

Figure 2

Russell traction.

in traction is that the tension in the taut connecting rope is everywhere the same. Thus, the forces exerted on the bone are the tensions T_1 , T_2 , T_3 , the weight of the leg w_L , and the force exerted by the muscles F_M . The first condition of equilibrium applied to the leg yields

$$\Sigma F_y = 0$$

= $T_1 \sin \theta + T_3 - T_2 \sin \theta - w_L = 0$ (5H.1)

The function of the pulleys is to change the direction of the force, but the tension in the rope is everywhere the same. But the ten-

The Language of Physics

Statics

That portion of the study of mechanics that deals with bodies in equilibrium (p. 121).

Equilibrium

A body is said to be in equilibrium under the action of several forces if the body has zero translational acceleration and no rotational motion (p. 121).

The first condition of equilibrium

For a body to be in equilibrium the vector sum of all the forces acting on the body must be zero. This can also be stated as: a body is in equilibrium if the sum of all the forces in the x-direction is equal to zero and the sum of all the forces in the ydirection is equal to zero (p. 121).

Torque

Torque is defined as the product of the force times the lever arm. Whenever an unbalanced torque acts on a body at rest, it will put that body into rotational motion (p. 127).

Lever arm

The lever arm is defined as the perpendicular distance from the axis of rotation to the direction or line of action of the force. If the force acts through the axis of rotation of the body, it has a zero lever arm and cannot cause a torque to act on the body (p. 127).

The second condition of equilibrium

In order for a body to be in rotational equilibrium, the sum of the torques acting on the body must be equal to zero. This can also be stated as: the necessary condition for a body to be in rotational equilibrium is that the sum of all the torques clockwise must be equal to the sum of all the torques counterclockwise (p. 130).

Center of gravity (cg)

The point that behaves as though the entire weight of the body is located at that point. For a body with a uniform mass distribution located in a uniform

sion T is supplied by the weight w that is hung from the end of the bed and is thus equal to the weight w. Hence,

$$T_1 = T_2 = T_3 = w (5H.2)$$

Equation 5H.1 now becomes

$$w\sin\theta + w - w\sin\theta - w_{\rm L} = 0$$

or

$$v = w_{\rm L}$$
 (5H.3)

Thus the weight w hung from the bottom of the bed must be equal to the weight of the leg w_L .

n

The second equation of the first condition of equilibrium is

0

$$\Sigma F_x = 0$$

- $T_1 \cos \theta - T_2 \cos \theta = 0$ (5H 4)

 $F_{\rm M} - T_1 \cos \theta - T$ Using equation 5H.2 this becomes

$$F_{\rm M} - w\cos\theta - w\cos\theta =$$

$$F_{\rm M} = w\cos\theta + w\cos\theta$$

Thus,

$$F_{\rm M} = 2w\cos\theta \tag{5H.5}$$

which says that by varying the angle θ , the force to overcome muscle contraction can be varied to any value desired. In this analysis, the force exerted to overcome the muscle contraction lies along the axis of the bone. Variations of this technique can be used if we want to have the traction force exerted at any angle because of the nature of the medical problem.

> gravitational field, the center of gravity is located at the geometrical center of the body (p. 134).

Center of mass (cm)

The point of a body at which all the mass of the body is assumed to be concentrated. For a body with a uniform mass distribution, the center of mass coincides with the geometrical center of the body. When external forces act on a body to put the body into translational motion, all the forces can be considered to act at the center of mass of the body. For a body in a uniform gravitational field, the center of gravity coincides with the center of mass of the body (p. 138).

Summary of Important Equations

First condition of equilibrium $\Sigma \mathbf{F} = 0$	(5.1)	Second condition of equilibrium $\Sigma \tau = 0$	(5.25)	$y_{\rm cg} = \frac{\sum w_i y_i}{W}$	(5.32)
First condition of equilibrium $\sum F_x = 0$	(5.4)	Second condition of equilibrium	Ageno groon in the solution of	Center of mass $x_{\rm cm} = \frac{\sum m_i x_i}{\sum} = \frac{\sum m_i x_i}{\sum}$	(5.34)
$Z F_y = 0$ Torque	(5.21)	$\Sigma \tau_{cw} = \Sigma \tau_{ccw}$ Center of gravity	(5.24)	$y_{\rm cm} = \frac{\sum m_i y_i}{\sum m_i} = \frac{\sum m_i y_i}{M}$	(5.35)
$\begin{aligned} \tau &= r_{\perp}r = rF_{\perp} \\ &= rF\sin\theta \end{aligned}$	(3.21)	$x_{\rm cg} = \frac{\Sigma w_i x_i}{W}$	(5.31)		

Questions for Chapter 5

- 1. Why can a body moving at constant velocity be considered as a body in equilibrium?
- 2. Why cannot an accelerated body be considered as in equilibrium?
- 3. Why can a point outside the body in equilibrium be considered as an axis to compute torques?
- 4. What is the difference between the center of mass of a body and its center of gravity?
- 5. A ladder is resting against a wall and a person climbs up the ladder. Is the ladder more likely to slip out at the bottom as the person climbs closer to the top of the ladder? Explain.
- 6. When flying an airplane a pilot frequently changes from the fuel tank in the right wing to the fuel tank in the left wing. Why does he do this?

- 7. Where would you expect the center of gravity of a sphere to be located? A cylinder?
- * 8. When lifting heavy objects why is it said that you should bend your knees and lift with your legs instead of your back? Explain.
 - 9. A short box and a tall box are sitting on the floor of a truck. If the truck makes a sudden stop, which box is more likely to tumble over? Why?
- **†10.** A person is sitting at the end of a row boat that is at rest in the middle of the lake. If the person gets up and walks toward the front of the boat, what will happen to the boat? Explain in terms of the center of mass of the system.

- 11. Is it possible for the center of gravity of a body to lie outside of the body? (*Hint*: consider a doughnut.)
- **†12.** Why does an obese person have more trouble with lower back problems than a thin person?
- 13. Describe how a lever works in terms of the concept of torque.
- **†14.** Describe how you could determine the center of gravity of an irregular body such as a plate, experimentally.
- **†15.** Engineers often talk about the moment of a force acting on a body. Is there any difference between the concept of a torque acting on a body and the moment of a force acting on a body?

Problems for Chapter 5

5.1 The First Condition of Equilibrium

- In a laboratory experiment on a force table, three forces are in equilibrium. One force of 0.300 N acts at an angle of 40.0°. A second force of 0.800 N acts at an angle of 120°. What is the magnitude and direction of the force that causes equilibrium?
- 2. Two ropes each 10.0 ft long are attached to the ceiling at two points located 15.0 ft apart. The ropes are tied together in a knot at their lower end and a load of 70.0 lb is hung on the knot. What is the tension in each rope?
- **3.** What force must be applied parallel to the plane to make the block move up the frictionless plane at constant speed?



4. Two ropes are attached to the ceiling as shown, making angles of 40.0° and 20.0°. A weight of 100 N is hung from the knot. What is the tension in each rope?



5. Find the force *F*, parallel to the frictionless plane, that will allow the system to move at constant speed.



- 6. A weightless rope is stretched horizontally between two poles 25.0 ft apart. Spiderman, who weighs 160 lb, balances himself at the center of the rope, and the rope is observed to sag 0.500 ft at the center. Find the tension in each part of the rope.
- 7. A weightless rope is stretched horizontally between two poles 25.0 ft apart. Spiderman, who weighs 160 lb, balances himself 5.00 ft from one end, and the rope is observed to sag 0.300 ft there. What is the tension in each part of the rope?
- 8. A force of 15.0 N is applied to a 15.0-N block on a rough inclined plane that makes an angle of 52.0° with the horizontal. The force is parallel to the plane. The block moves up the plane at constant velocity. Find the coefficient of kinetic friction between the block and the plane.
- 9. With what force must a 5.00-N eraser be pressed against a blackboard for it to be in static equilibrium? The coefficient of static friction between the board and the eraser is 0.250.
- 10. A traffic light, weighing 150 lb is hung from the center of a cable of negligible weight that is stretched horizontally between two poles that are 60.0 ft apart. The cable is observed to sag 2.00 ft. What is the tension in the cable?
- 11. A traffic light that weighs 600 N is hung from the cable as shown. What is the tension in each cable? Assume the cable to be massless.



12. Your car is stuck in a snow drift. You attach one end of a 50.0-ft rope to the front of the car and attach the other end to a nearby tree, as shown in the figure. If you can exert a force of 150 lb on the center of the rope, thereby displacing it 3.00 ft to the side, what will be the force exerted on the car?



†13. What force is indicated on the scale in part a and part b of the diagram if $m_1 = m_2 = 20.0 \text{ kg}$?





†14. Find the tension in each cord of the figure, if the block weighs 100 N.

(b)



5.2 The Concept of Torque

- **15.** A force of 10.0 lb is applied to a door knob perpendicular to a 30.0-in. door. What torque is produced to open the door?
- 16. A horizontal force of 10.0 lb is applied at an angle of 40.0° to the door knob of a 30.0-in. door. What torque is produced to open the door?
- 17. A horizontal force of 50.0 N is applied at an angle of 28.5° to a door knob of a 75.0-cm door. What torque is produced to open the door?
- 18. A door knob is placed in the center of a 30.0-in. door. If a force of 10.0 lb is exerted perpendicular to the door at the knob, what torque is produced to open the door?
- 19. Compute the net torque acting on the pulley in the diagram if the radius of the pulley is 0.250 m and the tensions are $T_1 = 50.0$ N and $T_2 = 30.0$ N.



20. Find the torque produced by the bicycle pedal in the diagram if the force F = 25.0 lb, the radius of the crank r = 7.00 inches, and angle $\theta = 37.0^{\circ}$.



5.5 Examples of Rigid Bodies in Equilibrium

Parallel Forces

21. Two men are carrying a 25.0-ft telephone pole that weighs 200 lb. If the center of gravity of the pole is 10.0 ft from the right end, and the men lift the pole at the ends, how much weight must each man support?

- 22. Two men are carrying a 9.00-m telephone pole that has a mass of 115 kg.*If the center of gravity of the pole is 3.00 m from the right end, and the men lift the pole at the ends, how much weight must each man support?
- 23. A uniform board that is 5.00 m long and weighs 450 N is supported by two wooden horses, 0.500 m from each end. If a 800-N person stands on the board 2.00 m from the right end, what force will be exerted on each wooden horse?
- 24. A 300-N boy and a 250-N girl sit at opposite ends of a 4.00-m seesaw. Where should another 250-N girl sit in order to balance the seesaw?
- 25. A uniform beam 10.0 ft long and weighing 15.0 lb carries a load of 20.0 lb at one end and 45.0 lb at the other end. It is held horizontal, while resting on a wooden horse 4.00 ft from the heavier load. What torque must be applied to keep it at rest in this position?
- 26. A uniform beam 3.50 m long and weighing 90.0 N carries a load of 110 N at one end and 225 N at the other end. It is held horizontal, while resting on a wooden horse 1.50 m from the heavier load. What torque must be applied to keep it at rest in this position?
- 27. A uniform pole 5.00 m long and weighing 100 N is to be carried at its ends by a man and his son. Where should a 250-N load be hung on the pole, such that the father will carry twice the load of his son?
- 28. A meter stick is hung from two scales that are located at the 20.0and 70.0-cm marks of the meter stick. Weights of 2.00 N are placed at the 10.0- and 40.0-cm marks, while a weight of 1.00 N is placed at the 90.0-cm mark. The weight of the uniform meter stick is 1.50 N. Determine the scale readings at A and B in the diagram.



Center of Gravity of a Body

- **29.** A tapered pole 10.0 ft long weighs 25.0 lb. The pole balances at its midpoint when a 5.00-lb weight hangs from the slimmer end. Where is the center of gravity of the pole?
- **†30.** A loaded wheelbarrow that weighs 75.0 lb has its center of gravity 2.00 ft from the front wheel axis. If the distance from the wheel axis to the end of the handles is 6.00 ft, how much of the weight of the wheelbarrow is supported by each arm?
- **†31.** Find the center of gravity of the carpenters square shown in the diagram.



- **32.** The front and rear axles of a 3000-lb car are 8.00 ft apart. If the center of gravity of the car is located 3.00 ft behind the front axle, find the load supported by the front and rear wheels of the car.
- **33.** The front and rear axles of a 1110-kg car are 2.50 m apart. If the center of gravity of the car is located 1.15 m behind the front axle, find the load supported by the front and rear wheels of the car.
- 34. A very bright but lonesome child decides to make a seesaw for one. The child has a large plank, and a wooden horse to act as a fulcrum. Where should the child place the fulcrum, such that the plank will balance, when the child is sitting on the end? The child weighs 60.0 lb and the plank weighs 40.0 lb and is 10.0 ft long. (*Hint:* find the center of gravity of the system.)

Center of Mass

- **35.** Four masses of 20.0, 40.0, 60.0, and 80.0 g are located at the respective distances of 10.0, 20.0, 30.0, and 40.0 cm from an origin. Find the center of mass of the system.
- 36. Three masses of 15.0, 45.0, and 25.0 g are located on the x-axis at 10.0, 25.0, and 45.0 cm. Two masses of 25.0 and 33.0 g are located on the y-axis at 35.0 and 50.0 cm, respectively. Find the center of mass of the system.

†37. A 1.00-kg circular metal plate of radius 0.500 m has attached to it a smaller circular plate of the same material of 0.100 m radius, as shown in the diagram. Find the center of mass of the combination with respect to the center of the large plate.



†38. This is the same problem as 37 except the smaller circle of material is removed from the larger plate. Where is the center of mass now?

Crane Boom Problems

39. A horizontal uniform boom that weighs 200 N and is 5.00 m long supports a load w_L of 1000 N, as shown in the figure. Find all the forces acting on the boom.



40. A horizontal, uniform boom 4.00 m long that weighs 200 N supports a load w_L of 1000 N. A guy wire that helps to support the boom, is attached 1.00 m in from the end of the boom. Find all the forces acting on the boom.



Diagram for problem 40.

- **41.** A horizontal, uniform boom 4.50 m long that weighs 250 N supports a 3500 N load w_L . A guy wire that helps to support the boom is attached 1.0 m in from the end of the boom, as in the diagram for problem 40. If the maximum tension that the cable can withstand is 3500 N, how far out on the boom can a 95.0-kg repairman walk without the cable breaking?
- 42. A uniform beam 4.00 m long that weighs 200 N is supported, as shown in the figure. The boom lifts a load w_L of 1000 N. Find all the forces acting on the boom.



†43. A uniform beam 4.00 m long that weighs 200 N is supported, as shown in the figure. The boom lifts a load w_L of 1000 N. Find all the forces acting on the boom.



44. An 80.0-lb sign is hung on a uniform steel pole that weighs 25.0 lb, as shown in the figure. Find all the forces acting on the boom.



Ladder Problems

- **45.** A uniform ladder 6.00 m long weighing 120 N leans against a frictionless wall. The base of the ladder is 1.00 m away from the wall. Find all the forces acting on the ladder.
- **46.** A uniform ladder 6.00 m long weighing 120 N leans against a frictionless wall. A girl weighing 400 N climbs three-fourths of the way up the ladder. If the base of the ladder makes an angle of 75.0° with the ground, find all the forces acting on the ladder. Compute all torques about the base of the ladder.
- **47.** Repeat problem 46, but compute all torques about the top of the ladder. Is there any difference in the results of the problem?
- **48.** A uniform ladder 15.0 ft long weighing 25.0 lb leans against a frictionless wall. If the base of the ladder makes an angle of 40.0° with the ground, what is the minimum coefficient of friction between the ladder and the ground such that the ladder will not slip out?
- **†49.** A uniform ladder 20.0 ft long weighing 35.0 lb leans against a frictionless wall. The base of the ladder makes an angle of 60.0° with the ground. If the coefficient of friction between the ladder and the ground is 0.300, how high can a 160-lb man climb the ladder before the ladder starts to slip?
- **50.** A uniform ladder 5.50 m long with a mass of 12.5 kg leans against a frictionless wall. The base of the ladder makes an angle of 48.0° with the ground. If the coefficient of friction between the ladder and the ground is 0.300, how high can a 82.3-kg man climb the ladder before the ladder starts to slip?

Applications to the Health Sciences

- 51. A weight lifter is lifting a dumbbell as in the example shown in figure 5.22 only now the forearm makes an angle of 30.0° with the horizontal. Using the same data as for that problem find the force $F_{\rm M}$ exerted by the biceps muscle and the reaction force at the elbow joint $F_{\rm J}$. Assume that the force $F_{\rm M}$ remains perpendicular to the arm.
- 52. Consider the weight lifter in the example shown in figure 5.23. Determine the forces $F_{\rm M}$ and $F_{\rm R}$ if the angle $\theta = 00.0^{\circ}$.
- **†53.** The weight of the upper body of the person in the accompanying diagram acts downward about 8.00 cm in front of the fifth lumbar vertebra. This weight produces a torque about the fifth lumbar vertebra. To counterbalance this torque the muscles in the lower back exert a force F_{M} that produces a counter torque. These muscles exert their force about 5.00 cm behind the fifth lumbar vertebra. If the person weighs 180 lb find the force exerted by the lower back muscles F_{M} and the reaction force $F_{\mathbf{R}}$ that the sacrum exerts upward on the fifth lumbar vertebra. The weight of the upper portion of the body is about 65% of the total body weight.



†54. Consider the same situation as in problem 53 except that the person is overweight. The center of gravity with the additional weight is now located 15.0 cm in front of the fifth lumbar vertebra instead of the previous 8.00 cm. Hence a greater torque will be exerted by this additional weight. The distance of the lower back muscles is only slightly greater at 6.00 cm. If the person weighs 240 lb find the force $F_{\rm R}$ on the fifth lumbar vertebra and the force $F_{\rm M}$ exerted by the lower back muscles.



†55. A 668-N person stands evenly on the balls of both feet. The Achilles tendon, which is located at the back of the ankle, provides a tension T_A to help balance the weight of the body as seen in the diagram. The distance from the ball of the foot to the Achilles tendon is approximately 18.0 cm. The tibia leg bone pushes down on the foot with a force $F_{\rm T}$. The distance from the tibia to the ball of the foot is about 14.0 cm. The ground exerts a reaction force F_N upward on the ball of the foot that is equal to half of the body weight. Draw a free body diagram of the forces acting and determine the force exerted by the Achilles tendon and the tibia.



Chapter 5 Equilibrium

Additional Problems

†56. If w weighs 100 N, find (a) the tension in ropes 1, 2, and 3 and (b) the tension in ropes 4, 5, and 6. The angle $\theta = 52.0^{\circ}$ and the angle $\phi = 33.0^{\circ}$.



†57. Block A rests on a table and is connected to another block B by a rope that is also connected to a wall. If $M_A = 15.0$ kg and $\mu_s = 0.200$, what must be the value of M_B to start the system into motion?



58. In the pulley system shown, what force F is necessary to keep the system in equilibrium?



†59. A sling is used to support a leg as shown in the diagram. The leg is elevated at an angle of 20.0°. The bed exerts a reaction force **R** on the thigh as shown. The weight of the thigh, leg, and ankle are given by w_T, w_L, and w_A, respectively, and the locations of these weights are as shown. The sling is located 27.0 in. from the point O in the diagram. A free body diagram is shown in part b of the diagram. Find the weight w that is necessary to put the leg into equilibrium.



- (b) ***60.** Find the tensions T_1 , T, and T_2 in the figure if $w_1 = 500$ N and $w_2 = 300$
 - figure if $w_1 = 500$ N and $w_2 = 300$ N. The angle $\theta = 35.0^\circ$ and the angle $\phi = 25.0^\circ$.



***61.** The steering wheel of an auto has a diameter of 18.0 in. The axle that it is connected to has a diameter of 2.00 in. If a force of 25.0 lb is exerted on the rim of the wheel, (a) what is the torque exerted on the steering wheel, (b) what is the torque exerted on the axle, and (c) what force is exerted on the rim of the axle?

- 62. One type of simple machine is called a wheel and axle. A wheel of radius 35.0 cm is connected to an axle of 2.00 cm radius. A force of $F_{in} =$ 10.0 N is applied tangentially to the wheel. What force F_{out} is exerted on the axle? The ratio of the output force F_{out} to the input force F_{in} is called the ideal mechanical advantage (IMA) of the system. Find the IMA of this system.
- **†63.** A box 1.00 m on a side rests on a floor next to a small piece of wood that is fixed to the floor. The box weighs 500 N. At what height h should a force of 400 N be applied so as to just tip the box?
- **†64.** A 200-N door, 0.760 m wide and 2.00 m long, is hung by two hinges. The top hinge is located 0.230 m down from the top, while the bottom hinge is located 0.330 m up from the bottom. Assume that the center of gravity of the door is at its geometrical center. Find the horizontal force exerted by each hinge on the door.
- **†65.** A uniform ladder 6.00 m long weighing 100 N leans against a frictionless wall. If the coefficient of friction between the ladder and the ground is 0.400, what is the smallest angle θ that the ladder can make with the ground before the ladder starts to slip?
- **†66.** If an 800-N man wants to climb a distance of 5.00 m up the ladder of problem 65, what angle θ should the ladder make with the ground such that the ladder will not slip?

- †67. A uniform ladder 20.0 ft long weighing 30.0 lb leans against a rough wall, that is, a wall where there is a frictional force between the top of the ladder and the wall. The coefficient of static friction is 0.400. If the base of the ladder makes an angle θ of 40.0° with the ground when the ladder begins to slip down the wall, find all the forces acting on the ladder. (Hint: With a rough wall there will be a vertical force f_s acting upward at the top of the ladder. In general, this force is unknown but we do know that it must be less than $\mu_{s}F_{N}$. At the moment the ladder starts to slip, this frictional force is known and is given by the equation of static friction, namely, $f_s = \mu_s F_N$ $= \mu_{s}F$. Although there are now four unknowns, there are also four equations to solve for them.)
- **†68.** A 225-lb person stands threequarters of the way up a stepladder. The step side weighs 20.0 lb, is 6.00 ft long, and is uniform. The rear side weighs 10.0 lb, is also uniform, and is also 6.00 ft long. A hinge connects the front and back of the ladder at the top. A weightless tie rod, 1.50 ft in length, is connected 2.00 ft from the top of the ladder. Find the forces exerted by the floor on the ladder and the tension in the tie rod.

Interactive Tutorials

- **Goldstate Goldstate Gold**
- **270.** A uniform beam of length L = 10.0m and mass m = 5.00 kg is held up at each end by a force F_A (at 0.00 m) and force F_B (at 10.0 m). If a weight W = 400 N is placed at the position x = 8.00 m, calculate forces F_A and F_B .
- 71. The crane boom. A uniform boom of weight $w_{\rm B} = 250$ N and length l =8.00 m is connected to the mast by a hinge pin at the point A in figure 5.20. A load $w_{\rm L} = 1200$ N is supported at the other end. A cable is connected at the end of the boom making an angle $\theta = 55.0^{\circ}$, as shown in the diagram. Find the tension T in the cable and the vertical V and horizontal H forces that the hinge pin exerts on the boom.
- □ 72. A uniform ladder of weight $w_1 = 100$ N and length L = 20.0 m leans against a frictionless wall at a base angle $\theta = 60.0^{\circ}$. A person weighing $w_p = 150$ N climbs the ladder a distance d = 6.00 m from the base of the ladder. Calculate the horizontal H and vertical V forces acting on the ladder, and the force F exerted by the wall on the top of the ladder.