 1.0-km-radius Fermilab synchrotron for 1.0-TeV protons?

32–2 to 32–6 Particle Interactions, Particle Exchange

15. (I) How much energy is released in the decay

$$\pi^+ \to \mu^+ + \nu_\mu?$$

See Table 32–2.

16. (I) About how much energy is released when a Λ^0 decays to $n + \pi^0$? (See Table 32–2.)

17. (I) How much energy is required to produce a neutron–antineutron pair?

18. (I) Estimate the range of the strong force if the mediating particle were the kaon instead of the pion.

19. (II) Two protons are heading toward each other with equal speeds. What minimum kinetic energy must each have if a π^0 meson is to be created in the process? (See Table 32–2.)

20. (II) What minimum kinetic energy must two neutrons each have if they are traveling at the same speed toward each other, collide, and produce a K^+K^- pair in addition to themselves? (See Table 32–2.)

21. (II) Estimate the range of the weak force using Eq. 32–3, given the masses of the W and Z particles as about 80 to 90 GeV/c^2.

22. (II) What are the wavelengths of the two photons produced when a proton and antiproton at rest annihilate?

23. (II) The Λ cannot decay by the following reactions. What conservation law is violated in each of the reactions?

(*a*) $\Lambda^0 \to n + \pi^-$
(*b*) $\Lambda^0 \to p + K^-$
(*c*) $\Lambda^0 \to \pi^+ + \pi^-$

24. (II) For the decay $\Lambda^0 \to p + \pi^-$, calculate (*a*) the *Q*-value (energy released), and (*b*) the kinetic energy of the p and π^-, assuming the Λ^0 decays from rest. (Use relativistic formulas.)

25. (II) (*a*) Show, by conserving momentum and energy, that it is impossible for an isolated electron to radiate only a single photon. (*b*) With this result in mind, how can you defend the photon exchange diagram in Fig. 32–7?

26. (II) What would be the wavelengths of the two photons produced when an electron and a positron, each with 420 keV of kinetic energy, annihilate head on?

27. (II) In the rare decay $\pi^+ \to e^+ + \nu_e$, what is the kinetic energy of the positron? Assume the π^+ decays from rest.

28. (II) Which of the following reactions and decays are possible? For those forbidden, explain what laws are violated.

(*a*) $\pi^- + p \to n + \eta^0$
(*b*) $\pi^+ + p \to n + \pi^0$
(*c*) $\pi^+ + p \to p + e^+$
(*d*) $p \to e^+ + \nu_e$
(*e*) $\mu^+ \to e^+ + \bar{\nu}_\mu$
(*f*) $p \to n + e^+ + \nu_e$

29. (II) Calculate the kinetic energy of each of the two products in the decay $\Xi^- \to \Lambda^0 + \pi^-$. Assume the Ξ^- decays from rest.

30. (III) Could a π^+ meson be produced if a 100-MeV proton struck a proton at rest? What minimum kinetic energy must the incoming proton have?

31. (III) Calculate the maximum kinetic energy of the electron in the decay $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$. [*Hint*: in what direction do the two neutrinos move relative to the electron in order to give the latter the maximum kinetic energy? Both energy and momentum are conserved; use relativistic formulas.]

32–7 to 32–11 Resonances, Standard Model, Quarks, QCD, GUT

32. (I) Use Fig. 32–11 to estimate the energy width and then the lifetime of the Δ resonance using the uncertainty principle.

33. (I) The measured width of the J/ψ meson is 88 keV. Estimate its lifetime.

34. (I) The measured width of the ψ (3685) meson is 277 keV. Estimate its lifetime.

35. (I) What is the energy width (or uncertainty) of (*a*) η^0, and (*b*) Σ^0? See Table 32–2.

36. (I) The B^- meson is a $b\bar{u}$ quark combination. (*a*) Show that this is consistent for all quantum numbers. (*b*) What are the quark combinations for B^+, B^0, $\bar{B}^0$?

37. (II) Which of the following decays are possible? For those that are forbidden, explain which laws are violated.

(*a*) $\Xi^0 \to \Sigma^+ + \pi^-$
(*b*) $\Omega^- \to \Sigma^0 + \pi^- + \nu$
(*c*) $\Sigma^0 \to \Lambda^0 + \gamma + \gamma$

38. (II) What are the quark combinations that can form (*a*) a neutron, (*b*) an antineutron, (*c*) a Λ^0, (*d*) a $\bar{\Sigma}^0$?

39. (II) What particles do the following quark combinations produce: (*a*) uud, (*b*) $\bar{u}\bar{u}\bar{s}$, (*c*) $\bar{u}s$, (*d*) $d\bar{u}$, (*e*) $\bar{c}s$?

40. (II) What is the quark combination needed to produce a D^0 meson ($Q = B = S = 0, c = +1$)?

41. (II) The D_S^+ meson has $S = c = +1$, $B = 0$. What quark combination would produce it?

42. (II) Draw a possible Feynman diagram using quarks (as in Fig. 32–13c) for the reaction $\pi^- + p \to \pi^0 + n$.

43. (II) Draw a Feynman diagram for the reaction $n + \nu_\mu \to p + \mu^-$.

General Problems

44. What is the total energy of a proton whose kinetic energy is 25 GeV? What is its wavelength?

45. Assume there are 5.0×10^{13} protons at 1.0 TeV stored in the 1.0-km-radius ring of the Tevatron. (*a*) How much current (amperes) is carried by this beam? (*b*) How fast would a 1500-kg car have to move to carry the same kinetic energy as this beam?

46. Protons are injected into the 1.0-km-radius Fermilab Tevatron with an energy of 150 GeV. If they are accelerated by 2.5 MV each revolution, how far do they travel and approximately how long does it take for them to reach 1.0 TeV?

47. (*a*) How much energy is released when an electron and a positron annihilate each other? (*b*) How much energy is released when a proton and an antiproton annihilate each other? (All particles KE $\approx$ 0.)

48. Which of the following reactions are possible, and by what interaction could they occur? For those forbidden, explain why.
(*a*) $\pi^- + p \rightarrow K^+ + \Sigma^-$
(*b*) $\pi^+ + p \rightarrow K^+ + \Sigma^+$
(*c*) $\pi^- + p \rightarrow \Lambda^0 + K^0 + \pi^0$
(*d*) $\pi^+ + p \rightarrow \Sigma^0 + \pi^0$
(*e*) $\pi^- + p \rightarrow p + e^- + \bar{\nu}_e$

49. Which of the following reactions are possible, and by what interaction could they occur? For those forbidden, explain why.
(*a*) $\pi^- + p \rightarrow K^0 + p + \pi^0$
(*b*) $K^- + p \rightarrow \Lambda^0 + \pi^0$
(*c*) $K^+ + n \rightarrow \Sigma^+ + \pi^0 + \gamma$
(*d*) $K^+ \rightarrow \pi^0 + \pi^0 + \pi^+$
(*e*) $\pi^+ \rightarrow e^+ + \nu_e$

50. One decay mode for a π^+ is $\pi^+ \rightarrow \mu^+ + \nu_\mu$. What would be the equivalent decay for a π^-? Check conservation rules.

51. Symmetry breaking occurs in the electroweak theory at about 10^{-18} m. Show that this corresponds to an energy that is on the order of the mass of the $W^\pm$.

52. The mass of a π^0 can be measured by observing the reaction $\pi^- + p \rightarrow \pi^0 + n$ at very low incident π^- kinetic energy (assume it is zero). The neutron is observed to be emitted with a kinetic energy of 0.60 MeV. Use conservation of energy and momentum to determine the π^0 mass.

53. Calculate the Q-value for each of the reactions, Eq. 32–4, for producing a pion.

54. Calculate the Q-value for the reaction $\pi^- + p \rightarrow \Lambda^0 + K^0$, when negative pions strike stationary protons. Estimate the minimum pion kinetic energy needed to produce this reaction. [*Hint*: assume Λ^0 and K^0 move off with the same velocity.]

55. How many fundamental fermions are there in a water molecule?

56. A proton and an antiproton annihilate each other at rest and produce two pions, π^- and π^+. What is the kinetic energy of each pion?

57. (*a*) Show that the so-called unification distance of 10^{-32} m in grand unified theory is equivalent to an energy of about 10^{16} GeV. Use the uncertainty principle, and also de Broglie's wavelength formula, and explain how they apply. (*b*) Calculate the temperature corresponding to 10^{16} GeV.

58. For the reaction $p + p \rightarrow 3p + \bar{p}$, where one of the initial protons is at rest, use relativistic formulas to show that the threshold energy is $6m_p c^2$, equal to three times the magnitude of the Q-value of the reaction, where m_p is the proton mass. [*Hint*: assume all final particles have the same velocity.]

59. The lifetimes listed in Table 32–2 are in terms of *proper time*, measured in a reference frame where the particle is at rest. If a tau lepton is created with a kinetic energy of 450 MeV, how long would its track be as measured in the lab, on average, ignoring any collisions?

60. Identify the missing particle in the following reactions.
(*a*) $p + p \rightarrow p + n + \pi^+ + ?$
(*b*) $p + ? \rightarrow n + \mu^+$

61. Use the quark model to describe the reaction

$$\bar{p} + n \rightarrow \pi^- + \pi^0.$$

62. What fraction of the speed of light c is the speed of a 7.0-TeV proton?

Answers to Exercises

A: 1.24×10^{-18} m.

B: $s\bar{u}$.