The Language of Physics

Scalar
A scalar quantity is a quantity that can be completely described by a magnitude, that is, by a number and a unit (p. 23).

Vector
A vector quantity is a quantity that needs both a magnitude and direction to completely describe it (p. 23).

Resultant
The vector sum of any number of vectors is called the resultant vector (p. 24).

Parallelogram method of vector addition
The main diagonal of a parallelogram is equal to the magnitude of the sum of the vectors that make up the sides of the parallelogram (p. 25).

Sine function
The ratio of the length of the opposite side to the length of the hypotenuse in a right triangle (p. 26).

Cosine function
The ratio of the length of the adjacent side to the length of the hypotenuse in a right triangle (p. 26).

Tangent function
The ratio of the length of the opposite side of a right triangle to the length of the adjacent side (p. 26).

Pythagorean theorem
The sum of the squares of the lengths of two sides of a right triangle is equal to the square of the length of the hypotenuse (p. 27).

Component of a vector
The projection of a vector onto a specified axis. The length of the projection of the vector onto the x-axis is called the x-component of the vector. The length of the projection of the vector onto the y-axis is called the y-component of the vector (p. 28).

The addition of vectors by the component method
The x-component of the resultant vector R_x is equal to the sum of the x-components of the individual vectors, while the y-component of the resultant vector R_y is equal to the sum of the y-components of the individual vectors. The magnitude of the resultant vector is then found by the Pythagorean theorem applied to the right triangle with sides R_x and R_y. The direction of the resultant vector is found by trigonometry (p. 31).

Summary of Important Equations

Vector addition is commutative
$$\mathbf{R} = \mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a} \quad (2.5)$$

Subtraction of vectors
$$\mathbf{a} - \mathbf{b} = \mathbf{a} + (-\mathbf{b}) \quad (2.6)$$

Addition of vectors
$$\mathbf{R} = \mathbf{a} + \mathbf{b} + \mathbf{c} + \mathbf{d} \quad (2.7)$$

Definition of the sine
$$\text{sine } \theta = \frac{\text{opposite side}}{\text{hypotenuse}} \quad (2.8)$$

Definition of the cosine
$$\text{cosine } \theta = \frac{\text{adjacent side}}{\text{hypotenuse}} \quad (2.10)$$

Definition of the tangent
$$\text{tangent } \theta = \frac{\text{opposite side}}{\text{adjacent side}} \quad (2.12)$$

Pythagorean theorem
$$c = \sqrt{a^2 + b^2} \quad (2.17)$$

x-component of a vector
$$a_x = a \cos \theta \quad (2.20)$$

y-component of a vector
$$a_y = a \sin \theta \quad (2.22)$$

Magnitude of a vector
$$a = \sqrt{a_x^2 + a_y^2} \quad (2.24)$$

Direction of a vector
$$\theta = \tan^{-1}\frac{a_y}{a_x} \quad (2.26)$$

x-component of resultant vector
$$R_x = a_x + b_x + c_x + d_x \quad (2.35)$$

y-component of resultant vector
$$R_y = a_y + b_y + c_y + d_y \quad (2.36)$$

Magnitude of resultant vector
$$R = \sqrt{R_x^2 + R_y^2} \quad (2.37)$$

Direction of resultant vector
$$\theta = \tan^{-1}\frac{R_y}{R_x} \quad (2.39)$$

Questions for Chapter 2

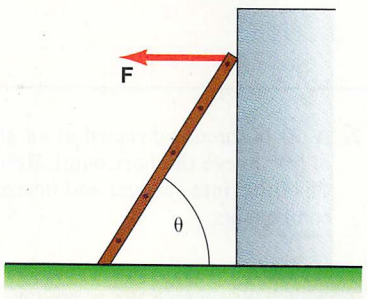
1. Give an example of some quantities that are scalars and vectors other than those listed in section 2.1.
2. Can a vector ever be zero? What does a zero vector mean?
† 3. Since time seems to pass from the past to the present and then to the future, can you say that time has a direction and therefore could be represented as a vector quantity?
4. Does the subtraction of two vectors obey the commutative law?
5. What happens if you multiply a vector by a scalar?
6. What happens if you divide a vector by a scalar?
7. If a person walks around a block that is 800 ft on each side and ends up at the starting point, what is the person's displacement?
8. How can you add three vectors of equal magnitude in a plane such that their resultant is zero?
9. When are two vectors **a** and **b** equal?
†10. If a coordinate system is rotated, what does this do to the vector? to the components?
†11. Why are all the fundamental quantities scalars?
12. A vector equation is equivalent to how many component equations?
13. If the components of a vector **a** are a_x and a_y, what are the components of the vector $\mathbf{b} = -5\mathbf{a}$?
14. If $|\mathbf{a} + \mathbf{b}| = |\mathbf{a} - \mathbf{b}|$, what is the angle between **a** and **b**?

Problems for Chapter 2

2.7 Resolution of a Vector into Its Components and *2.8* Determination of a Vector from Its Components

1. A strong child pulls a sled with a force of 100 lb at an angle of 35° above the horizontal. Find the vertical and horizontal components of this pull.
2. A 50-N force is directed at an angle of 50° above the horizontal. Resolve this force into vertical and horizontal components.

3. A girl wants to hold a 25-lb sled at rest on a snow covered hill. The hill makes an angle of 15° with the horizontal. What force must she exert parallel to the slope? What is the force perpendicular to the surface of the hill that presses the sled against the hill?
4. A boy wants to hold a 68.0-N sled at rest on a snow-covered hill. The hill makes an angle of 27.5° with the horizontal. (a) What force must he exert parallel to the slope? (b) What is the force perpendicular to the surface of the hill that presses the sled against the hill?
5. A displacement vector, at an angle of 35° with respect to a specified direction, has a y-component equal to 150 ft. What is the magnitude of the displacement vector?
6. A plane is traveling northeast at 200 km/hr. What is (a) the northward component of its velocity, and (b) the eastward component of its velocity?
7. While taking off, an airplane climbs at an 8° angle with respect to the ground. If the aircraft's speed is 200 km/hr, what are the vertical and horizontal components of its velocity?
8. A car that weighs 3200 lb is parked on a hill that makes an angle of 23° with the horizontal. Find the component of the car's weight parallel to the hill and perpendicular to the hill.
9. A car that weighs 8900 N is parked on a hill that makes an angle of 43° with the horizontal. Find the component of the car's weight parallel to the hill and perpendicular to the hill.
10. A girl pushes a lawn mower with a force of 90 N. The handle of the mower makes an angle of 40° with the ground. What are the vertical and horizontal components of this force and what are their physical significances? What effect does raising the handle to 50° have?
11. A missile is launched with a speed of 1000 m/s at an angle of 73° above the horizontal. What are the horizontal and vertical components of the missile's velocity?
12. When a ladder leans against a smooth wall, the wall exerts a horizontal force **F** on the ladder, as shown in the diagram. If F is equal to 50 N and θ is equal to 63°, find the component of the force perpendicular to the ladder and the component parallel to the ladder.



2.9 The Addition of Vectors by the Component Method

13. Find the resultant of the following three displacements; 3 mi due east, 6 mi east-northeast, and 7 mi northwest.
14. A girl drives 3 km north, then 12 km to the northwest, and finally 5 km south-southwest. How far has she traveled? What is her displacement?
15. An airplane flies due east at 200 mph straight from city A to city B. A northeast wind of 40 mph is blowing. (Note that all winds are defined in terms of the direction from which the wind blows. Hence, a northeast wind blows out of the northeast and blows toward the southwest.) What is the resultant velocity of the plane with respect to the ground?
16. An airplane flies due north at 380 km/hr straight from city A to city B. A southeast wind of 75 km/hr is blowing. (Note that all winds are defined in terms of the direction from which the wind blows. Hence, a southeast wind blows out of the southeast and blows toward the northwest.) What is the resultant velocity of the plane with respect to the ground?
17. Find the resultant of the following forces: (a) 30 N at an angle of 40° with respect to the x-axis, (b) 120 N at an angle of 135°, and (c) 60 N at an angle of 260°.
18. Find the resultant of the following set of forces. (a) $\mathbf{F}_1$ of 200 N at an angle of 53° with respect to the x-axis. (b) $\mathbf{F}_2$ of 300 N at an angle of 150° with respect to the x-axis. (c) $\mathbf{F}_3$ of 200 N at an angle of 270° with respect to the x-axis. (d) $\mathbf{F}_4$ of 350 N at an angle of 310° with respect to the x-axis.

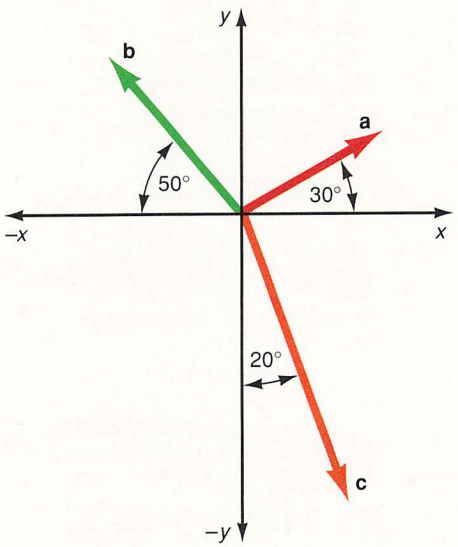
Additional Problems

19. A heavy trunk weighing 150 lb is pulled along a smooth station platform by a 50 lb force making an angle of 37° above the horizontal. Find (a) the horizontal component of the force, (b) the vertical component of the force, and (c) the resultant downward force on the floor.
20. A heavy trunk weighing 800 N is pulled along a smooth station platform by a 210-N force making an angle of 53° above the horizontal. Find (a) the horizontal component of the force, (b) the vertical component of the force, and (c) the resultant downward force on the floor.
21. Vector **A** has a magnitude of 50 ft and points in a direction of 50° north of east. What are the magnitudes and directions of the vectors, (a) $2\mathbf{A}$, (b) $0.5\mathbf{A}$, (c) $-\mathbf{A}$, (d) $-5\mathbf{A}$, (e) $\mathbf{A} + 4\mathbf{A}$, (f) $\mathbf{A} - 4\mathbf{A}$?
22. Given the two force vectors $\mathbf{F}_1 = 20.0$ N at an angle of 30.0° with the positive x-axis and $\mathbf{F}_2 = 40.0$ N at an angle of 150.0° with the positive x-axis, find the magnitude and direction of a third force that when added to $\mathbf{F}_1$ and $\mathbf{F}_2$ gives a zero resultant.
23. When vector **A**, of magnitude 5.00 m/s at an angle of 120° with respect to the positive x-axis, is added to a second vector **B**, the resultant vector has a magnitude $R = 8.00$ m/s and is at an angle of 85.0° with the positive x-axis. Find the vector **B**.
24. A car travels 100 mi due west and then 45 mi due north. How far is the car from its starting point? Solve graphically and analytically.
25. Find the resultant of the following forces graphically and analytically: 5 lb at an angle of 33° above the horizontal and 20 lb at an angle of 97° counterclockwise from the horizontal.
26. Find the resultant of the following forces graphically and analytically: 25 N at an angle of 53° above the horizontal and 100 N at an angle of 117° counterclockwise from the horizontal.
†27. The velocity of an aircraft is 200 km/hr due west. A northwest wind of 50 km/hr is blowing. (a) What is the velocity of the aircraft relative to the ground? (b) If the pilot's destination is due west, at what angle should he point his plane to get there? (c) If his destination is 400 km due west, how long will it take him to get there?
28. A plane flies east for 50.0 km, then at an angle of 30.0° north of east for 75.0 km. In what direction should it now fly and how far, such that it will be 200 km northwest of its original position?
†29. The current in a river flows north at 5 mph. A boat starts straight across the river at 8 mph relative to the water. (a) What is the speed of the boat relative to the land? (b) If the river is 2 mi wide, how long does it take the boat to cross the river? (c) If the boat sets out straight for the opposite side, how far north will it reach the opposite shore? (d) If we want to have the boat go straight across the river, at what angle should the boat be headed?

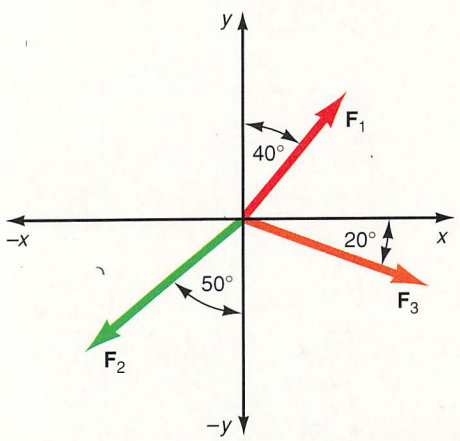
30. The current in a river flows south at 7 km/hr. A boat starts straight across the river at 19 km/hr relative to the water. (a) What is the speed of the boat relative to the land? (b) If the river is 1.5 km wide, how long does it take the boat to cross the river? (c) If the boat sets out straight for the opposite side, how far south will it reach the opposite shore? (d) If we want to have the boat go straight across the river, at what angle should the boat be headed?

†31. Show that if the angle between vectors **a** and **b** is an acute angle, then the sum **a** + **b** becomes the main diagonal of the parallelogram and the difference **a** − **b** becomes the minor diagonal of the parallelogram. Also show that if the angle is obtuse the results are reversed.

32. Find the resultant of the following three vectors. The magnitudes of the vectors are $a = 5.00$ km, $b = 10.0$ km, and $c = 20.0$ km.



33. Find the resultant of the following three forces. The magnitudes of the forces are $F_1 = 2.00$ N, $F_2 = 8.00$ N, and $F_3 = 6.00$ N.



†34. Show that for three nonparallel vectors all in the same plane, any one of them can be represented as a linear sum of the other two.

†35. A unit vector is a vector that has a magnitude of one unit and is in a specified direction. If a unit vector **i** is defined to be in the x-direction, and a unit vector **j** is defined to be in the y-direction, show that any vector **a** can be written in the form

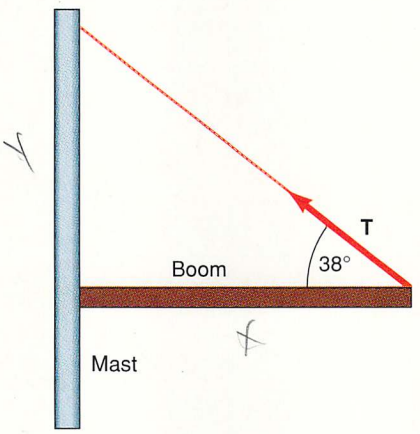
$$\mathbf{a} = a_x\mathbf{i} + a_y\mathbf{j}$$

†36. Prove that $|\mathbf{a} + \mathbf{b}| \leq |\mathbf{a}| + |\mathbf{b}|$.

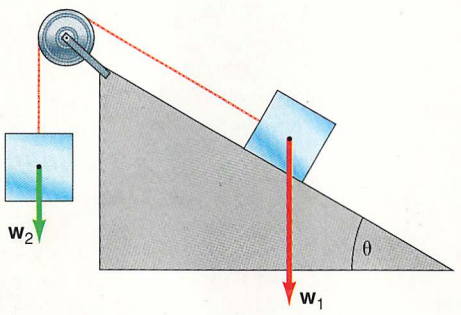
37. An airplane flies due east at 200 mph straight from city A to city B a distance of 200 mi. A wind of 40 mph from the northwest is blowing. If the pilot doesn't make any corrections, where will the plane be in 1 hr?

38. Given vectors **a** and **b**, where $a = 50$, $\theta_1 = 33°$, $b = 80$, and $\theta_2 = 128°$, find (a) **a** + **b**, (b) **a** − **b**, (c) **a** − 2**b**, (d) 3**a** + **b**, (e) 2**a** − **b**, and (f) 2**b** − **a**.

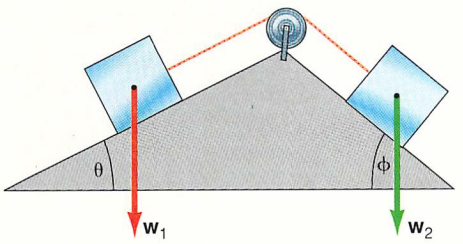
39. In the accompanying figure the tension T in the cable is 200 N. Find the vertical component T_y and the horizontal component T_x of this tension.



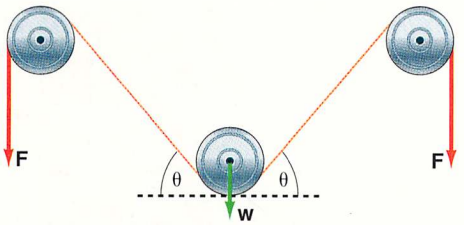
†40. In the accompanying diagram w_1 is 5 lb and w_2 is 3 lb. Find the angle θ such that the component of w_1 parallel to the incline is equal to w_2.



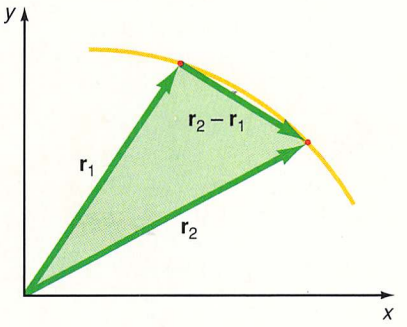
†41. In the accompanying diagram $w_1 = 2$ N, $w_2 = 5$ N, and $\theta = 65°$. Find the angle ϕ such that the components of the two forces parallel to the inclines are equal.



†42. In the accompanying diagram $w = 50$ lb, and $\theta = 10°$. What must be the value of F such that w will be held in place? What happens if the angle is doubled to 20°?



†43. In projectile motion in two dimensions the projectile is located by the displacement vector $\mathbf{r}_1$ at the time t_1 and by the displacement vector $\mathbf{r}_2$ at t_2, as shown in the diagram. If $r_1 = 20$ m, $\theta_1 = 60°$, $r_2 = 25$ m, and $\theta_2 = 25°$, find the magnitude and direction of the vector $\mathbf{r}_2 - \mathbf{r}_1$.



Interactive Tutorials

44. A 50.0-N force is directed at an angle of 50° above the horizontal. Resolve this force into vertical and horizontal components.

45. Find the resultant of any number of force vectors (up to five vectors).