Summary

When a rigid object rotates about a fixed axis, each point of the object moves in a circular path. Lines drawn perpendicularly from the rotation axis to various points in the object all sweep out the same angle θ in any given time interval.

Angles are conveniently measured in **radians**, where one radian is the angle subtended by an arc whose length is equal to the radius, or

$$2\pi \text{ rad} = 360^\circ$$
$$1 \text{ rad} \approx 57.3^\circ.$$

Angular velocity, ω, is defined as the rate of change of angular position:

$$\omega = \frac{\Delta\theta}{\Delta t}. \tag{8–2}$$

All parts of a rigid object rotating about a fixed axis have the same angular velocity at any instant.

Angular acceleration, α, is defined as the rate of change of angular velocity:

$$\alpha = \frac{\Delta\omega}{\Delta t}. \tag{8–3}$$

The linear velocity v and acceleration a of a point fixed at a distance r from the axis of rotation are related to ω and α by

$$v = r\omega, \tag{8–4}$$
$$a_{\tan} = r\alpha, \tag{8–5}$$
$$a_R = \omega^2 r, \tag{8–6}$$

where $a_{\tan}$ and a_R are the tangential and radial (centripetal) components of the linear acceleration, respectively.

The frequency f is related to ω by

$$\omega = 2\pi f, \tag{8–7}$$

and to the period T by

$$T = 1/f. \tag{8–8}$$

The equations describing uniformly accelerated rotational motion (α = constant) have the same form as for uniformly accelerated linear motion:

$$\omega = \omega_0 + \alpha t, \qquad \theta = \omega_0 t + \tfrac{1}{2}\alpha t^2,$$
$$\omega^2 = \omega_0^2 + 2\alpha\theta, \qquad \overline{\omega} = \frac{\omega + \omega_0}{2}. \tag{8–9}$$

The dynamics of rotation is analogous to the dynamics of linear motion. Force is replaced by **torque** τ, which is defined as the product of force times lever arm (perpendicular distance from the line of action of the force to the axis of rotation):

$$\tau = rF\sin\theta = r_\perp F = rF_\perp. \tag{8–10}$$

Mass is replaced by **moment of inertia** I, which depends not only on the mass of the object, but also on how the mass is distributed about the axis of rotation. Linear acceleration is replaced by angular acceleration. The rotational equivalent of Newton's second law is then

$$\Sigma\tau = I\alpha. \tag{8–14}$$

The **rotational kinetic energy** of an object rotating about a fixed axis with angular velocity ω is

$$\text{KE} = \tfrac{1}{2}I\omega^2. \tag{8–15}$$

For an object both translating and rotating, the total kinetic energy is the sum of the translational kinetic energy of the object's center of mass plus the rotational kinetic energy of the object about its center of mass:

$$\text{KE} = \tfrac{1}{2}Mv_{\text{CM}}^2 + \tfrac{1}{2}I_{\text{CM}}\omega^2 \tag{8–16}$$

as long as the rotation axis is fixed in direction.

The **angular momentum** L of an object about a fixed rotation axis is given by

$$L = I\omega. \tag{8–18}$$

Newton's second law, in terms of angular momentum, is

$$\Sigma\tau = \frac{\Delta L}{\Delta t}. \tag{8–19}$$

If the net torque on the object is zero, $\Delta L/\Delta t = 0$, so L = constant. This is the **law of conservation of angular momentum** for a rotating object.

The following Table summarizes angular (or rotational) quantities, comparing them to their translational analogs.

Translation	Rotation	Connection
x	θ	$x = r\theta$
v	ω	$v = r\omega$
a	α	$a = r\alpha$
m	I	$I = \Sigma mr^2$
F	τ	$\tau = rF\sin\theta$
$\text{KE} = \frac{1}{2}mv^2$	$\frac{1}{2}I\omega^2$	
$p = mv$	$L = I\omega$	
$W = Fd$	$W = \tau\theta$	
$\Sigma F = ma$	$\Sigma\tau = I\alpha$	
$\Sigma F = \frac{\Delta p}{\Delta t}$	$\Sigma\tau = \frac{\Delta L}{\Delta t}$	

Questions

1. A bicycle odometer (which measures distance traveled) is attached near the wheel hub and is designed for 27-inch wheels. What happens if you use it on a bicycle with 24-inch wheels?
2. Suppose a disk rotates at constant angular velocity. Does a point on the rim have radial and/or tangential acceleration? If the disk's angular velocity increases uniformly, does the point have radial and/or tangential acceleration? For which cases would the magnitude of either component of linear acceleration change?
3. Could a nonrigid body be described by a single value of the angular velocity ω? Explain.
4. Can a small force ever exert a greater torque than a larger force? Explain.

5. If a force $\vec{\mathbf{F}}$ acts on an object such that its lever arm is zero, does it have any effect on the object's motion? Explain.
6. Why is it more difficult to do a sit-up with your hands behind your head than when your arms are stretched out in front of you? A diagram may help you to answer this.
7. A 21-speed bicycle has seven sprockets at the rear wheel and three at the pedal cranks. In which gear is it harder to pedal, a small rear sprocket or a large rear sprocket? Why? In which gear is it harder to pedal, a small front sprocket or a large front sprocket? Why?
8. Mammals that depend on being able to run fast have slender lower legs with flesh and muscle concentrated high, close to the body (Fig. 8–34). On the basis of rotational dynamics, explain why this distribution of mass is advantageous.

FIGURE 8–34 Question 8. A gazelle.

FIGURE 8–35 Question 9.

9. Why do tightrope walkers (Fig. 8–35) carry a long, narrow beam?
10. If the net force on a system is zero, is the net torque also zero? If the net torque on a system is zero, is the net force zero?
11. Two inclines have the same height but make different angles with the horizontal. The same steel ball is rolled down each incline. On which incline will the speed of the ball at the bottom be greater? Explain.
12. Two solid spheres simultaneously start rolling (from rest) down an incline. One sphere has twice the radius and twice the mass of the other. Which reaches the bottom of the incline first? Which has the greater speed there? Which has the greater total kinetic energy at the bottom?
13. A sphere and a cylinder have the same radius and the same mass. They start from rest at the top of an incline. Which reaches the bottom first? Which has the greater speed at the bottom? Which has the greater total kinetic energy at the bottom? Which has the greater rotational KE?
14. We claim that momentum and angular momentum are conserved. Yet most moving or rotating objects eventually slow down and stop. Explain.
15. If there were a great migration of people toward the Earth's equator, how would this affect the length of the day?
16. Can the diver of Fig. 8–29 do a somersault without having any initial rotation when she leaves the board?
17. The moment of inertia of a rotating solid disk about an axis through its center of mass is $\frac{1}{2}MR^2$ (Fig. 8–21c). Suppose instead that the axis of rotation passes through a point on the edge of the disk. Will the moment of inertia be the same, larger, or smaller?
18. Suppose you are sitting on a rotating stool holding a 2-kg mass in each outstretched hand. If you suddenly drop the masses, will your angular velocity increase, decrease, or stay the same? Explain.
19. Two spheres look identical and have the same mass. However, one is hollow and the other is solid. Describe an experiment to determine which is which.

* 20. In what direction is the Earth's angular velocity vector as it rotates daily about its axis?

* 21. The angular velocity of a wheel rotating on a horizontal axle points west. In what direction is the linear velocity of a point on the top of the wheel? If the angular acceleration points east, describe the tangential linear acceleration of this point at the top of the wheel. Is the angular speed increasing or decreasing?

* 22. Suppose you are standing on the edge of a large freely rotating turntable. What happens if you walk toward the center?

* 23. A shortstop may leap into the air to catch a ball and throw it quickly. As he throws the ball, the upper part of his body rotates. If you look quickly you will notice that his hips and legs rotate in the opposite direction (Fig. 8–36). Explain.

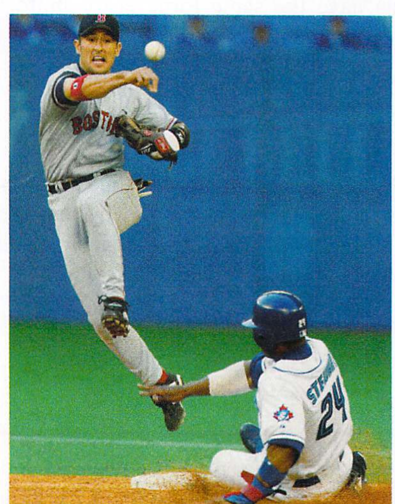


FIGURE 8–36 Question 23. A shortstop in the air, throwing the ball.

* 24. On the basis of the law of conservation of angular momentum, discuss why a helicopter must have more than one rotor (or propeller). Discuss one or more ways the second propeller can operate to keep the helicopter stable.

Problems

8–1 Angular Quantities

1. (I) Express the following angles in radians: (*a*) 30°, (*b*) 57°, (*c*) 90°, (*d*) 360°, and (*e*) 420°. Give as numerical values and as fractions of π.
2. (I) Eclipses happen on Earth because of an amazing coincidence. Calculate, using the information inside the Front Cover, the angular diameters (in radians) of the Sun and the Moon, as seen on Earth.
3. (I) A laser beam is directed at the Moon, 380,000 km from Earth. The beam diverges at an angle θ (Fig. 8–37) of 1.4×10^{-5} rad. What diameter spot will it make on the Moon?

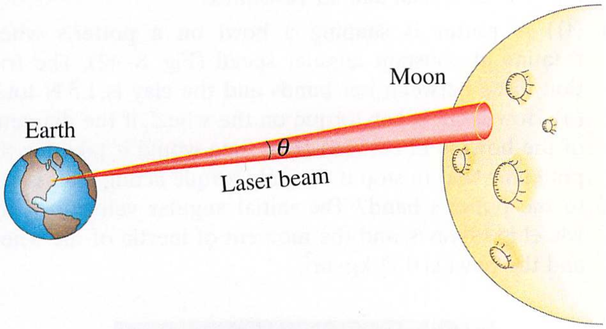


FIGURE 8–37 Problem 3.

4. (I) The blades in a blender rotate at a rate of 6500 rpm. When the motor is turned off during operation, the blades slow to rest in 3.0 s. What is the angular acceleration as the blades slow down?
5. (II) A child rolls a ball on a level floor 3.5 m to another child. If the ball makes 15.0 revolutions, what is its diameter?
6. (II) A bicycle with tires 68 cm in diameter travels 8.0 km. How many revolutions do the wheels make?
7. (II) (*a*) A grinding wheel 0.35 m in diameter rotates at 2500 rpm. Calculate its angular velocity in rad/s. (*b*) What are the linear speed and acceleration of a point on the edge of the grinding wheel?
8. (II) A rotating merry-go-round makes one complete revolution in 4.0 s (Fig. 8–38). (*a*) What is the linear speed of a child seated 1.2 m from the center? (*b*) What is her acceleration (give components)?

FIGURE 8–38 Problem 8.

9. (II) Calculate the angular velocity of the Earth (*a*) in its orbit around the Sun, and (*b*) about its axis.
10. (II) What is the linear speed of a point (*a*) on the equator, (*b*) on the Arctic Circle (latitude 66.5° N), and (*c*) at a latitude of 45.0° N, due to the Earth's rotation?
11. (II) How fast (in rpm) must a centrifuge rotate if a particle 7.0 cm from the axis of rotation is to experience an acceleration of 100,000 *g*'s?
12. (II) A 70-cm-diameter wheel accelerates uniformly about its center from 130 rpm to 280 rpm in 4.0 s. Determine (*a*) its angular acceleration, and (*b*) the radial and tangential components of the linear acceleration of a point on the edge of the wheel 2.0 s after it has started accelerating.
13. (II) A turntable of radius R_1 is turned by a circular rubber roller of radius R_2 in contact with it at their outer edges. What is the ratio of their angular velocities, ω_1/ω_2?
14. (III) In traveling to the Moon, astronauts aboard the *Apollo* spacecraft put themselves into a slow rotation to distribute the Sun's energy evenly. At the start of their trip, they accelerated from no rotation to 1.0 revolution every minute during a 12-min time interval. The spacecraft can be thought of as a cylinder with a diameter of 8.5 m. Determine (*a*) the angular acceleration, and (*b*) the radial and tangential components of the linear acceleration of a point on the skin of the ship 5.0 min after it started this acceleration.

8–2 and 8–3 Constant Angular Acceleration; Rolling

15. (I) A centrifuge accelerates uniformly from rest to 15,000 rpm in 220 s. Through how many revolutions did it turn in this time?
16. (I) An automobile engine slows down from 4500 rpm to 1200 rpm in 2.5 s. Calculate (*a*) its angular acceleration, assumed constant, and (*b*) the total number of revolutions the engine makes in this time.
17. (I) Pilots can be tested for the stresses of flying high-speed jets in a whirling "human centrifuge," which takes 1.0 min to turn through 20 complete revolutions before reaching its final speed. (*a*) What was its angular acceleration (assumed constant), and (*b*) what was its final angular speed in rpm?
18. (II) A wheel 33 cm in diameter accelerates uniformly from 240 rpm to 360 rpm in 6.5 s. How far will a point on the edge of the wheel have traveled in this time?
19. (II) A cooling fan is turned off when it is running at 850 rev/min. It turns 1500 revolutions before it comes to a stop. (*a*) What was the fan's angular acceleration, assumed constant? (*b*) How long did it take the fan to come to a complete stop?
20. (II) A small rubber wheel is used to drive a large pottery wheel, and they are mounted so that their circular edges touch. The small wheel has a radius of 2.0 cm and accelerates at the rate of 7.2 rad/s^2, and it is in contact with the pottery wheel (radius 25.0 cm) without slipping. Calculate (*a*) the angular acceleration of the pottery wheel, and (*b*) the time it takes the pottery wheel to reach its required speed of 65 rpm.

21. (II) The tires of a car make 65 revolutions as the car reduces its speed uniformly from 95 km/h to 45 km/h. The tires have a diameter of 0.80 m. (*a*) What was the angular acceleration of the tires? (*b*) If the car continues to decelerate at this rate, how much more time is required for it to stop?

8–4 Torque

22. (I) A 55-kg person riding a bike puts all her weight on each pedal when climbing a hill. The pedals rotate in a circle of radius 17 cm. (*a*) What is the maximum torque she exerts? (*b*) How could she exert more torque?

23. (I) A person exerts a force of 55 N on the end of a door 74 cm wide. What is the magnitude of the torque if the force is exerted (*a*) perpendicular to the door, and (*b*) at a 45° angle to the face of the door?

24. (II) Calculate the net torque about the axle of the wheel shown in Fig. 8–39. Assume that a friction torque of 0.40 m·N opposes the motion.

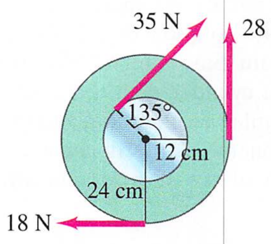


FIGURE 8–39 Problem 24.

25. (II) Two blocks, each of mass m, are attached to the ends of a massless rod which pivots as shown in Fig. 8–40. Initially the rod is held in the horizontal position and then released. Calculate the magnitude and direction of the net torque on this system.

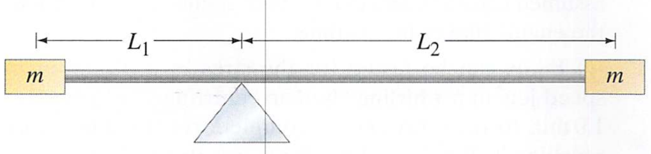


FIGURE 8–40 Problem 25.

26. (II) The bolts on the cylinder head of an engine require tightening to a torque of 88 m·N. If a wrench is 28 cm long, what force perpendicular to the wrench must the mechanic exert at its end? If the six-sided bolt head is 15 mm in diameter, estimate the force applied near each of the six points by a socket wrench (Fig. 8–41).

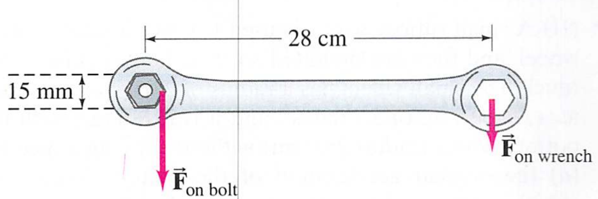


FIGURE 8–41 Problem 26.

8–5 and 8–6 Rotational Dynamics

27. (I) Determine the moment of inertia of a 10.8-kg sphere of radius 0.648 m when the axis of rotation is through its center.

28. (I) Calculate the moment of inertia of a bicycle wheel 66.7 cm in diameter. The rim and tire have a combined mass of 1.25 kg. The mass of the hub can be ignored (why?).

29. (II) A small 650-gram ball on the end of a thin, light rod is rotated in a horizontal circle of radius 1.2 m. Calculate (*a*) the moment of inertia of the ball about the center of the circle, and (*b*) the torque needed to keep the ball rotating at constant angular velocity if air resistance exerts a force of 0.020 N on the ball. Ignore the rod's moment of inertia and air resistance.

30. (II) A potter is shaping a bowl on a potter's wheel rotating at constant angular speed (Fig. 8–42). The friction force between her hands and the clay is 1.5 N total. (*a*) How large is her torque on the wheel, if the diameter of the bowl is 12 cm? (*b*) How long would it take for the potter's wheel to stop if the only torque acting on it is due to the potter's hand? The initial angular velocity of the wheel is 1.6 rev/s, and the moment of inertia of the wheel and the bowl is 0.11 kg·m^2.

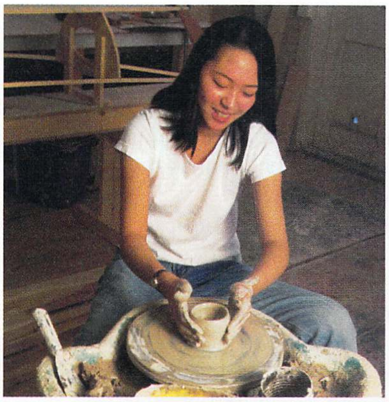


FIGURE 8–42 Problem 30.

31. (II) Calculate the moment of inertia of the array of point objects shown in Fig. 8–43 about (*a*) the vertical axis, and (*b*) the horizontal axis. Assume $m = 1.8$ kg, $M = 3.1$ kg, and the objects are wired together by very light, rigid pieces of wire. The array is rectangular and is split through the middle by the horizontal axis. (*c*) About which axis would it be harder to accelerate this array?

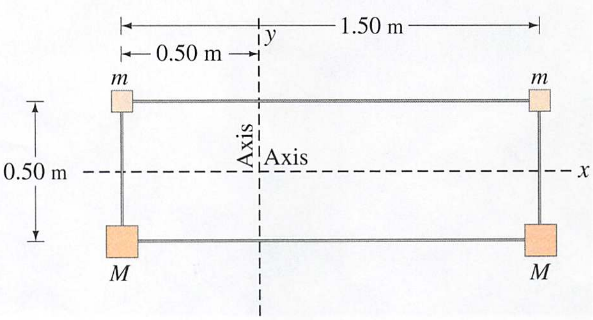


FIGURE 8–43 Problem 31.

32. (II) An oxygen molecule consists of two oxygen atoms whose total mass is 5.3×10^{-26} kg and whose moment of inertia about an axis perpendicular to the line joining the two atoms, midway between them, is $1.9 \times 10^{-46}\ \mathrm{kg \cdot m^2}$. From these data, estimate the effective distance between the atoms.

33. (II) To get a flat, uniform cylindrical satellite spinning at the correct rate, engineers fire four tangential rockets as shown in Fig. 8–44. If the satellite has a mass of 3600 kg and a radius of 4.0 m, what is the required steady force of each rocket if the satellite is to reach 32 rpm in 5.0 min?

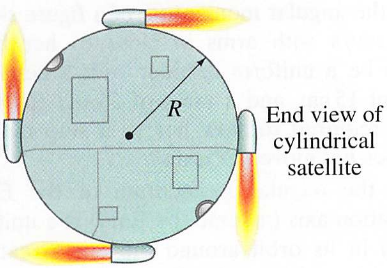


FIGURE 8–44 Problem 33.

34. (II) A grinding wheel is a uniform cylinder with a radius of 8.50 cm and a mass of 0.580 kg. Calculate (*a*) its moment of inertia about its center, and (*b*) the applied torque needed to accelerate it from rest to 1500 rpm in 5.00 s if it is known to slow down from 1500 rpm to rest in 55.0 s.

35. (II) A softball player swings a bat, accelerating it from rest to 3.0 rev/s in a time of 0.20 s. Approximate the bat as a 2.2-kg uniform rod of length 0.95 m, and compute the torque the player applies to one end of it.

36. (II) A teenager pushes tangentially on a small hand-driven merry-go-round and is able to accelerate it from rest to a frequency of 15 rpm in 10.0 s. Assume the merry-go-round is a uniform disk of radius 2.5 m and has a mass of 760 kg, and two children (each with a mass of 25 kg) sit opposite each other on the edge. Calculate the torque required to produce the acceleration, neglecting frictional torque. What force is required at the edge?

37. (II) A centrifuge rotor rotating at 10,300 rpm is shut off and is eventually brought uniformly to rest by a frictional torque of $1.20\ \mathrm{m \cdot N}$. If the mass of the rotor is 4.80 kg and it can be approximated as a solid cylinder of radius 0.0710 m, through how many revolutions will the rotor turn before coming to rest, and how long will it take?

38. (II) The forearm in Fig. 8–45 accelerates a 3.6-kg ball at $7.0\ \mathrm{m/s^2}$ by means of the triceps muscle, as shown. Calculate (*a*) the torque needed, and (*b*) the force that must be exerted by the triceps muscle. Ignore the mass of the arm.

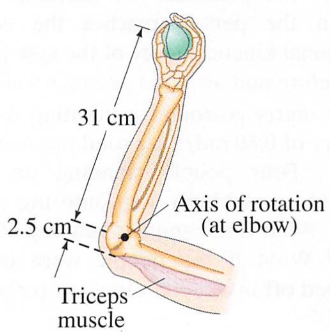


FIGURE 8–45 Problems 38 and 39.

39. (II) Assume that a 1.00-kg ball is thrown solely by the action of the forearm, which rotates about the elbow joint under the action of the triceps muscle, Fig. 8–45. The ball is accelerated uniformly from rest to 10.0 m/s in 0.350 s, at which point it is released. Calculate (*a*) the angular acceleration of the arm, and (*b*) the force required of the triceps muscle. Assume that the forearm has a mass of 3.70 kg and rotates like a uniform rod about an axis at its end.

40. (II) A helicopter rotor blade can be considered a long thin rod, as shown in Fig. 8–46. (*a*) If each of the three rotor helicopter blades is 3.75 m long and has a mass of 160 kg, calculate the moment of inertia of the three rotor blades about the axis of rotation. (*b*) How much torque must the motor apply to bring the blades up to a speed of 5.0 rev/s in 8.0 s?

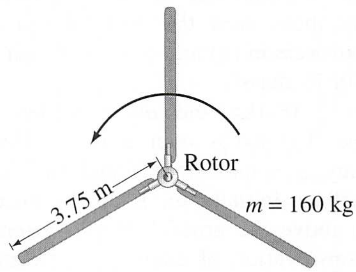


FIGURE 8–46 Problem 40.

41. (III) An *Atwood's machine* consists of two masses, m_1 and m_2, which are connected by a massless inelastic cord that passes over a pulley, Fig. 8–47. If the pulley has radius R and moment of inertia I about its axle, determine the acceleration of the masses m_1 and m_2, and compare to the situation in which the moment of inertia of the pulley is ignored. [*Hint*: The tensions F_{T1} and F_{T2} are not equal. We discussed this situation in Example 4–13, assuming $I = 0$ for the pulley.]

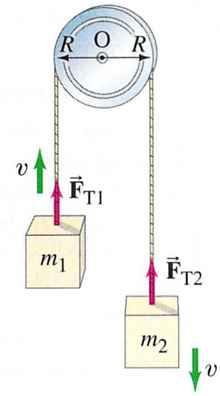


FIGURE 8–47 Problems 41 and 49. Atwood's machine.

42. (III) A hammer thrower accelerates the hammer (mass = 7.30 kg) from rest within four full turns (revolutions) and releases it at a speed of 28.0 m/s. Assuming a uniform rate of increase in angular velocity and a horizontal circular path of radius 1.20 m, calculate (*a*) the angular acceleration, (*b*) the (linear) tangential acceleration, (*c*) the centripetal acceleration just before release, (*d*) the net force being exerted on the hammer by the athlete just before release, and (*e*) the angle of this force with respect to the radius of the circular motion.

8–7 Rotational Kinetic Energy

43. (I) A centrifuge rotor has a moment of inertia of $3.75 \times 10^{-2}\ \mathrm{kg \cdot m^2}$. How much energy is required to bring it from rest to 8250 rpm?

44. (II) An automobile engine develops a torque of $280\ \mathrm{m \cdot N}$ at 3800 rpm. What is the power in watts and in horsepower?

45. (II) A bowling ball of mass 7.3 kg and radius 9.0 cm rolls without slipping down a lane at 3.3 m/s. Calculate its total kinetic energy.

46. (II) Estimate the kinetic energy of the Earth with respect to the Sun as the sum of two terms, (*a*) that due to its daily rotation about its axis, and (*b*) that due to its yearly revolution about the Sun. [Assume the Earth is a uniform sphere with mass = 6.0×10^{24} kg and radius = 6.4×10^{6} m, and is 1.5×10^{8} km from the Sun.]

47. (II) A merry-go-round has a mass of 1640 kg and a radius of 7.50 m. How much net work is required to accelerate it from rest to a rotation rate of 1.00 revolution per 8.00 s? Assume it is a solid cylinder.

48. (II) A sphere of radius 20.0 cm and mass 1.80 kg starts from rest and rolls without slipping down a 30.0° incline that is 10.0 m long. (*a*) Calculate its translational and rotational speeds when it reaches the bottom. (*b*) What is the ratio of translational to rotational KE at the bottom? Avoid putting in numbers until the end so you can answer: (*c*) do your answers in (*a*) and (*b*) depend on the radius of the sphere or its mass?

49. (III) Two masses, $m_1 = 18.0$ kg and $m_2 = 26.5$ kg, are connected by a rope that hangs over a pulley (as in Fig. 8–47). The pulley is a uniform cylinder of radius 0.260 m and mass 7.50 kg. Initially, m_1 is on the ground and m_2 rests 3.00 m above the ground. If the system is now released, use conservation of energy to determine the speed of m_2 just before it strikes the ground. Assume the pulley is frictionless.

50. (III) A 2.30-m-long pole is balanced vertically on its tip. It starts to fall and its lower end does not slip. What will be the speed of the upper end of the pole just before it hits the ground? [*Hint*: Use conservation of energy.]

8–8 Angular Momentum

51. (I) What is the angular momentum of a 0.210-kg ball rotating on the end of a thin string in a circle of radius 1.10 m at an angular speed of 10.4 rad/s?

52. (I) (*a*) What is the angular momentum of a 2.8-kg uniform cylindrical grinding wheel of radius 18 cm when rotating at 1500 rpm? (*b*) How much torque is required to stop it in 6.0 s?

53. (II) A person stands, hands at his side, on a platform that is rotating at a rate of 1.30 rev/s. If he raises his arms to a horizontal position, Fig. 8–48, the speed of rotation decreases to 0.80 rev/s. (*a*) Why? (*b*) By what factor has his moment of inertia changed?

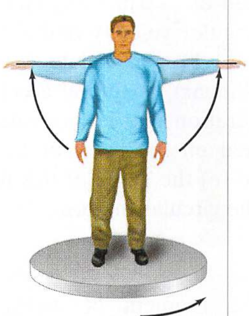


FIGURE 8–48
Problem 53.

54. (II) A diver (such as the one shown in Fig. 8–29) can reduce her moment of inertia by a factor of about 3.5 when changing from the straight position to the tuck position. If she makes 2.0 rotations in 1.5 s when in the tuck position, what is her angular speed (rev/s) when in the straight position?

55. (II) A figure skater can increase her spin rotation rate from an initial rate of 1.0 rev every 2.0 s to a final rate of 3.0 rev/s. If her initial moment of inertia was $4.6\ \text{kg}\cdot\text{m}^2$, what is her final moment of inertia? How does she physically accomplish this change?

56. (II) A potter's wheel is rotating around a vertical axis through its center at a frequency of 1.5 rev/s. The wheel can be considered a uniform disk of mass 5.0 kg and diameter 0.40 m. The potter then throws a 3.1-kg chunk of clay, approximately shaped as a flat disk of radius 8.0 cm, onto the center of the rotating wheel. What is the frequency of the wheel after the clay sticks to it?

57. (II) (*a*) What is the angular momentum of a figure skater spinning at 3.5 rev/s with arms in close to her body, assuming her to be a uniform cylinder with a height of 1.5 m, a radius of 15 cm, and a mass of 55 kg? (*b*) How much torque is required to slow her to a stop in 5.0 s, assuming she does *not* move her arms?

58. (II) Determine the angular momentum of the Earth (*a*) about its rotation axis (assume the Earth is a uniform sphere), and (*b*) in its orbit around the Sun (treat the Earth as a particle orbiting the Sun). The Earth has mass = 6.0×10^{24} kg and radius = 6.4×10^{6} m, and is 1.5×10^{8} km from the Sun.

59. (II) A nonrotating cylindrical disk of moment of inertia I is dropped onto an identical disk rotating at angular speed ω. Assuming no external torques, what is the final common angular speed of the two disks?

60. (II) A uniform disk turns at 2.4 rev/s around a frictionless spindle. A nonrotating rod, of the same mass as the disk and length equal to the disk's diameter, is dropped onto the freely spinning disk, Fig. 8–49. They then both turn around the spindle with their centers superposed. What is the angular frequency in rev/s of the combination?

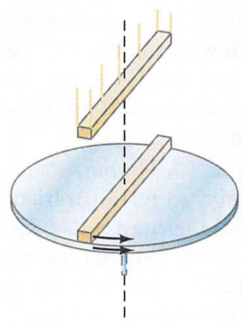


FIGURE 8–49
Problem 60.

61. (II) A person of mass 75 kg stands at the center of a rotating merry-go-round platform of radius 3.0 m and moment of inertia $920\ \text{kg}\cdot\text{m}^2$. The platform rotates without friction with angular velocity 2.0 rad/s. The person walks radially to the edge of the platform. (*a*) Calculate the angular velocity when the person reaches the edge. (*b*) Calculate the rotational kinetic energy of the system of platform plus person before and after the person's walk.

62. (II) A 4.2-m-diameter merry-go-round is rotating freely with an angular velocity of 0.80 rad/s. Its total moment of inertia is $1760\ \text{kg}\cdot\text{m}^2$. Four people standing on the ground, each of mass 65 kg, suddenly step onto the edge of the merry-go-round. What is the angular velocity of the merry-go-round now? What if the people were on it initially and then jumped off in a radial direction (relative to the merry-go-round)?

63. (II) Suppose our Sun eventually collapses into a white dwarf, losing about half its mass in the process, and winding up with a radius 1.0% of its existing radius. Assuming the lost mass carries away no angular momentum, what would the Sun's new rotation rate be? (Take the Sun's current period to be about 30 days.) What would be its final KE in terms of its initial KE of today?

64. (III) Hurricanes can involve winds in excess of 120 km/h at the outer edge. Make a crude estimate of (*a*) the energy, and (*b*) the angular momentum, of such a hurricane, approximating it as a rigidly rotating uniform cylinder of air (density $1.3\ \mathrm{kg/m^3}$) of radius 100 km and height 4.0 km.

65. (III) An asteroid of mass 1.0×10^5 kg, traveling at a speed of 30 km/s relative to the Earth, hits the Earth at the equator tangentially, and in the direction of Earth's rotation. Use angular momentum to estimate the percent change in the angular speed of the Earth as a result of the collision.

* 8–9 Angular Quantities as Vectors

* 66. (II) A person stands on a platform, initially at rest, that can rotate freely without friction. The moment of inertia of the person plus the platform is I_P. The person holds a spinning bicycle wheel with its axis horizontal. The wheel has moment of inertia I_W and angular velocity ω_W. What will be the angular velocity ω_P of the platform if the person moves the axis of the wheel so that it points (*a*) vertically upward, (*b*) at a 60° angle to the vertical, (*c*) vertically downward? (*d*) What will ω_P be if the person reaches up and stops the wheel in part (*a*)?

* 67. (III) Suppose a 55-kg person stands at the edge of a 6.5-m diameter merry-go-round turntable that is mounted on frictionless bearings and has a moment of inertia of $1700\ \mathrm{kg \cdot m^2}$. The turntable is at rest initially, but when the person begins running at a speed of 3.8 m/s (with respect to the turntable) around its edge, the turntable begins to rotate in the opposite direction. Calculate the angular velocity of the turntable.

General Problems

68. A large spool of rope rolls on the ground with the end of the rope lying on the top edge of the spool. A person grabs the end of the rope and walks a distance L, holding onto it, Fig. 8–50. The spool rolls behind the person without slipping. What length of rope unwinds from the spool? How far does the spool's center of mass move?

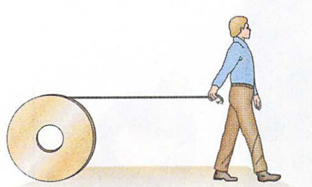


FIGURE 8–50 Problem 68.

69. The Moon orbits the Earth such that the same side always faces the Earth. Determine the ratio of the Moon's spin angular momentum (about its own axis) to its orbital angular momentum. (In the latter case, treat the Moon as a particle orbiting the Earth.)

70. A cyclist accelerates from rest at a rate of $1.00\ \mathrm{m/s^2}$. How fast will a point on the rim of the tire (diameter = 68 cm) at the top be moving after 3.0 s? [*Hint*: At any moment, the lowest point on the tire is in contact with the ground and is at rest—see Fig. 8–51.]

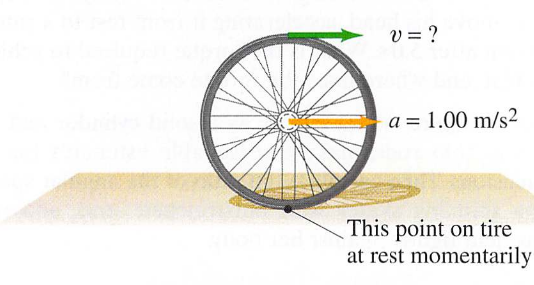


FIGURE 8–51 Problem 70.

71. A 1.4-kg grindstone in the shape of a uniform cylinder of radius 0.20 m acquires a rotational rate of 1800 rev/s from rest over a 6.0-s interval at constant angular acceleration. Calculate the torque delivered by the motor.

72. (*a*) A yo-yo is made of two solid cylindrical disks, each of mass 0.050 kg and diameter 0.075 m, joined by a (concentric) thin solid cylindrical hub of mass 0.0050 kg and diameter 0.010 m. Use conservation of energy to calculate the linear speed of the yo-yo when it reaches the end of its 1.0-m-long string, if it is released from rest. (*b*) What fraction of its kinetic energy is rotational?

73. (*a*) For a bicycle, how is the angular speed of the rear wheel (ω_R) related to that of the pedals and front sprocket (ω_F), Fig. 8–52? That is, derive a formula for ω_R/ω_F. Let N_F and N_R be the number of teeth on the front and rear sprockets, respectively. The teeth are spaced equally on all sprockets so that the chain meshes properly. (*b*) Evaluate the ratio ω_R/ω_F when the front and rear sprockets have 52 and 13 teeth, respectively, and (*c*) when they have 42 and 28 teeth.

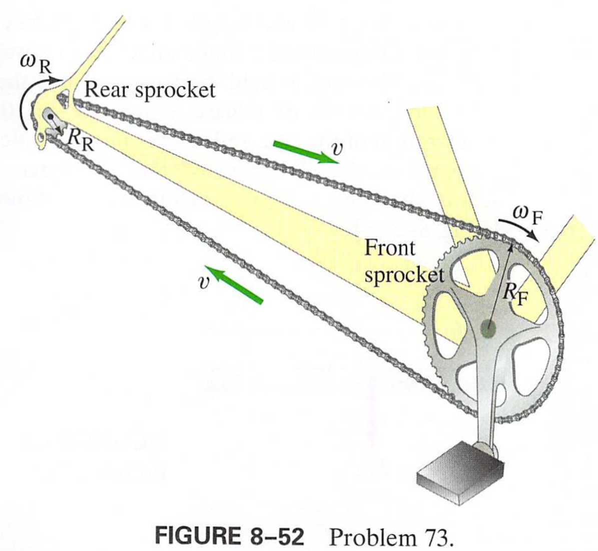


FIGURE 8–52 Problem 73.

74. Suppose a star the size of our Sun, but with mass 8.0 times as great, were rotating at a speed of 1.0 revolution every 12 days. If it were to undergo gravitational collapse to a neutron star of radius 11 km, losing three-quarters of its mass in the process, what would its rotation speed be? Assume that the star is a uniform sphere at all times, and that the lost mass carries off no angular momentum.

75. One possibility for a low-pollution automobile is for it to use energy stored in a heavy rotating flywheel. Suppose such a car has a total mass of 1400 kg, uses a uniform cylindrical flywheel of diameter 1.50 m and mass 240 kg, and should be able to travel 350 km without needing a flywheel "spinup." (*a*) Make reasonable assumptions (average frictional retarding force = 450 N, twenty acceleration periods from rest to 95 km/h, equal uphill and downhill, and that energy can be put back into the flywheel as the car goes downhill), and show that the total energy needed to be stored in the flywheel is about 1.7×10^8 J. (*b*) What is the angular velocity of the flywheel when it has a full "energy charge"? (*c*) About how long would it take a 150-hp motor to give the flywheel a full energy charge before a trip?

76. Figure 8–53 illustrates an H_2O molecule. The O–H bond length is 0.96 nm and the H–O–H bonds make an angle of 104°. Calculate the moment of inertia for the H_2O molecule about an axis passing through the center of the oxygen atom (*a*) perpendicular to the plane of the molecule, and (*b*) in the plane of the molecule, bisecting the H–O–H bonds.

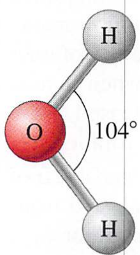


FIGURE 8–53 Problem 76.

77. A hollow cylinder (hoop) is rolling on a horizontal surface at speed $v = 3.3$ m/s when it reaches a 15° incline. (*a*) How far up the incline will it go? (*b*) How long will it be on the incline before it arrives back at the bottom?

78. A uniform rod of mass M and length L can pivot freely (i.e., we ignore friction) about a hinge attached to a wall, as in Fig. 8–54. The rod is held horizontally and then released. At the moment of release, determine (*a*) the angular acceleration of the rod, and (*b*) the linear acceleration of the tip of the rod. Assume that the force of gravity acts at the center of mass of the rod, as shown. [*Hint:* See Fig. 8–21g.]

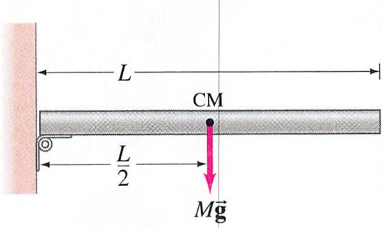


FIGURE 8–54 Problem 78.

79. A wheel of mass M has radius R. It is standing vertically on the floor, and we want to exert a horizontal force F at its axle so that it will climb a step against which it rests (Fig. 8–55). The step has height h, where $h < R$. What minimum force F is needed?

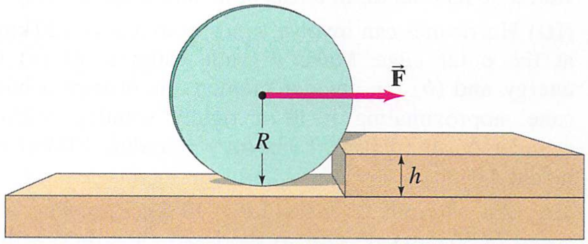
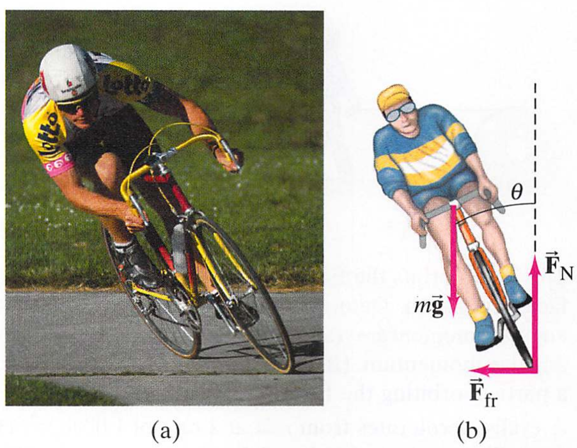


FIGURE 8–55 Problem 79.

80. A bicyclist traveling with speed $v = 4.2$ m/s on a flat road is making a turn with a radius $r = 6.4$ m. The forces acting on the cyclist and cycle are the normal force ($\vec{\mathbf{F}}_N$) and friction force ($\vec{\mathbf{F}}_{fr}$) exerted by the road on the tires, and $m\vec{\mathbf{g}}$, the total weight of the cyclist and cycle (see Fig. 8–56). (*a*) Explain carefully why the angle θ the bicycle makes with the vertical (Fig. 8–56) must be given by $\tan\theta = F_{fr}/F_N$ if the cyclist is to maintain balance. (*b*) Calculate θ for the values given. (*c*) If the coefficient of static friction between tires and road is $\mu_s = 0.70$, what is the minimum turning radius?



(a) (b)

FIGURE 8–56 Problem 80.

81. Suppose David puts a 0.50-kg rock into a sling of length 1.5 m and begins whirling the rock in a nearly horizontal circle above his head, accelerating it from rest to a rate of 120 rpm after 5.0 s. What is the torque required to achieve this feat, and where does the torque come from?

82. Model a figure skater's body as a solid cylinder and her arms as thin rods, making reasonable estimates for the dimensions. Then calculate the ratio of the angular speeds for a spinning skater with outstretched arms, and with arms held tightly against her body.

83. You are designing a clutch assembly which consists of two cylindrical plates, of mass $M_A = 6.0\,\text{kg}$ and $M_B = 9.0\,\text{kg}$, with equal radii $R = 0.60\,\text{m}$. They are initially separated (Fig. 8–57). Plate M_A is accelerated from rest to an angular velocity $\omega_1 = 7.2\,\text{rad/s}$ in time $\Delta t = 2.0\,\text{s}$. Calculate (*a*) the angular momentum of M_A, and (*b*) the torque required to have accelerated M_A from rest to ω_1. (*c*) Plate M_B, initially at rest but free to rotate without friction, is allowed to fall vertically (or pushed by a spring), so it is in firm contact with plate M_A (their contact surfaces are high-friction). Before contact, M_A was rotating at constant ω_1. After contact, at what constant angular velocity ω_2 do the two plates rotate?

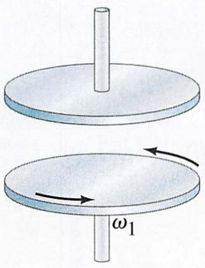


FIGURE 8–57 Problem 83.

84. A marble of mass m and radius r rolls along the looped rough track of Fig. 8–58. What is the minimum value of the vertical height h that the marble must drop if it is to reach the highest point of the loop without leaving the track? Assume $r \ll R$, and ignore frictional losses.

85. Repeat Problem 84, but do not assume $r \ll R$.

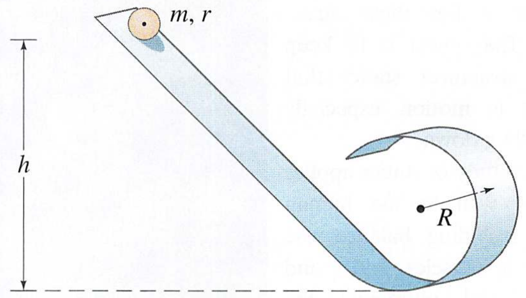


FIGURE 8–58 Problems 84 and 85.

86. The tires of a car make 85 revolutions as the car reduces its speed uniformly from 90.0 km/h to 60.0 km/h. The tires have a diameter of 0.90 m. (*a*) What was the angular acceleration of each tire? (*b*) If the car continues to decelerate at this rate, how much more time is required for it to stop?

Answers to Exercises

A: $f = 0.076\,\text{Hz}$; $T = 13\,\text{s}$.

B: $\vec{F}_A$.

C: Yes; she does work to pull in her arms.

D: Work was done in pulling the string and decreasing the circle's radius.