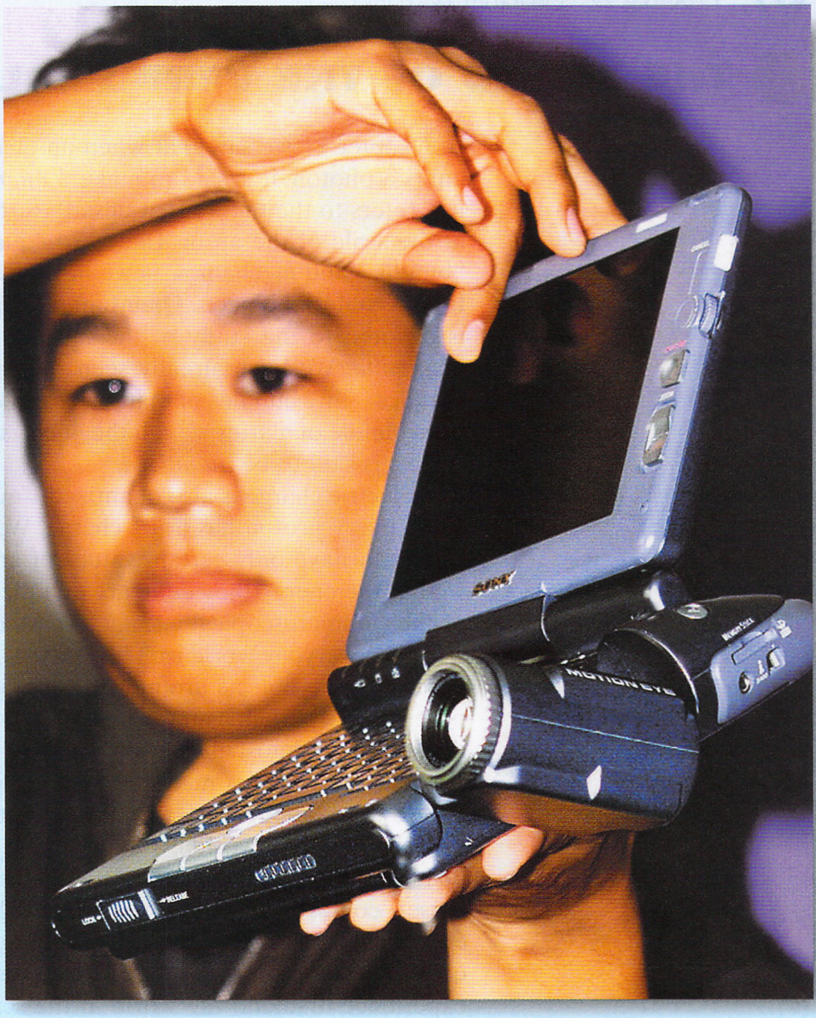
22 Quantum Mechanics



KEY IDEA

At the subatomic scale, physical quantities are quantized; any measurement at that scale significantly alters the object being measured.

PHYSICS AROUND US . . . Quantum Mechanics in Your Life

Have you used a computer lately? How about a digital camera? A car radio? A calculator?

All these devices, and many more, operate according to the laws of a strange and wonderful area of physics known as *quantum mechanics,* which governs the behavior of individual atoms and subatomic particles. Take the digital camera as an example. At the heart of every digital camera is a plate of light-sensitive material called a "photoelectric device"—the same kind of material that converts the Sun's energy into electricity in a solar cell and measures brightness in a light meter. Light hitting this plate causes the release of individual electrons, which are detected by receivers in the camera. The pattern of electrons leaving the plate ultimately produces the picture you see. The interaction involved between light and electrons cannot be explained correctly by classical physics—you need quantum mechanics to understand it.

In the same way, the building block of every computer is a device known as a "transistor." As this device is turned on and off (a process we describe in more detail in Chapter 25), your computer takes in, processes, and outputs information in the form of individual electric charges. Again, the behavior of these charges cannot be described correctly by classical physics. The entire information revolution depends on the laws of quantum mechanics. In fact, all the everyday objects listed above, and many more, are practical consequences of the laws of the quantum world.

THE WORLD OF THE VERY SMALL

In Chapter 21 we saw that when an electron moves between energy levels and emits a photon, it is said to make a *quantum leap.* The term "quantum mechanics" refers to the theory that describes this process, as well as many other events at the scale of the atom. The word "quantum" comes from the Latin word for "bundle," and mechanics, as we have seen in Chapter 4, is the study of the motion of material objects. **Quantum mechanics,** then, is the branch of physics that is devoted to the study of the motion of objects that come in small bundles, or *quanta.* We have already seen that material inside the atom comes in little bundles—tiny bundles of matter we call *electrons* travel in orbits around another little bundle of matter we call the *nucleus.* In the language of physicists, the atom's matter is said to be *quantized.*

Electric charge is also quantized—every electron has a charge of exactly -1, while every proton has a $+1$ charge. We have seen that photons emitted by an atom can have only certain values of energy, so that energy levels in atoms, as well as the energy they emit, are quantized. In fact, inside the atom, in the world of the submicroscopically small, *everything* comes in quantized bundles.

Our everyday world isn't like this at all. Although we've been told since childhood that the objects around us are made up of atoms, for all intents and purposes we experience matter as if it were smooth, continuous, and infinitely divisible. For almost all everyday human activities, there's no advantage to knowing that the world is made of atoms. For example, when was the last time you had to think about matter in terms of atoms?

The quantum world is foreign to our senses. All the intuition that we've built up about the way the world operates—all the gut feelings we have about the universe—comes from our experiences with large-scale objects made up of apparently continuous material. If it should turn out (as it does) that the world of the quantum doesn't match our intuition, we shouldn't be surprised. We can't see or experience firsthand the world at the scale of the atom, so we have no particular reason based on everyday observations to believe that it should behave one way or the other.

This warning may not make you feel much better as you learn just how strange and different the quantum world really is, but it might help you come to intellectual grips with a most fascinating part of our physical universe.

Measurement and Observation in the Quantum World

Every measurement in the physical world incorporates three essential components:

1. A sample—a piece of matter to study
2. A source of energy—light or heat or kinetic energy that interacts with the sample
3. A detector to observe and measure that interaction

When you look at a piece of matter such as this book, you can see it because light bounces off the book and comes to your eye, which is a very sophisticated detector of electromagnetic radiation (see Chapter 19). When you examine a piece of fruit at the grocery store, you apply energy by squeezing it to detect if it feels ripe. You may listen to sound waves generated by a CD before you buy it—a process that involves having the CD interact with a laser beam.

A radar antenna sends out microwaves that interact with flying airplanes, are reflected, and are detected on their return. This allows air traffic controllers to keep track of where airplanes are in the sky.

Many professions employ sophisticated devices to make measurements. Air traffic controllers reflect microwaves (radar) off airplanes to determine their positions, oceanographers bounce sound waves (sonar) off deep-ocean sediments to map the sea floor, and dentists pass X rays through your teeth and gums to look for cavities. In our everyday world we assume that such interactions of matter and energy do not change the objects being measured in any appreciable way. Microwaves don't alter an airplane's flight path, nor do sound waves disturb the topography of the ocean's bottom. And although prolonged exposure to X rays can be harmful, the dentist's brief exploratory X-ray photograph has no immediate effect on the tooth. Our experience tells us that a measurement can usually be made on a macroscopic object—something large enough to be seen without a microscope—without altering that object because the energy of the probe is much less than the energy of the object.

The situation is rather different in the quantum world. If you want to "see" an electron, you have to bounce energy off it so that the information can be carried to your detectors. But nothing at your disposal can interact with the electron without simultaneously affecting it. You can bounce a photon off it, but in the process the electron's energy changes. You can bounce another particle off it, but the electron will recoil like a billiard ball. No matter what you try, the energy of the probe is too close to the energy of the electron being measured. The electron cannot fail to be altered by the interaction.

Many everyday analogies illustrate the process of measurement in the quantum world. For example, it's like trying to locate the position of a bowling ball by bouncing other bowling balls off it. The act of measurement in the quantum world poses a dilemma analogous to trying to discover if there is a car in a dark tunnel when the only means of finding out is to send another car into the tunnel and listen for a crash. With this technique you can certainly discover whether the first car is there. You can probably even find out where it is by measuring the time it takes the probe car to crash. What you *cannot* do, however, is assume that the first car is the same after the interaction as it was before. In the same way, nothing in the quantum world can be the same after the interaction associated with a measurement as it was before.

In principle, this argument applies to any interaction, whether it involves photons and electrons or photons and bowling balls. However, as we demonstrate in Example 22-1 later in this chapter, the effects of the interaction for large-scale objects are so tiny that they can be ignored, while in the case of interactions at the atomic level they cannot be ignored. This fundamental difference between the quantum and macroscopic worlds is what makes quantum mechanics quite different from the classical mechanics of Isaac Newton. Remember that every experiment, be it on planets or fruit or quantum objects, involves interactions of one sort or another. The consequences of small-scale interactions make the quantum world different, not the fact that a measurement is being made.

The Heisenberg Uncertainty Principle

In 1927, a young German physicist, Werner Heisenberg (1901–1976), put the idea of limitations on quantum-scale measurements into precise mathematical form. His work, which was one of the first results to come from the new science of quantum mechanics, is called the *Heisenberg uncertainty principle* in his honor. The central concept of the uncertainty principle is simple:

Werner Heisenberg

At the quantum scale, any measurement significantly alters the object being measured.

For example, suppose that you have a particle such as an electron in an atom and want to know where it is *and* how fast it's moving. The uncertainty principle tells us that it is impossible to measure both the position and the velocity with infinite accuracy at the same time so that there is always an uncertainty to our knowledge of some aspects of the subatomic world.

The reason for this state of affairs is that every measurement changes the object being measured. Just as the car in the tunnel could not be the same after the first measurement was made on it, so too does the quantum object change. The result is that as you measure one property such as position more and more exactly, your knowledge of a property such as velocity gets fuzzier and fuzzier.

The uncertainty principle doesn't say that we can't know a particle's location with great precision. It's possible, at least in principle, for the uncertainty in position to be zero, which would mean that we know the exact location of a quantum particle. In this case, however, the uncertainty in the velocity has to be infinite. Thus, at the point in time when we know exactly where the particle is, we have no idea whatsoever how fast it is moving. By the same token, if we know exactly how fast the quantum particle is moving, we cannot know where it is. It could, quite literally, be in the room with us or in China.

In practice, every quantum measurement involves trade-offs. We accept some fuzziness in the location of the particle and some fuzziness in the knowledge of the velocity, playing the two off against one another to get the best solution to whatever problem it is we're working on. We cannot have precise knowledge of both quantities at the same time, but we can know either one as accurately as we like at any time.

Let's look a little more closely at the differences between the world of our intuition and the quantum world. Our intuition, based on experience in the macroscopic world, suggests that a measurement doesn't affect an object being measured. According to that view, we should be able to have exact, simultaneous knowledge of both the position and velocity of an object such as a car or a baseball. In the quantum world, we cannot.

Heisenberg put his notion into a simple mathematical relationship, which is a complete and exact statement of the **uncertainty principle.** (Note that this relationship is given in terms of momentum p rather than velocity v, where $p = mv$, and m is the particle's mass.)

1. In words:

The error or uncertainty in the measurement of an object's position, multiplied by the error or uncertainty in that object's momentum, must be greater than a constant (called Planck's constant).

2. In an equation with words:

$$(\text{Uncertainty in position}) \times (\text{Uncertainty in momentum}) > h$$

where h is a number known as *Planck's constant.*

3. In an equation with symbols:

$$\Delta x \times \Delta p > h$$

where x represents the position of the particle and p its momentum. The Greek letter Δ (delta) is customarily used to represent the spread of values that a variable can have, and hence the uncertainty in our knowledge of that value.

This equation is a precise, shorthand way of saying that you can never know both the position and momentum (or velocity) of an object with perfect accuracy.

The difference between our everyday world and the world inside the atom hangs on the question of the numerical value of Planck's constant h, the number on the right side of Heisenberg's equation. In SI units (see Appendix A), Planck's constant has a value of 6.63×10^{-34} joule-seconds. This is a very small number, which is why we never notice the uncertainty principle in daily life.

The important point about the Heisenberg relationship is not the exact value of Planck's constant h, but the fact that h is greater than zero. Look at it this way. If you make more and more precise measurements about the location of a particle, you determine its position more and more exactly, and the uncertainty in position, Δx, must get smaller and smaller. In this situation, it follows that the uncertainty in velocity, Δv, has to get bigger and bigger (note that $\Delta p = m\Delta v$). In fact, we can use the uncertainty principle to calculate exactly our uncertainty in velocity for a given uncertainty in position, and vice versa.

Develop Your Intuition: Uncertainties in Time and Energy

Heisenberg developed a second form of the uncertainty principle, which said that the uncertainty in an object's energy multiplied by the uncertainty in the time interval for measuring that energy is always greater than Planck's constant h.

$$\Delta E \times \Delta t > h$$

Use this relation to determine how long you could own a new car that's created out of nothing.

The amazing thing about quantum mechanics is that you can get a real answer to this question. The first step is to estimate how much mass your car probably has. We won't be greedy and ask for a luxury limousine, so let's say 1000 kilograms will do. We know that mass can be converted to energy according to Einstein's equation $E = mc^2$ (Chapter 12). If you work out the numbers with $m = 1000$ kg and $c = 3 \times 10^8$ m/s, you get an energy E of 9×10^{19} joules or about 10^{20} J.

That's a huge amount of energy, but according to the uncertainty principle you could have an uncertainty in energy of this amount for a very short time. That means that you could (in theory) produce a car out of nothing for this short time. How short a time? The time interval would be Planck's constant, 6.63×10^{-34} joule-seconds, divided by the uncertainty in energy, which we just calculated to be about 10^{20} J. So this uncertainty in energy could exist for about 6×10^{-54} seconds. Hardly long enough to wait around for.

This example may seem pretty far-fetched and ridiculous, and in some respects it is. But the idea of something created out of nothing for a very short time is not at all ridiculous; in fact, it happens pretty often. In high-energy experiments, radiation fields can occur with such high energy that tiny particles appear, formed as mass converted from the energy. If the mass is small enough and the energy is high enough, these particles exist long enough to be detected before they disappear back into radiation. Physicists call them "virtual particles" since they are there but not there. Despite their brief existence, physicists have measured their effect on real objects. Virtual particles are just one part of the strange world of quantum mechanics.

Uncertainty in the Macroscopic and Microscopic Worlds

The best way to understand why we do not have to worry about the uncertainty principle in our everyday lives is to calculate the uncertainty of measurements in two separate situations: large objects and very small objects.

Small Uncertainties with Large Objects

EXAMPLE 22-1

A moving automobile with a mass of 1000 kilograms is located in an intersection that is 5 meters across. How precisely can you know how fast the car is traveling?

REASONING AND SOLUTION: We can solve this problem by noting that if the car is somewhere in an intersection 5 m across, then the uncertainty in position of the car is about equal to 5 m. We know the car's mass and uncertainty in position, so we can calculate the uncertainty in velocity:

$$\Delta x \text{ (Uncertainty in position)} \times \Delta p \text{ (Uncertainty in momentum)} > h$$

$$\Delta x \text{ (Uncertainty in position)} \times [\Delta v \text{ (Uncertainty in velocity)} \times m \text{ (mass)}] > h$$

We rearrange this equation to solve for uncertainty in velocity:

$$\begin{aligned}\Delta v &> \frac{h/m}{\Delta x} \\ &> \frac{(6.63 \times 10^{-34}\ \text{J-s})/1000\ \text{kg}}{5\ \text{m}} \\ &> \frac{(6.63 \times 10^{-37}\ \text{J-s})/\text{kg}}{5\text{m}} \\ &> 1.33 \times 10^{-37}\ \text{m/s}\end{aligned}$$

Thus the uncertainty in the velocity of the automobile must be greater than 1.33×10^{-37} m/s (note that the unit J-s/kg-m is equivalent to m/s; see Problem 2 at the end of the chapter). This uncertainty is extremely small. Theoretically, we could know the velocity of the car to an accuracy of 37 decimal places! In practice, however, we have no method of measuring velocities with present or foreseeable human technology to an accuracy remotely approaching this. The uncertainty is for all practical purposes indistinguishable from zero. Therefore, for objects with significant mass such as automobiles, the effects of the uncertainty principle are totally negligible. The equation confirms our experience that Newtonian mechanics works perfectly well in dealing with everyday objects. ●

Large Uncertainty with a Small Object

EXAMPLE 22-2

Contrast Example 22-1 with the uncertainty in velocity of an electron in an atom, located within a volume about 10^{-10} meters on a side. To what accuracy can we measure the velocity of that electron?

REASONING AND SOLUTION: The mass of an electron is 9.11×10^{-31} kg. If we take the uncertainty in position to be 10^{-10} m, then according to the uncertainty principle,

$$\begin{aligned}\Delta v &> \frac{h/m}{\Delta x} \\ &> \frac{(6.63 \times 10^{-34}\ \text{J-s})/(9.11 \times 10^{-31}\ \text{kg})}{10^{-10}\ \text{m}} \\ &> 7.3 \times 10^{6}\ \text{m/s}\end{aligned}$$

This uncertainty is very large indeed. The mere fact that we know that an electron is somewhere in an atom means that we cannot know its velocity to within a million meters per second—almost 10% of the speed of light!

For ordinary-sized objects such as cars and bowling balls, whose mass is measured in kilograms, the number on the right side of the uncertainty relation is so small that we can treat it as zero. Only when the masses get very small, as they do for particles such as the electron, does the number on the right get big enough to make a practical difference. ●

PROBABILITIES

The uncertainty principle has consequences that go far beyond simple statements about measurement. In the quantum world, we must radically change how we describe events. Consider an everyday example in which the uncertainties are much easier to picture than those associated with Heisenberg's equation. Think of a batter hitting a ball during a nighttime baseball game.

Imagine yourself at a big-league ball game under the lights of a great stadium. Cheering fans fill the stands, roving vendors sell their food and drink, and the pitcher and batter play out their classic duel. The pitcher stares the batter down, winds up, and hurls a fastball. But the batter is ready and pounces on the pitch. The ball leaps off the bat with a sharp crack. And then all the lights go out.

Where will the ball be in 5 seconds? If you were an outfielder, this would be more than a philosophical question. You would need to know where to go to make your catch, even in the dark. In a Newtonian world, you would have no problem doing this. If you knew the position and velocity of the ball at the instant the lights went out, a few calculations could tell you exactly where the ball would be at any time in the future.

If you were a quantum outfielder in an atom-sized ball field, on the other hand, you would have a much harder time of it. You couldn't know both the position and velocity of the quantum ball when the lights go out; at best you could put some bounds on them. For example, you might be able to say something like, "The ball is somewhere inside this 3-foot circle around home plate, traveling with a horizontal speed between 30 and 70 feet per second." This result means that when you have to guess where it would be in 5 seconds, you wouldn't be able to do so exactly. In Newtonian terms, if it were traveling 30 feet per second and located at the far end of the 3 foot circle, then it would be 147 feet (147 feet = 30 ft/s $\times$ 5 s $-$ 3 ft) from the plate. If, on the other hand, it were traveling 70 feet per second and located at the near end of the circle, then it would be 353 ft (353 ft = 70 ft/s $\times$ 5 s + 3 ft) from the plate. Hence, it could be anywhere between 147 and 353 feet from the plate. The best you could do would be to predict the likelihood, or **probability,** that the ball would be anywhere in the outfield. You could present these probabilities on a graph such as the one shown in Figure 22-1. In this graph, the most likely place to find the baseball is near the spot where the probability is highest, but the ball could be some other place instead.

This example shows that the uncertainty principle requires a description of quantum-scale events in terms of probabilities. Just like the baseball in our example of the darkened stadium, there must be uncertainties in the position and velocity for every quantum object when we first start observing it, and hence there are uncertainties at the end—uncertainties that can be dealt with by reporting probabilities.

The graph of probabilities shown in Figure 22-1 can be thought of as a wave where the amplitude of the wave corresponds to the probability of finding a particle at a specific point. For this reason, such a set of probabilities is referred to as a *wave function.* (Technically, the square of the amplitude of the wave at some point gives the probability of finding a particle at that point.) The equations of quantum mechanics, in fact, take this resemblance a step further and actually describe the way that a probability wave changes over time. In the case of our baseball stadium, for example, they would describe how the wave evolved from one describing the probabilities of finding the ball near home plate to the one shown in Figure 22-1. For this reason, quantum mechanics is sometimes called "wave mechanics."

The Austrian physicist Erwin Schrödinger (1887–1961) first wrote down the ground-breaking equation that describes the probability wave, and so it is called the *Schrödinger equation.* While the precise form of this equation is complex, Schrödinger's equation plays the same role in quantum mechanics that Newton's second law does for ordinary mechanics and Maxwell's equations do for electricity and magnetism. It describes quantitatively how a physical system evolves over time in response to outside influences.

This aspect of Schrödinger's equation is extremely important. It tells us that we cannot think of quantum events in the same way that we think of normal events

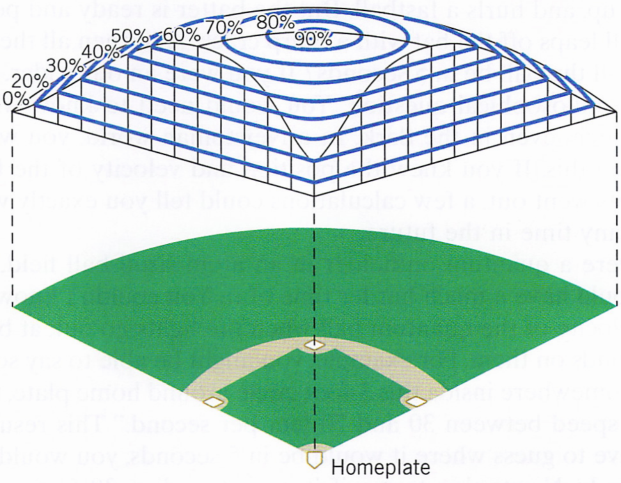


FIGURE 22-1. The wave function that represents the position of the quantum baseball. The height of the surface represents the probability of finding the baseball at a given point. Each contour represents the probability of finding the baseball within the area enclosed. For example, there is a 90% probability that the baseball will be found within the curve labeled 90%, an 80% probability that it will be within the curve labeled 80%, and so on. The most likely place to find the baseball is near the peak of the wave function, where the probability is highest.

in our everyday world. In particular, we have to rethink what it means to talk about concepts such as regularity, predictability, and causality at the quantum level.

WAVE-PARTICLE DUALITY

It turns out that quantum objects sometimes act like particles and sometimes act like waves. This dichotomy is known as the problem of *wave-particle duality,* and it has puzzled some of the best minds in science. To understand wave-particle duality, think about how particles and waves behave in our macroscopic world.

The Double-Slit Test

In our everyday world, energy travels either as a wave (see Chapter 14) or as a particle. Particles transfer energy through collisions, while waves transfer energy through collective motion of the media or electromagnetic fields. Every aspect of the everyday world can be neatly divided into particles or waves, and many experiments can be used to determine whether a phenomenon is a particle or a wave. The most famous of these experiments uses a double-slit apparatus, which consists of a barrier that has two slits in it, each of which will allow a particle (or a wave) to pass through the barrier (Figure 22-2). If particles such as baseballs are thrown from the left side, a few will make it through the slits, but most will

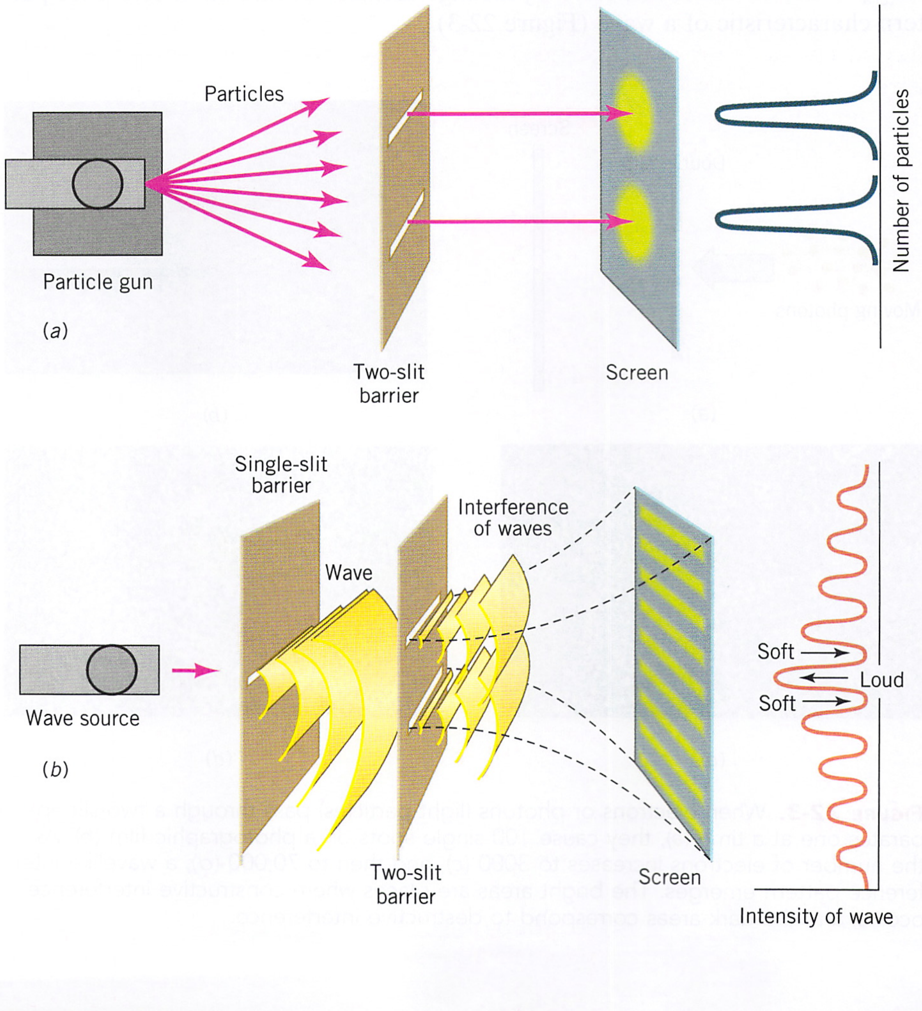


FIGURE 22-2. (*a*) The two-slit experiment may be used to determine whether something is a wave or a particle. A stream of particles striking the barrier will accumulate in the two regions directly behind the slits. (*b*) When waves converge on two narrow slits, however, constructive and destructive interference results in a series of peaks.

bounce off. If you were standing on the other side of the barrier you would expect to see the baseballs coming through more or less in the two places shown, accumulating in two areas behind the barrier. You wouldn't expect to see many particles (baseballs) winding up between the slits.

However, if waves of sound were coming from the left side, you would expect to see the results of constructive and destructive interference (see Chapter 14). Rather than the two areas of baseballs, we would see perhaps half a dozen regions of loud sound beyond the barrier, interspersed with regions of soft sound, a situation illustrated by the wave height shown in Figure 22-2*b*.

Now, let's use the same arrangement to see whether light behaves as a particle or a wave. In Chapter 21 we learned that light is emitted in discrete bundles of energy called *photons*. Photons behave like particles in the sense that they can be localized in space. You can set up experiments in which a photon is emitted at one point and then received somewhere else after an appropriate lapse of time. However, if you shine light—a flood of photons—on the two-slit apparatus, you will definitely get an interference pattern, like the one shown in Figure 22-2*b*. In that experiment, photons act like waves. The big question: How can photons sometimes act like waves and sometimes act like particles?

You can make the problem even more puzzling by setting up the apparatus so that only one photon at a time comes through the slits. If you do this, you find that each photon arrives at a specific point at the screen behind the slits—behavior you would expect of a particle. If you allow photons to accumulate over long periods of time, however, they arrange themselves into an interference pattern characteristic of a wave (Figure 22-3).

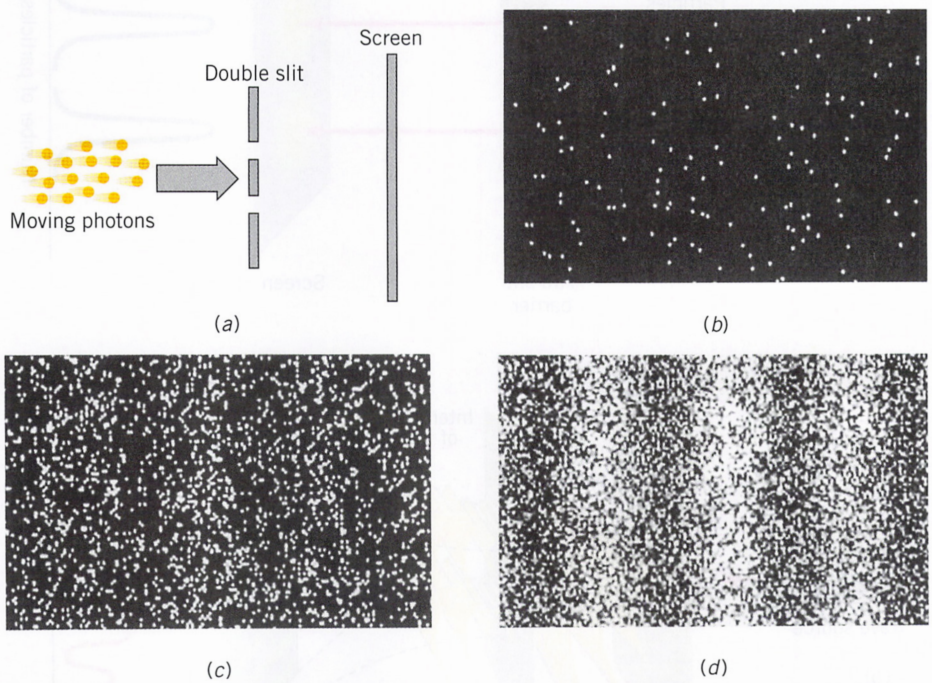


FIGURE 22-3. When electrons or photons (light particles) pass through a two-slit apparatus one at a time (*a*), they cause 100 single spots on a photographic film (*b*). As the number of electrons increases to 3000 (*c*), and then to 70,000 (*d*), a wavelike interference pattern emerges. The bright areas are places where constructive interference occurs, and the dark areas correspond to destructive interference.

The French physicist Louis de Broglie (1892–1987) put this wavelike feature of quantum objects into mathematical form. He asked a simple question: If we are to think about quantum objects as both waves and particles, how are the particlelike properties (such as momentum) related to the wavelike properties (such as wavelength)? The result of his work is known as the *de Broglie relation.*

1. In words:

The higher the momentum an object has (if we think of it as a particle), the shorter its wavelength (if we think of it as a wave).

2. In an equation with words:

The momentum of a quantum object is inversely proportional to its wavelength.

3. In an equation with symbols:

$$p = \frac{h}{\lambda}$$

where p is the momentum, λ is the wavelength, and h is Planck's constant.

You could do a similar series of experiments with any quantum object—electrons, for example, or even atoms. They all exhibit the properties of both particles and waves, depending on what sort of experiment is done. If you perform an experiment that tests the particle properties of these things, they look like particles. If you perform an experiment to test their wave properties, they look like waves. Whether you see quantum objects as particles or waves seems to depend on the experiment that you do.

Some experimenters have gone so far as to try to trick quantum particles such as electrons into revealing their true identity by using modern fast electronics to decide whether a particle- or wave-type experiment is being done *after* the quantum object is already on its way into the apparatus. Scientists who do these experiments find that the quantum object seems to "know" what experiment is being done, because the particle experiments always turn up particle properties and the wave experiments always turn up wave properties.

At the quantum level, the objects that we talk about are neither particles nor waves in the classical sense. In fact, we can't really visualize them at all because we have never encountered anything like them in our everyday experience. They are a third kind of object, neither particle nor wave, but exhibiting the properties of both. If you persist in thinking about them as if they were baseballs or surf coming onto a beach, you will quickly lose yourself in confusion.

It's a little bit like finding someone who has seen only the colors red and green in her entire life. If she has decided that everything in the world has to be either red or green, she will be totally confused by seeing the color blue. What she has to realize is that the problem is not in nature, but in her assumption that everything has to be either red or green.

In the same way, the problem of wave-particle duality arises from our assumption that everything has to be either a wave or a particle. If we allow ourselves the possibility that quantum objects are entities that we have never encountered before and that they therefore might have unencountered properties, then the puzzle vanishes. However, it vanishes only if we agree that we won't try to draw a picture of these objects or pretend that we can actually visualize what they are.

Connection
Interference of Electrons

The classic test for deciding if energy is being transmitted by particles or by waves is to see if you can detect interference effects. If you can, then you're looking at a wave; if you can't, you're looking at particles. Now, we've just learned that particles such as electrons can behave as if they're waves. So can they exhibit constructive and destructive interference?

Yes, they can. The first experiments that detected interference of electron beams were done in 1927 by C. Davisson and L. Germer and in 1928 by G. P. Thomson. (It's a lovely coincidence of history that G. P. Thomson shared a Nobel Prize in physics for demonstrating that electrons could act as waves, and his father, J. J. Thomson, received a Nobel Prize for discovering electrons as particles.) And the wave nature of electrons has been applied in a more practical way than just in laboratory experiments: it is the basis for the electron microscope.

In an electron microscope, electron beams are bent by electric and magnetic fields in the same way that light rays are bent by lenses. Optical microscopes use wavelengths of about 500 nm, which means they can resolve details of a specimen of about a few hundred nanometers—that corresponds to the length of a few thousand atoms side by side. However, by using de Broglie's relation, we can determine that an electron moving at about 10% of the speed of light (3×10^8 m/s) has a wavelength in the neighborhood of 0.1 nm, which is 100 times smaller than the typical size of an atom.

There are two main kinds of electron microscopes (Figure 22-4). Transmission electron microscopes (TEMs) send electron beams through a thin slice of a specimen, such as a biological cell wall. Scanning electron microscopes (SEMs) bounce electron beams off a specimen and collect the beams reflected at various angles, producing more of a three-dimensional effect. Both kinds of instruments are now regular parts of research in biology and materials science. ●

Connection
The Photoelectric Effect

When photons of sufficient energy strike some materials, their energy can be absorbed by electrons, which are shaken loose from their home atoms. If the material in question is in the form of a thin sheet, then when light strikes one side, electrons are observed coming out of the other side. This phenomenon is called the *photoelectric effect* and it finds applications in numerous everyday devices. (See Physics Around Us on page 471.)

One aspect of the photoelectric effect played a major role in the history of quantum mechanics. The time between the arrival of the light and the appearance of the electrons is extremely short—far too short to be explained by the gradual buildup of electromagnetic wave energy that shakes the electron loose. In fact, Albert Einstein pointed out that this rapid response depends on the particlelike nature of the photon. He argued that the interaction between the light and the electron is something like the collision between two billiard balls, with one ball shooting out instantly after the collision. It was this work, which led to our modern concept of the photon, that was the basis for Einstein's Nobel Prize in 1921.

The conversion of light energy into electric current is used in many familiar devices. For example, in a digital camera one photoelectric device measures the

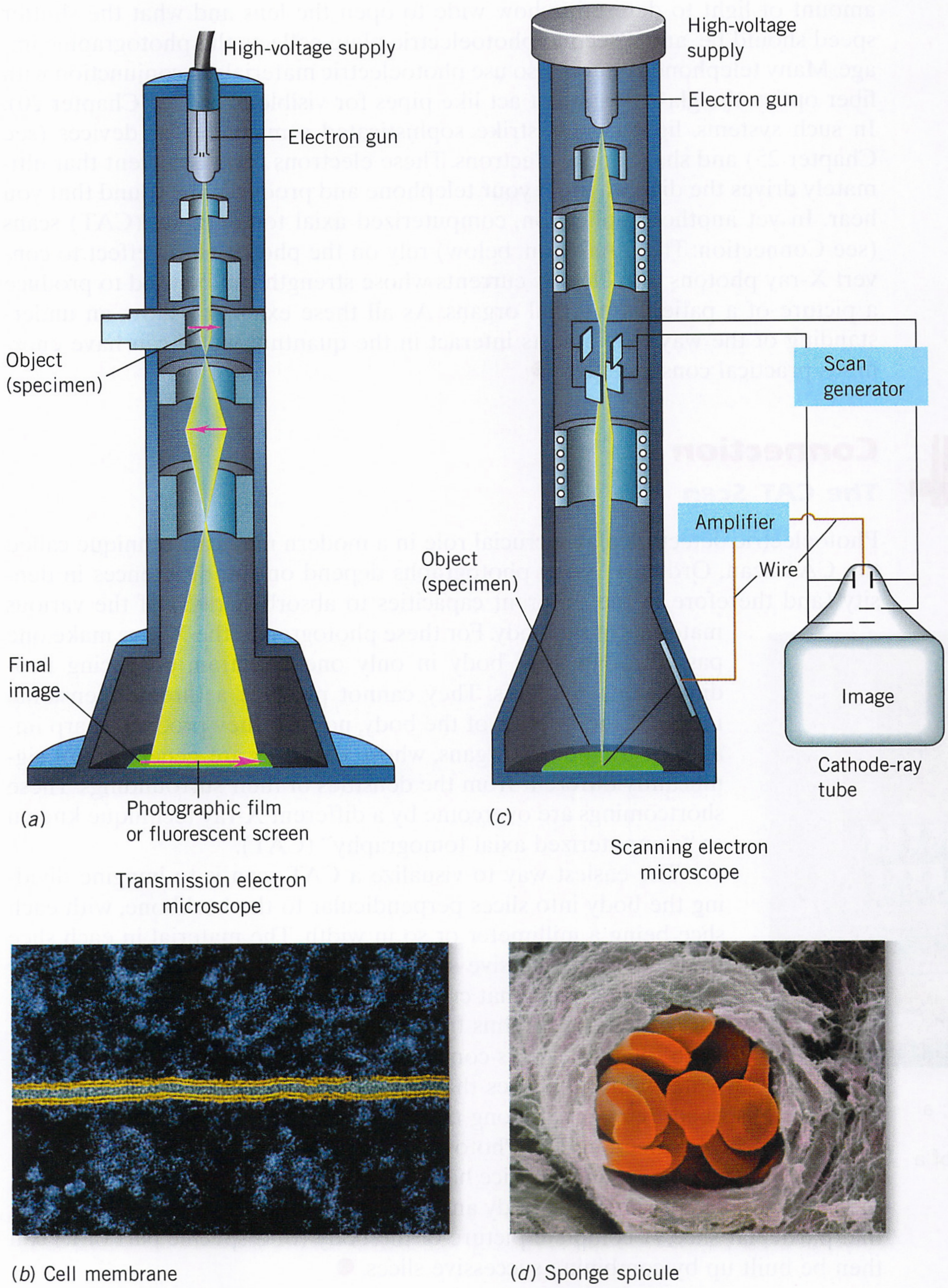


FIGURE 22-4. Two main kinds of electron microscopes. (*a*) Transmission electron microscopes (TEMs) send electron beams through a thin slice of a specimen such as a biological cell wall (*b*). In contrast, (*c*) scanning electron microscopes (SEMs) bounce electron beams off a specimen and collect the beams reflected at various angles, producing more of a three-dimensional effect for microscopic samples such as a sponge spicule (*d*).

amount of light to determine how wide to open the lens and what the shutter speed should be, and a second photoelectric plate collects the photographic image. Many telephone systems also use photoelectric materials in conjunction with fiber optics, the glass fibers that act like pipes for visible light (see Chapter 20). In such systems, light signals strike sophisticated semiconductor devices (see Chapter 25) and shake loose electrons. These electrons form a current that ultimately drives the diaphragm in your telephone and produces the sound that you hear. In yet another application, computerized axial tomography (CAT) scans (see Connection: The CAT Scan, below) rely on the photoelectric effect to convert X-ray photons into electric currents whose strength can be used to produce a picture of a patient's internal organs. As all these examples show, an understanding of the way that objects interact in the quantum world can have enormous practical consequences. ●

Connection
The CAT Scan

Photoelectric detectors play a crucial role in a modern medical technique called the CAT scan. Ordinary X-ray photographs depend on the differences in density (and therefore in the different capacities to absorb X rays) of the various materials in the body. For these photographs, the X rays make one pass through your body in only one direction, producing two-dimensional pictures. They cannot produce a three-dimensional image of the interior of the body, nor can they produce sharp images of the body's organs, whose densities are generally not significantly different from the densities of their surroundings. These shortcomings are overcome by a different X-ray technique known as "computerized axial tomography" (CAT).

The easiest way to visualize a CAT scan is to imagine dividing the body into slices perpendicular to the backbone, with each slice being a millimeter or so in width. The material in each slice is probed by successive short bursts of X rays, lasting only a few milliseconds each, that cross the slice in different directions. Each part of the slice is thus traversed by many different X-ray bursts. Each burst of X rays contains the same number of photons when it starts, and the ones that go all the way through the body (i.e., those not absorbed by material along their path) are measured by a photoelectric device (see Connection: The Photoelectric Effect, above).

A boy having a CAT scan; a video monitor is in the foreground. A CAT scan of a human skull and brain is shown on the right.

Once all the data on a given slice have been obtained, a computer works out the density of each point of the body and produces a detailed cross section along that particular slice. A complete picture of the body (or a specific part of it) can then be built up by combining successive slices. ●

WAVE-PARTICLE DUALITY AND THE BOHR ATOM

Treating electrons as waves helps explain why only certain orbits are allowed in atoms (see Chapter 21). As we have seen, every quantum object displays a simple relationship between its speed (when we think of it as a particle) and its wavelength (when we think of it as a wave). It turns out that for electrons, protons, and other quantum objects, a faster speed always corresponds to a shorter,

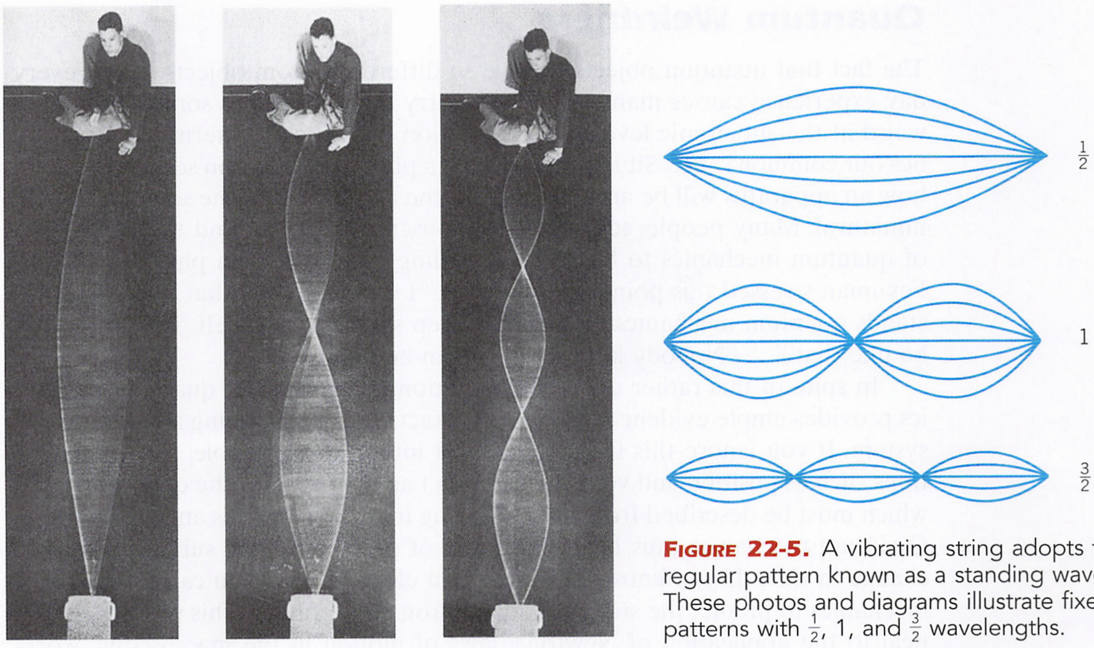


FIGURE 22-5. A vibrating string adopts the regular pattern known as a standing wave. These photos and diagrams illustrate fixed patterns with $\frac{1}{2}$, 1, and $\frac{3}{2}$ wavelengths.

more energetic wavelength (or a higher frequency). This idea is given quantitative form in the de Broglie relationship.

If you think of an electron as a particle, then you can treat its motion around an atom's nucleus in the same Newtonian way that you treat the motion of Earth in orbit around the Sun. That is, for any given distance from the nucleus, the electron must have a precise velocity to stay in a stable orbit. Provided it is moving at such a velocity, it stays in that orbit, just as Earth stays in a stable orbit around the Sun. Any slower and it must adopt a higher orbit; any faster and it moves closer to the nucleus.

However, if we choose to think about the electron as a wave, a different set of criteria can be used to decide how to put the electron into its orbit. A wave on a straight string (on a guitar, for example) vibrates only at certain frequencies that depend on the length of the string (Figure 22-5). These frequencies correspond to fitting $\frac{1}{2}$, 1, and $\frac{3}{2}$ wavelengths on the string in the figure. Now imagine bending the guitar string around into a circle. In this case, you will be able to fit only certain standing waves on the circular string, as shown in Figure 22-6.

You can now ask a simple question: Are there any orbits for which the wave and particle descriptions are consistent? In other words, are there orbits for which the velocity of the electron (when we think of it as a particle) is appropriate to the orbit, while at the same time the electron wave (when we think of it as a wave) fits onto the orbit, given the relation between wavelength and velocity?

When you do the mathematics, you find that the only orbits that satisfy these twin conditions are the Bohr orbits. That is to say, the only orbits allowed in the atom are those for which it makes no difference whether we think of the electron as a particle or a wave. In a sense, then, the wave-particle duality exists in our minds, and not in nature—nature has arranged things so that what we think doesn't matter.

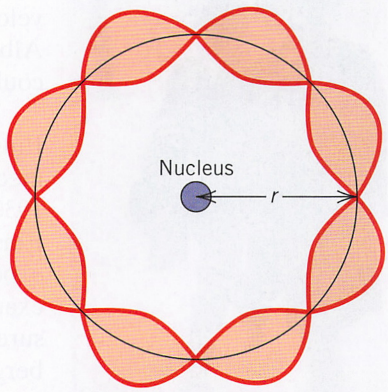


FIGURE 22-6. An electron in orbit about an atom adopts a standing wave like a vibrating string. This illustration shows a standing wave with four wavelengths fitting into the orbit's circumference.

Quantum Weirdness

The fact that quantum objects behave so differently from objects in our everyday experience causes many people to worry that nature has somehow become weird at the subatomic level. The description of particles in terms of waves defies our common sense. Situations in which a photon or electron seems to "know" how an apparatus will be arranged before the arranging is done seem wrong and unnatural. Many people, scientists and nonscientists alike, find the conclusions of quantum mechanics to be quite unsettling. The American physicist Richard Feynman stressed this point when he said, "I can safely say that nobody understands quantum mechanics. . . . Do not keep saying to yourself, 'But how can it be like that?' . . . Nobody knows how it can be like that."

In spite of this rather disturbing situation, the success of quantum mechanics provides ample evidence that it is a correct way of describing an atomic-scale system. If you ignore this fact, you can get into a lot of trouble. Newtonian notions such as position and velocity just aren't appropriate for the quantum world, which must be described from the beginning in terms of waves and probabilities. Quantum mechanics thus becomes a way of predicting how subatomic objects change in time. If you know the state of an electron now, you can use quantum mechanics to predict the state of that electron in the future. This process is identical to the application of Newton's laws of motion in the macroscopic world. The only difference is that in the quantum world, the "state" of the system is described in terms of a probability.

In the view of most working scientists, quantum mechanics is a marvelous tool that allows us to do all sorts of experiments and build all manner of new and important pieces of equipment. The fact that we can't visualize the quantum world in familiar terms seems a small price to pay for all the benefits we receive.

Physics in the Making

A Famous Exchange

Niels Bohr and Albert Einstein discuss quantum mechanics during a physics conference in Brussels, Belgium, in October 1930.

Many people are disturbed by the fact that nature at the subatomic level must be described in terms of probabilities. When quantum mechanics was first developed in the early twentieth century, many physicists were also troubled. Even Albert Einstein, who contributed one of the key ideas of quantum mechanics, could not accept what it was telling us about the world.

Einstein and Bohr were lifelong friends as well as colleagues. However, Einstein spent a good part of the last half of his life trying to refute quantum mechanics, while Bohr defended it. At major physics conferences in 1927 and in 1930, the two men exchanged ideas every day. As other physicists described it, Einstein would come down to breakfast every morning with a beautiful thought experiment he had devised for which quantum mechanics would not work. For example, he might come up with a situation in which he thought he could measure a particle's position and velocity with complete accuracy despite the Heisenberg uncertainty relation. Bohr would think about the problem all day, talking to people, trying to find flaws with Einstein's reasoning. Every evening at dinner, he would have the solution worked out and show Einstein how the uncertainty relation held or other ideas of quantum mechanics still worked.

Einstein's most famous statement from this period was, "I cannot believe that God plays dice with the universe." Confronted once too often with this aphorism, Bohr is supposed to have replied, "Albert, stop telling God what to do."

THINKING MORE ABOUT

Quantum Mechanics: Uncertainty and Human Beings

The ultimate Newtonian view of the universe was the concept of the Divine Calculator (see Chapter 5). Given the position and velocity of every particle in the universe, this imaginary being could predict every future state of those particles. The difficulty with this concept is that if the future of the universe is laid out with clockwork precision, it allows no room for human action. No one can make a choice about what he or she will do because that choice is already determined and exists (in the mind of the Divine Calculator) before it is made.

Quantum mechanics gives us one way to get out of this particular bind. Heisenberg tells us that although we might be able to predict the future if we knew the position and velocity of every particle exactly, we can never actually get those two numbers at the same time. The Divine Calculator in a quantum world is doomed to wait forever for the input data with which to start the calculation.

One area where the uncertainty principle is starting to play a somewhat unexpected role is in the old philosophical argument about the connection between the mind and the brain. The brain is a physical object, an incredibly complex organ that processes information in the form of nerve impulses. The problem: What is the connection between the physical reality of the brain—the atoms and structures that compose it—and the consciousness that we all experience?

Many scientists and philosophers have argued that the brain is no more than a physical structure. These thinkers have run into a problem, however, because if the brain is purely a physical object, its future states should be predictable. Recently, scientists (most notably Roger Penrose of Cambridge University) have argued that quantum mechanics can introduce a kind of unpredictability that squares better with our perceptions of our own minds.

Think about how the workings of the brain might be unpredictable at the quantum level. Why might that uncertainty make it difficult (or even impossible) to make precise predictions of the future state of the brain?

Summary

Matter and energy at the atomic scale come in discrete packets called quanta. The rules of **quantum mechanics,** the laws that allow us to describe and predict events in the quantum world, are disturbingly different from Newton's laws of motion.

At the quantum scale, unlike our everyday experience, any measurement of the position or velocity of a particle causes the particle to change in unpredictable ways. The mere act of measurement alters the thing being measured. Werner Heisenberg quantified this situation in the **uncertainty principle,** which states that the uncertainty in the position of a particle multiplied by the uncertainty in its momentum must be greater than a small positive number. Unlike the Newtonian world, you can never know the exact position and velocity of a quantum particle.

These uncertainties preclude us from describing atomic-scale particles in the classical way. Instead, quantum descriptions are given in terms of **probabilities** that an object is in one state or another. Furthermore, quantum objects are not simply particles or waves, a dichotomy familiar to us in the macroscopic world. They represent something completely different from our experience, incorporating properties of both particles and waves.

Key Terms

probability The likelihood that a certain event or outcome will occur. (p. 478)

quantum mechanics The branch of physics devoted to the study of very small systems in which physical quantities come in discrete bundles called quanta. (p. 472)

uncertainty principle A physical law that places limits on the accuracy to which certain quantities (momentum and position, for example) can be measured simultaneously. (p. 474)

Key Equations

Δx (Uncertainty in position) $\times$ Δp (Uncertainty in momentum) $> h$

$$p = \frac{h}{\lambda}$$

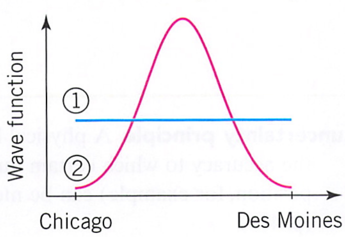
Review

1. What does "quantum mechanics" mean?
2. Give three examples of properties that are quantized at the scale of an electron.
3. What are the three essential parts of every physical measurement?
4. In what way is a measurement at the quantum scale of an electron different from a measurement at the large scale of everyday objects?
5. What is the Heisenberg uncertainty principle?
6. Under what circumstances can you know the position of an electron with great accuracy?
7. Under what conditions can you know the velocity of an electron with great accuracy?
8. The equation form of the uncertainty principle is $\Delta x \times \Delta v > h/m$. What does each variable stand for? Restate this equation in your own words.
9. Why is quantum mechanics sometimes called "wave mechanics"?
10. What is a wave function?
11. What role does probability play in describing subatomic events?
12. What is wave-particle duality? Give an everyday example.
13. Which properties of electrons are particlelike? Which are wavelike?
14. Light is emitted in discrete bundles called photons. Does a photon behave like a particle or a wave? Explain.
15. What is a double-slit experiment? What is the difference between a baseball that goes through a slit and a wave that goes through?
16. How is the momentum of a quantum object related to its wavelength?
17. What is the de Broglie relation?
18. Explain how the photoelectric effect works. Does it depend on the wave nature or the particle nature of light?
19. Give several real-life examples of devices that depend on the photoelectric effect.
20. How does a CAT scan work? How does it differ from an ordinary X-ray photograph?
21. What is an allowed orbit? What two conditions must be satisfied for an electron to be in an allowed orbit?
22. How does wave-particle duality explain the allowed orbits of electrons in atoms?
23. What is quantum weirdness?
24. Why did Albert Einstein use playing dice as an analogy for quantum mechanics?
25. What is the Divine Calculator? Are its predictions consistent with quantum mechanics? Explain.

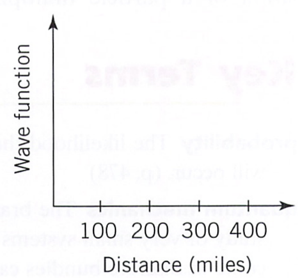
Questions

1. John measures the position of an electron (Δx) to an accuracy of $\pm 10^{-9}$ m, while Jill measures the position of another electron to an accuracy of $\pm 10^{-10}$ m. After these measurements, who is more unsure of the electron's speed? Explain.

2. Your friends, John and Jean, are both driving from Chicago to Des Moines. You know that Jean is on the road, and you know when she left Chicago. On the other hand, you know that John is on the road, but you have no idea when he left. The figure shows two wave functions. Which one is for John and which is for Jean? Explain.



3. A freight train leaves Dallas, traveling at about 100 miles per hour in a straight line. It makes a 400-mile trip, stopping at 100, 200, and 300 miles to unload freight. These stops take about 1 hour each. Reproduce the figure and make

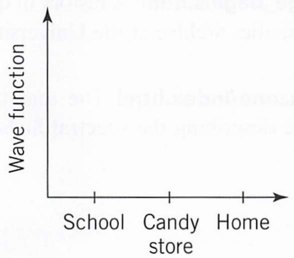


an approximate graph of the wave function for the following three cases:

a. You know that the train has left Dallas, but you do not know when. You are aware, however, that it has not finished its trip.
b. You know that the train is about 4 hours into its trip.
c. You know that the train is about 6 hours into its trip.

4. Little Annie walks to school every day. There is a candy store on the way. Reproduce the figure and graph Annie's wave function for the following three cases:

a. It's the middle of the day and Annie is supposed to be at school, but it is known that she occasionally sneaks out and goes to the candy store.
b. It is nighttime and she is sleeping at home.
c. She is on her way home, but since she has no money we know she will not stop at the candy store.



5. Sketch a possible probability diagram for the final resting position of a golf ball on a driving range. Assume that the golf tee is the starting point and that an average drive is 250 feet.

6. Chaotic systems are, for all practical purposes, unpredictable (see Chapter 5). How does this sort of unpredictability differ from that associated with quantum mechanics?

7. If you threw baseballs through a large two-slit apparatus, would you produce a diffraction pattern? Why or why not?

8. There was once a humorous poster showing a picture of a bed with the caption, "Heisenberg may have slept here." In what way is this an inaccurate representation of Heisenberg's uncertainty principle?

9. A hydrogen atom and a uranium atom are moving at the same speed. Which one has the longer wavelength?

10. An electron and a proton are traveling at the same speed. Which one has more momentum? Which one has a longer wavelength?

11. A small amount of water is brought to a boil in a microwave oven and then removed. An accurate mercury thermometer is taken from the refrigerator and used to take the water's temperature. The reading on the thermometer is 98°C even though the temperature of the water was 100°C. Why was this measurement not accurate? How is this measurement similar to what happens in quantum mechanics?

12. Johnny spends most of his time indoors. Other than an occasional trip to the bathroom, he spends his time sleeping and watching TV in his bedroom. He never visits the kitchen or the living room. If Johnny were a quantum object, how would you describe his wave function?

13. Why are ultraviolet photons more effective at inducing the photoelectric effect than visible light photons?

Problems

1. A ball (mass 0.1 kg) is thrown with a speed between 20.0 and 20.1 m/s. How accurately can we determine its position?

2. In Example 22-1, we converted the unit J-s/kg-m to the unit of velocity (m/s) without comment. Demonstrate the equivalence of these two units.

3. An atom of iron (mass 10^{-25} kg) travels at a speed between 20.0 and 20.1 m/s. How accurately can we determine its position? Could we ever actually measure a position to this accuracy? How does the uncertainty in position compare to the size of an atom? Of a nucleus?

Investigations

1. Look up the doctrine of predestination in an encyclopedia. Does it have a logical connection to the notion of the Divine Calculator? Which came first historically?

2. Werner Heisenberg was a central, and ultimately controversial, figure in German science of the 1930s and 1940s. Read a biography of Heisenberg. Discuss how his early work in quantum mechanics influenced his prominent scientific role in Nazi Germany.

3. What changes in artistic movements were taking place during the period around 1900 (just before the discoveries of quantum mechanics) and in the mid-twentieth century? Are there any connections between the artistic and scientific movements of those times?

4. Some people interpret the Heisenberg uncertainty principle to mean that you can never really know anything for certain. Do you agree or disagree?

WWW Resources

See the *Physics Matters* home page at **www.wiley.com/college/trefil** for valuable web links.

1. **http://www.aip.org/history/heisenberg/** Online exhibit on Werner Heisenberg and the Uncertainty Principle by the American Institute of Physics.
2. **http://www.colorado.edu/physics/PhysicsInitiative/Physics2000/atomic_lab.html** A presentation of the classic experiments of quantum physics as animated tutorials and simulations. This section includes quantum interference and the Bose-Einstein condensate.
3. **http://www.colorado.edu/physics/PhysicsInitiative/Physics2000/quantumzone/photoelectric.html** Presentation of the classic experiments of quantum physics as animated tutorials and simulations. This section includes the photoelectric effect.
4. **http://www.aip.org/history/einstein/** An online exhibit of the American Institute of Physics describing the life and legacy of Albert Einstein.
5. **http://www.aip.org/history/electron/** An online exhibit describing the discovery of the electron by J.J. Thompson from the American Institute of Physics.
6. **http://www-groups.dcs.st-and.ac.uk/~history/HistTopics/The_Quantum_age_begins.html** A history of quantum mechanics with biographies of the major scientists from the MacTutor History of Mathematics archive at the University of St Andrews, Scotland. Includes entries on Bohr, Einstein, and other figures.
7. **http://www.colorado.edu/physics/PhysicsInitiative/Physics2000/quantumzone/index.html** The quantum atom: a lavishly illustrated and Java-rich (and often slow) site includes a detailed storyline describing the spectral lines, the Bohr atom, and related topics.