The Language of Physics

Kinematics
The branch of mechanics that describes the motion of a body without regard to the cause of that motion (p. 39).

Average velocity
The average rate at which the displacement vector changes with time. Since a displacement is a vector, the velocity is also a vector (p. 39).

Average speed
The distance that a body moves per unit time. Speed is a scalar quantity (p. 41).

Constant velocity
A body moving in one direction in such a way that it always travels equal distances in equal times (p. 42).

Acceleration
The rate at which the velocity of a moving body changes with time (p. 43).

Instantaneous velocity
The velocity at a particular instant of time. It is defined as the limit of the ratio of the change in the displacement of the body to the change in time, as the time interval approaches zero. The magnitude of the instantaneous velocity is the instantaneous speed of the moving body (p. 45).

Kinematic equations of linear motion
A set of equations that gives the displacement and velocity of the moving body at any instant of time, and the velocity of the moving body at any displacement, if the acceleration of the body is a constant (p. 46).

Freely falling body
Any body that is moving under the influence of gravity only. Hence, any body that is dropped or thrown on the surface of the earth is a freely falling body (p. 51).

Acceleration due to gravity
If air friction is ignored, all objects that are dropped near the surface of the earth, are accelerated toward the center of the earth with an acceleration of 9.80 m/s^2 or 32 ft/s^2 (p. 52).

Projectile motion
The motion of a body thrown or fired with an initial velocity v_0 in a gravitational field (p. 55).

Trajectory
The path through space followed by a projectile (p. 56).

Range of a projectile
The horizontal distance from the point where the projectile is launched to the point where it returns to its launch height (p. 64).

Summary of Important Equations

Average velocity
$$\mathbf{v}_{avg} = \frac{\Delta \mathbf{r}}{\Delta t} = \frac{\mathbf{r}_2 - \mathbf{r}_1}{t_2 - t_1} \qquad (3.32)$$

Acceleration
$$\mathbf{a} = \frac{\Delta \mathbf{v}}{\Delta t} = \frac{\mathbf{v} - \mathbf{v}_0}{t} \qquad (3.33)$$

Instantaneous velocity in two or more directions, which is a generalization of the instantaneous velocity in one dimension
$$\mathbf{v} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{r}}{\Delta t} \qquad (3.8)$$
$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

Velocity at any time
$$\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t \qquad (3.35)$$

Displacement at any time
$$\mathbf{r} = \mathbf{v}_0 t + \tfrac{1}{2}\mathbf{a}t^2 \qquad (3.34)$$

Velocity at any displacement in the x-direction
$$v^2 = v_0^2 + 2ax \qquad (3.16)$$

Velocity at any displacement in the y-direction
$$v^2 = v_0^2 + 2ay \qquad (3.16)$$

For Projectile Motion

x-displacement
$$x = v_{0x} t \qquad (3.38)$$

y-displacement
$$y = v_{0y} t - \tfrac{1}{2} g t^2 \qquad (3.39)$$

x-component of velocity
$$v_x = v_{0x} \qquad (3.40)$$

y-component of velocity
$$v_y = v_{0y} - gt \qquad (3.41)$$

y-component of velocity at any height y
$$v_y^2 = v_{0y}^2 - 2gy \qquad (3.48)$$

Range
$$R = \frac{v_0^2 \sin 2\theta}{g} \qquad (3.47)$$

Questions for Chapter 3

1. Discuss the difference between distance and displacement.
2. Discuss the difference between speed and velocity.
3. Discuss the difference between average speed and instantaneous speed.

† 4. Although speed is the magnitude of the instantaneous velocity, is the average speed equal to the magnitude of the average velocity?

5. Why can the kinematic equations be used only for motion at constant acceleration?
6. In dealing with average velocities discuss the statement, "Straight line motion at 60 mph for 1 hr followed by motion in the same direction at 30 mph for 2 hr does not give an average of 45 mph but rather 40 mph."
7. What effect would air resistance have on the velocity of a body that is dropped near the surface of the earth?
8. What is the acceleration of a projectile when its instantaneous vertical velocity is zero at the top of its trajectory?
9. Can an object have zero velocity at the same time that it has an acceleration? Explain and give some examples.
10. Can the velocity of an object be in a different direction than the acceleration? Give some examples.
11. Can you devise a means of using two clocks to measure your reaction time?

†12. A person on a moving train throws a ball straight upward. Describe the motion as seen by a person on the train and by a person on the station platform.

13. You are in free fall, and you let go of your watch. What is the relative velocity of the watch with respect to you?

†14. What kind of motion is indicated by a graph of displacement versus time, if the slope of the curve is (a) horizontal, (b) sloping upward to the right, and (c) sloping downward?

†15. What kind of motion is indicated by a graph of velocity versus time, if the slope of the curve is (a) horizontal, (b) sloping upward at a constant value, (c) sloping upward at a changing rate, (d) sloping downward at a constant value, and (e) sloping downward at a changing rate?

Hints for Problem Solving

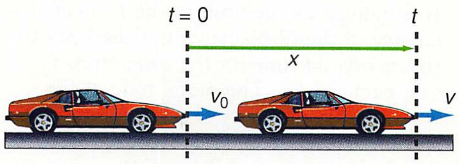
To be successful in a physics course it is necessary to be able to solve problems. The following procedure should prove helpful in solving the physics problems assigned. First, as a preliminary step, read the appropriate topic in the textbook. Do not attempt to solve the problems before doing this. Look at the appropriate illustrative problems to see how they are solved. With this background, now read the assigned problem. Now continue with the following procedure.

1. Draw a small picture showing the details of the problem. This is very useful so that you do not lose sight of the problem that you are trying to solve.
2. List all the information that you are given.
3. List all the answers you are expected to find.
4. From the summary of important equations or the text proper, list the equations that are appropriate to this topic.
5. Pick the equation that relates the variables that you are given.
6. Place a check mark (✓) over each variable that is given and a question mark (?) over each variable that you are looking for.
7. Solve the equation for the unknown variable.
8. When the answer is obtained check to see if the answer is reasonable.

Let us apply this technique to the following example.

A car is traveling at 30 ft/s when it starts to accelerate at 10 ft/s². Find (a) the velocity and (b) the displacement of the car at the end of 5 s.

1. Draw a picture of the problem.



2. Given: $v_0 = 30$ ft/s

$a = 10$ ft/s²

$t = 5$ s

3. Find: $v = ?$

$x = ?$

4. The problem is one in kinematics and the kinematic equations apply. That is,

$$(1) \quad x = v_0 t + \tfrac{1}{2}at^2$$

$$(2) \quad v = v_0 + at$$

$$(3) \quad v^2 = v_0^2 + 2ax$$

5. Part a of the problem. To solve for the velocity v, we need an equation containing v. Equation 1 does not contain a velocity term v, and hence can not be used to solve for the velocity. Equations 2 and 3, on the other hand, both contain v. Thus, we can use one or possibly both of these equations to solve for the velocity.
6. Write down the equation and place a check mark over the known terms and a question mark over the unknown terms:

$$(2) \quad \overset{?}{v} = \overset{\checkmark}{v_0} + \overset{\checkmark\checkmark}{at}$$

The only unknown in equation 2 is the velocity v and we can now solve for it.

7. The velocity after 5 s, found from equation 2 is

$$\begin{aligned} v &= v_0 + at \\ &= 30 \text{ ft/s} + (10 \text{ ft/s}^2)(5 \text{ s}) \\ &= 30 \text{ ft/s} + 50 \text{ ft/s} \\ &= 80 \text{ ft/s} \end{aligned}$$

Notice what would happen if we tried to use equation 3 at this time:

$$(3) \quad \overset{?}{v^2} = \overset{\checkmark}{v_0^2} + \overset{\checkmark\checkmark ?}{2ax}$$

We can not solve for the velocity v from equation 3 because there are two unknowns, both v and x. However, if we had solved part b of the problem for x first, then we could have used this equation.

5. Part b of the problem. To solve for the displacement x, we need an equation containing x. Notice that equation 2 does not contain x, so we can not use it. Equations 1 and 3, on the other hand, do contain x, and we can use either to solve for x.
6. Looking at equation 1, we have

$$(1) \quad \overset{?}{x} = \overset{\checkmark\checkmark}{v_0 t} + \overset{\checkmark\checkmark\checkmark}{\tfrac{1}{2}at^2}$$

7. Solving for the only unknown in equation 1, x, we get

$$\begin{aligned} x &= v_0 t + \tfrac{1}{2}at^2 \\ &= (30 \text{ ft/s})(5 \text{ s}) + \tfrac{1}{2}(10 \text{ ft/s}^2)(5 \text{ s})^2 \\ &= 150 \text{ ft} + 125 \text{ ft} \\ &= 275 \text{ ft} \end{aligned}$$

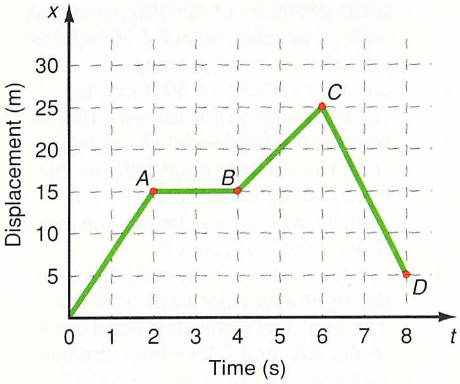
Note that at this point we could also have used equation 3 to determine x, because we already found the velocity v in part a of the problem.

Problems for Chapter 3

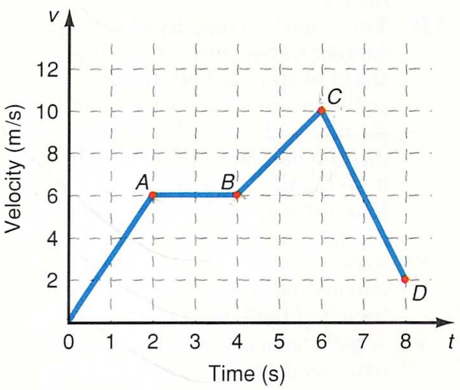
3.1 Introduction

1. A driver travels 300 mi in 5 hr and 25 min. What is his average speed in (a) mph, (b) ft/s, (c) km/hr, and (d) m/s?
2. A car travels at 40 mph for 2 hr and 60 mph for 3 hr. What is its average speed?
3. A man hears the sound of thunder 5 s after he sees the lightning flash. If the speed of sound in air is 343 m/s, how far away is the lightning? Assume that the speed of light is so large that the lightning was seen essentially at the same time that it was created.
4. The earth-moon distance is 3.84×10^8 m. If it takes 3 days to get to the moon, what is the average speed?
5. Electronic transmission is broadcast at the speed of light, which is 3.00×10^8 m/s. How long would it take for a radio transmission from earth to an astronaut orbiting the planet Mars? Assume that at the time of transmission the distance from earth to Mars is 7.80×10^7 km.
6. In the game of baseball, some excellent fast-ball pitchers have managed to pitch a ball at approximately 100 mph. If the pitcher's mound is 60.5 ft from home plate, how long does it take the ball to get to home plate? If the pitcher then throws a change-of-pace ball (a slow ball) at 60 mph, how long will it now take the ball to get to the plate?
7. Two students are having a race on a circular track. Student 1 is on the inside track, which has a radius of curvature $r_1 = 250$ m, and is moving at the speed $v_1 = 4.50$ m/s. With what speed must student 2 run to keep up with student 1 if student 2 is on the outside track of radius of curvature $r_2 = 255$ m?

8. A plot of the displacement of a car as a function of time is shown in the diagram. Find the velocity of the car along the paths (a) *O-A*, (b) *A-B*, (c) *B-C*, and (d) *C-D*.



9. A plot of the velocity of a car as a function of time is shown in the diagram. Find the acceleration of the car along the paths (a) *O-A*, (b) *A-B*, (c) *B-C*, and (d) *C-D*.



10. If an airplane is traveling at 110 knots, what is its velocity in (a) mph, (b) km/hr, (c) ft/s, and (d) m/s? A knot is a nautical mile per hour, and a nautical mile is equal to 6076 ft.

3.6 The Kinematic Equations in One Dimension

11. A girl who is initially running at 1.00 m/s increases her velocity to 2.50 m/s in 5.00 s. Find her acceleration.

12. A car is traveling at 95.0 km/hr. The driver steps on the brakes and the car comes to a stop in 60.0 m. What is the car's deceleration?

13. A train accelerates from an initial velocity of 20.0 mph to a final velocity of 35.0 mph in 11.8 s. Find its acceleration and the distance the train travels during this time.

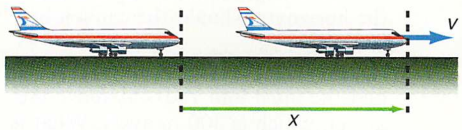
14. A train accelerates from an initial velocity of 25.0 km/hr to a final velocity of 65.0 km/hr in 8.50 s. Find its acceleration and the distance the train travels during this time.

15. An airplane travels 1000 ft at a constant acceleration while taking off. If it starts from rest, and takes off in 25.0 s, what is its takeoff velocity?

16. An airplane travels 450 km at a constant acceleration while taking off. If it starts from rest, and takes off in 30.0 s, what is its takeoff velocity?

17. A car starts from rest and acquires a velocity of 20.0 mph in 10.0 s. Where is the car located and what is its velocity at 10.0, 15.0, 20.0, and 25.0 s?

18. A jet airplane goes from rest to a velocity of 250 ft/s in a distance of 400 ft. What is the airplane's average acceleration in ft/s²?



19. An electron in a vacuum tube acquires a velocity of 5.3×10^8 cm/s in a distance of 0.25 cm. Find the acceleration of the electron.

20. A driver traveling at 60.0 mph tries to stop the car and finds that the brakes have failed. The emergency brake is then pulled and the car comes to a stop in 456 ft. Find the car's deceleration.

21. An airplane has a touchdown velocity of 75.0 knots and comes to rest in 400 ft. What is the airplane's average deceleration? How long does it take the plane to stop?

22. A pitcher gives a baseball a horizontal velocity of 110 ft/s by moving his arm through a distance of approximately 3.00 ft. What is the average acceleration of the ball during this throwing process?

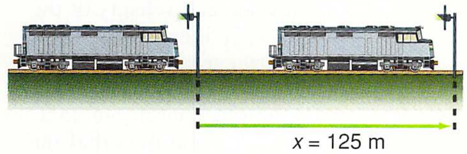
23. The speedometer of a car reads 60.0 mph when the brakes are applied. The car comes to rest in 4.55 s. How far does the car travel before coming to rest?

†24. A body with unknown initial velocity moves with constant acceleration. At the end of 8.00 s, it is moving at a velocity of 50.0 m/s and it is 200 m from where it started. Find the body's acceleration and its initial velocity.

†25. A driver traveling at 25.0 mph sees the light turn red at the intersection. If her reaction time is 0.600 s, and the car can decelerate at 18.0 ft/s², find the stopping distance of the car. What would the stopping distance be if the car were moving at 50.0 mph?

26. A driver traveling at 30.0 km/hr sees the light turn red at the intersection. If his reaction time is 0.600 s, and the car can decelerate at 4.50 m/s², find the stopping distance of the car. What would the stopping distance be if the car were moving at 90.0 km/hr?

†27. A uniformly accelerating train passes a green light signal at 25.0 km/hr. It passes a second light 125 m farther down the track, 12.0 s later. What is the train's acceleration? What is the train's velocity at the second light?



28. A car accelerates from 50.0 mph to 80.0 mph in 26.9 s. Find its acceleration and the distance the car travels in this time.

†29. A motorcycle starts from rest and accelerates at 4.00 m/s² for 5.00 s. It then moves at constant velocity for 25.0 s, and then decelerates at 2.00 m/s² until it stops. Find the total distance that the motorcycle has moved.

†30. A car starts from rest and accelerates at a constant rate of 3.00 m/s² until it is moving at 18.0 m/s. The car then decreases its acceleration to 0.500 m/s² and continues moving for an additional distance of 250 m. Find the total time taken.

3.7 The Freely Falling Body

31. A passenger, in abandoning a sinking ship, steps over the side. The deck is 15.0 m above the water surface. With what velocity does the passenger hit the water?

32. How long does it take for a stone to fall from a bridge to the water 30.0 m below? With what velocity does the stone hit the water?

33. An automobile traveling at 60.0 mph hits a stone wall. From what height would the car have to fall to acquire the same velocity?

34. A rock is dropped from the top of a building and hits the ground 8.00 s later. How high is the building?

35. A stone is dropped from a bridge 100 ft high. How long will it take for the stone to hit the water below?

36. A ball is dropped from a building 50.0 meters high. How long will it take the ball to hit the ground below?

†37. A girl is standing in an elevator that is moving upward at a velocity of 12.0 ft/s when she drops her handbag. If she was originally holding the bag at a height of 4.00 ft above the elevator floor, how long will it take the bag to hit the floor?

3.9 Projectile Motion in One Dimension

38. A ball is thrown vertically upward with an initial velocity of 130 ft/s. Find its position and velocity at the end of 2, 4, 6, and 8 s and sketch these positions and velocities on a piece of graph paper.

39. A projectile is fired vertically upward with an initial velocity of 40.0 m/s. Find the position and velocity of the projectile at 1, 3, 5, and 7 s.

†40. A ball is thrown vertically upward from the top of a building 40.0 m high with an initial velocity of 25.0 m/s. What is the total time that the ball is in the air?

41. A stone is thrown vertically upward from a bridge 100 ft high at an initial velocity of 50.0 ft/s. How long will it take for the stone to hit the water below?

†42. A stone is thrown vertically downward from a bridge 100 ft high at an initial velocity of −50.0 ft/s. How long will it take for the stone to hit the water below?

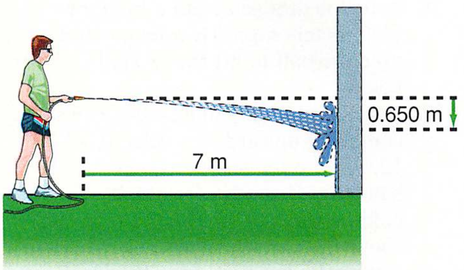
43. A rock is thrown vertically downward from a building 40.0 m high at an initial velocity of −15.0 m/s. (a) What is the rock's velocity as it strikes the ground? (b) How long does it take for the rock to hit the ground?

44. A baseball batter fouls a ball vertically upward. The ball is caught right behind home plate at the same height that it was hit. How long was the baseball in flight if it rose a distance of 100 ft? What was the initial velocity of the baseball?

3.11 Projectile Motion in Two Dimensions

45. A projectile is thrown from the top of a building with a horizontal velocity of 15.0 m/s. The projectile lands on the street 85.0 m from the base of the building. How high is the building?

46. To find the velocity of water issuing from the nozzle of a garden hose, the nozzle is held horizontally and the stream is directed against a vertical wall. If the wall is 7.00 m from the nozzle and the water strikes the wall 0.650 m below the horizontal, what is the velocity of the water?



47. A bomb is dropped from an airplane in level flight at a velocity of 970 km/hr. The altitude of the aircraft is 2000 m. At what horizontal distance from the initial position of the aircraft will the bomb land?

†48. A cannon is placed on a hill 20.0 m above level ground. A shell is fired horizontally at a muzzle velocity of 300 m/s. At what horizontal distance from the cannon will the shell land? How long will this take? What will be the shell's velocity as it strikes its target?

49. A shell is fired from a cannon at a velocity of 300 m/s to hit a target 3000 m away. At what angle above the horizontal should the cannon be aimed?

50. In order to hit a target, a marksman finds he must aim 10.0 cm above the target, which is 300 m away. What is the initial speed of the bullet?

51. A golf ball is hit with an initial velocity of 175 ft/s at an angle of 50.0° above the horizontal. (a) How high will the ball go? (b) What is the total time the ball is in the air? (c) How far will the ball travel horizontally before it hits the ground?

52. A projectile is thrown from the ground with an initial velocity of 20.0 m/s at an angle of 40.0° above the horizontal. Find (a) the projectile's maximum height, (b) the time required to reach its maximum height, (c) its velocity at the top of the trajectory, (d) the range of the projectile, and (e) the total time of flight.

Additional Problems

53. A missile has a velocity of 10,000 mph at "burn-out," which occurs 2 min after ignition. Find the average acceleration in (a) ft/s^2, (b) m/s^2, and (c) in terms of g, the acceleration due to gravity at the surface of the earth.

54. A block slides down a smooth inclined plane that makes an angle of 25.0° with the horizontal. Find the acceleration of the block. If the plane is 10.0 meters long and the block starts from rest, what is its velocity at the bottom of the plane? How long does it take for the block to get to the bottom?

†55. At the instant that the traffic light turns green, a car starting from rest with an acceleration of 7.00 ft/s^2 is passed by a truck moving at a constant velocity of 30.0 mph. (a) How long will it take for the car to overtake the truck? (b) How far from the starting point will the car overtake the truck? (c) At what velocity will the car be moving when it overtakes the truck?

56. At the instant that the traffic light turns green, a car starting from rest with an acceleration of 2.50 m/s^2 is passed by a truck moving at a constant velocity of 60.0 km/hr. (a) How long will it take for the car to overtake the truck? (b) How far from the starting point will the car overtake the truck? (c) At what velocity will the car be moving when it overtakes the truck?

†57. A boat passes a buoy while moving to the right at a velocity of 8.00 m/s. The boat has a constant acceleration to the left, and 10.0 s later the boat is found to be moving at a velocity of −3.00 m/s. Find (a) the acceleration of the boat, (b) the distance from the buoy when the boat reversed direction, (c) the time for the boat to return to the buoy, and (d) the velocity of the boat when it returns to the buoy.

†58. Two trains are initially at rest on parallel tracks with train 1 50.0 m ahead of train 2. Both trains accelerate simultaneously, train 1 at the rate of 2.00 m/s^2 and train 2 at the rate of 2.50 m/s^2. How long will it take train 2 to overtake train 1? How far will train 2 travel before it overtakes train 1?

†59. Repeat problem 58 but with train 1 initially moving at 5.00 m/s and train 2 initially moving at 7.00 m/s.

†60. A policeman driving at 55.0 mph observes a car 200 ft ahead of him speeding at 80.0 mph. If the county line is 1200 ft away from the police car, what must the acceleration of the police car be, in order to catch the speeder before he leaves the county?

61. A policewoman driving at 80.0 km/hr observes a car 50.0 m ahead of her speeding at 120 km/hr. If the county line is 400 m away from the police car, what must the acceleration of the police car be in order to catch the speeder before he leaves the county?

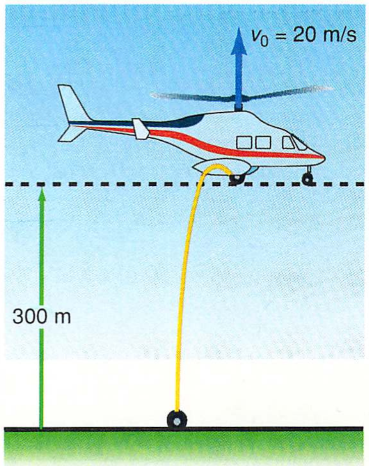
†62. Two trains are approaching each other along a straight and level track. The first train is heading east at 70.0 mph, while the second train is heading west at 45.0 mph. When they are 1.50 miles apart they see each other and start to decelerate. Train 1 decelerates at 5.00 ft/s^2, while train 2 decelerates at 3.00 ft/s^2. Will the trains be able to stop or will there be a collision?

63. Two trains are approaching each other along a straight and level track. The first train is heading south at 125 km/hr, while the second train is heading north at 80.0 km/hr. When they are 2.00 km apart, they see each other and start to decelerate. Train 1 decelerates at 2.00 m/s², while train 2 decelerates at 1.50 m/s². Will the trains be able to stop or will there be a collision?

†64. A boy in an elevator, which is descending at the constant velocity of −5.00 m/s, jumps to a height of 0.500 m above the elevator floor. How far will the elevator descend before the boy returns to the elevator floor?

65. The acceleration due to gravity on the moon is 1.62 m/s². If an astronaut on the moon throws a ball straight upward, with an initial velocity of 25.0 m/s, how high will the ball rise?

†66. A helicopter, at an altitude of 300 m, is rising vertically at 20.0 m/s when a wheel falls off. How high will the wheel go with respect to the ground? How long will it take for the wheel to hit the ground below? At what velocity will the wheel hit the ground?



†67. A ball is dropped from the roof of a building 40.0 m high. Simultaneously, another ball is thrown upward from the ground and collides with the first ball at half the distance to the roof. What was the initial velocity of the ball that was thrown upward?

†68. A ball is dropped from the top of a 40.0-m high building. At what initial velocity must a second ball be thrown from the top of the building 2.00 s later, such that both balls arrive at the ground at the same time?

†69. Show that the range of a projectile is the same for either a projection angle of 45.0° $+\theta$ or an angle of 45.0° $-\theta$.

70. A projectile hits a target 1.50 km away 10.5 s after it was fired. Find (a) the elevation angle of the gun and (b) the initial velocity of the projectile.

71. A football is kicked with an initial velocity of 70.0 ft/s at an angle of 65.0° above the horizontal. Find (a) how long the ball is in the air, (b) how far down field the ball lands, (c) how high the ball rises, and (d) the velocity of the ball when it strikes the ground.

†72. A baseball is hit at an initial velocity of 110 ft/s at an angle of 45.0° above the horizontal. Will the ball clear a 10.0 ft fence 300 ft from home plate for a home run? If so, by how much will it clear the fence?

†73. A ball is thrown from a bridge 100 m high at an initial velocity of 30.0 m/s at an angle of 50.0° above the horizontal. Find (a) how high the ball goes, (b) the total time the ball is in the air, (c) the maximum horizontal distance that the ball travels, and (d) the velocity of the ball as it strikes the ground.

74. A ball is thrown at an angle of 35.5° below the horizontal at a speed of 22.5 m/s from a building 20.0 m high. (a) How long will it take for the ball to hit the ground below? (b) How far from the building will the ball land?

†75. Using the kinematic equations for the x- and y-components of the displacement, find the equation of the trajectory for two-dimensional projectile motion. Compare this equation with the equation for a parabola expressed in its standard form.

†76. Using the kinematic equations, prove that if two balls are released simultaneously from a table, one with zero velocity and the other with a horizontal velocity v_{0x}, they will both reach the ground at the same time.

Interactive Tutorials

77. A train accelerates from an initial velocity of 20.0 m/s to a final velocity of 35.0 m/s in 11.8 s. Find its acceleration and the distance the train travels in this time.

78. A ball is dropped from a building 50.0 m high. How long will it take the ball to hit the ground below and with what final velocity?

79. A golf ball is hit with an initial velocity $v_0 = 53.0$ m/s at an angle $\theta = 50.0°$ above the horizontal. (a) How high will the ball go? (b) What is the total time the ball is in the air? (c) How far will the ball travel horizontally before it hits the ground?

80. Instantaneous velocity. If the equation for the displacement x of a body is known, the average velocity throughout an interval can be computed by the formula

$$v_{avg} = (\Delta x)/(\Delta t)$$

The instantaneous velocity is defined as the limit of the average velocity as Δt approaches zero. That is,

$$v = \lim_{\Delta t \to 0} (\Delta x)/(\Delta t)$$

For an acceleration with a displacement given by $x = 0.5\,at^2$, use different values of Δt to see how the average velocity approaches the instantaneous velocity. Compare this to the velocity determined by the equation $v = at$, and determine the percentage error. Plot the average velocity, $(\Delta x)/(\Delta t)$, versus Δt.

81. Free-fall and generalized one-dimensional projectile motion. A projectile is fired from a height y_0 above the ground with an initial velocity v_0 in a vertical direction. Find (a) the time t_r for the projectile to rise to its maximum height, (b) the total time t_t the ball is in the air, (c) the maximum height y_{max} of the projectile, (d) the velocity v_g of the projectile as it strikes the ground, and (e) the location and velocity of the projectile at any time t. (f) Plot a picture of the motion as a function of time.

82. Generalized two-dimensional projectile motion. A projectile is fired from a height y_0 above the horizontal with an initial velocity v_0 at an angle θ. Find (a) the time t_r for the projectile to rise to its maximum height; (b) the total time t_t the ball is in the air; (c) the maximum distance the ball travels in the x-direction, x_{max} before it hits the ground; (d) the maximum height y_{max} of the projectile; (e) the velocity v_g of the projectile as it strikes the ground; and (f) the location and velocity of the projectile at any time t. (g) Plot a picture of the trajectory.