



Solution

- a. This is an example of a perfectly inelastic collision in two dimensions. The law of conservation of momentum yields

$$\mathbf{p}_f = \mathbf{p}_i$$

$$(m_1 + m_2)\mathbf{V}_f = m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} \quad (8.61)$$

Resolving this equation into its x - and y -component equations, we get for the x -component:

$$(m_1 + m_2)V_f \cos \theta = m_1v_{1i} \quad (8.62)$$

and for the y -component:

$$(m_1 + m_2)V_f \sin \theta = m_2v_{2i} \quad (8.63)$$

Dividing the y -component equation by the x -component equation we get

$$\frac{(m_1 + m_2)V_f \sin \theta}{(m_1 + m_2)V_f \cos \theta} = \frac{m_2v_{2i}}{m_1v_{1i}}$$

$$\frac{\sin \theta}{\cos \theta} = \frac{m_2v_{2i}}{m_1v_{1i}}$$

$$\tan \theta = \frac{m_2v_{2i}}{m_1v_{1i}}$$

$$\tan \theta = \frac{(1400 \text{ kg})(100 \text{ km/hr})}{(1200 \text{ kg})(95.0 \text{ km/hr})}$$

$$\theta = 50.8^\circ$$

- b. The combined final speed, found by solving for V_f in equation 8.62, is

$$V_f = \frac{m_1v_{1i}}{(m_1 + m_2)\cos \theta}$$

$$= \frac{(1200 \text{ kg})(95.0 \text{ km/hr})}{(1200 \text{ kg} + 1400 \text{ kg})\cos 50.8^\circ}$$

$$= 69.4 \text{ km/hr}$$

The Language of Physics

Linear momentum

The product of the mass of the body in motion times its velocity (p. 215).

Newton's second law in terms of linear momentum

When a resultant applied force acts on a body, it causes the linear momentum of that body to change with time (p. 216).

External forces

Forces that originate outside the system and act on the system (p. 216).

Internal forces

Forces that originate within the system and act on the particles within the system (p. 216).

Law of conservation of linear momentum

If the total external force acting on a system is equal to zero, then the final value of the total momentum of the system is

equal to the initial value of the total momentum of the system. Thus, the total momentum is a constant, or as usually stated, the total momentum is conserved. The law of conservation of momentum is a consequence of Newton's third law (p. 217).

Impulse

The product of the force that is acting and the time that the force is acting. The impulse acting on a body is equal to the change in momentum of the body (p. 222).

Perfectly elastic collision

A collision in which no kinetic energy is lost, that is, the kinetic energy is conserved. Momentum is conserved in all collisions for which there are no external forces. In this type of collision, the velocity of separation of the two bodies is equal to the velocity of approach (p. 223).

Inelastic collision

A collision in which some kinetic energy is lost. The velocity of separation of the two bodies in this type of collision is not equal to the velocity of approach. The coefficient of restitution is a measure of the inelastic collision (p. 223).

Perfectly inelastic collision

A collision in which the two objects stick together during the collision. A great deal of kinetic energy is usually lost in this type of collision (p. 223).

Coefficient of restitution

The measure of the amount of the inelastic collision. It is equal to the ratio of the velocity of separation of the two bodies to the velocity of approach (p. 228).

Summary of Important Equations

Definition of momentum

$$\mathbf{p} = m\mathbf{v} \quad (8.1)$$

Newton's second law in terms of momentum

$$\mathbf{F} = \frac{\Delta \mathbf{p}}{\Delta t} \quad (8.5)$$

Law of conservation of momentum for

$$\mathbf{F}_{\text{net}} = 0$$

$$\mathbf{p}_f = \mathbf{p}_i \quad (8.7)$$

Recoil speed of a gun

$$v_G = \frac{m_B}{m_G} v_B \quad (8.14)$$

Impulse

$$\mathbf{J} = \mathbf{F}\Delta t \quad (8.18)$$

Impulse is equal to the change in momentum

$$\mathbf{J} = \Delta \mathbf{p} \quad (8.19)$$

Conservation of momentum in a collision

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \quad (8.22)$$

Conservation of momentum in scalar form, both bodies in motion in same direction, and $v_{1i} > v_{2i}$.

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \quad (8.23)$$

Conservation of energy in a perfectly elastic collision

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \quad (8.26)$$

Final velocity of ball 1 in a perfectly elastic collision

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2} \right) v_{2i} \quad (8.30)$$

Final velocity of ball 2 in a perfectly elastic collision

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2} \right) v_{1i} - \left(\frac{m_1 - m_2}{m_1 + m_2} \right) v_{2i} \quad (8.31)$$

The velocity of approach

$$v_{1i} - v_{2i} = V_A \quad (8.38)$$

The velocity of separation

$$v_{2f} - v_{1f} = V_S \quad (8.39)$$

For any collision

$$V_S = e V_A \quad (8.42)$$

For a perfectly elastic collision

$$e = 1$$

For an inelastic collision

$$0 < e < 1 \quad (8.43)$$

For a perfectly inelastic collision

$$e = 0$$

Perfectly inelastic collision

$$V_f = \left(\frac{m_1}{m_1 + m_2} \right) v_{1i} + \left(\frac{m_2}{m_1 + m_2} \right) v_{2i} \quad (8.54)$$

Questions for Chapter 8

- If the velocity of a moving body is doubled, what does this do to the kinetic energy and the momentum of the body?
- Why is Newton's second law in terms of momentum more appropriate than the form $F = ma$?
- State and discuss the law of conservation of momentum and show its relation to Newton's third law of motion.
- Discuss what is meant by an isolated system and how it is related to the law of conservation of momentum.
- Is it possible to have a collision in which all the kinetic energy is lost? Describe such a collision.
- An airplane is initially flying at a constant velocity in plane and level flight. If the throttle setting is not changed, explain what happens to the plane as it continues to burn its fuel?
- † In the early days of rocketry it was assumed by many people that a rocket would not work in outer space because there was no air for the exhaust gases to push against. Explain why the rocket does work in outer space.
- Discuss the possibility of a fourth type of collision, a super elastic collision, in which the particles have more kinetic energy after the collision than before. As an example, consider a car colliding with a truck loaded with dynamite.
- If the net force acting on a body is equal to zero, what happens to the center of mass of the body?
- † A bird is sitting on a swing in an enclosed bird cage that is resting on a mass balance. If the bird leaves the swing and flies around the cage without touching anything, does the balance show any change in its reading?
- From the point of view of impulse, explain why an egg thrown against a wall will break, while an egg thrown against a loose vertical sheet will not.

Problems for Chapter 8

8.1 Momentum

- What is the momentum of a 3200-lb car traveling at 55.0 mph?
- A 3200-lb car traveling at 55.0 mph collides with a tree and comes to a stop in 0.100 s. What is the change in momentum of the car? What average force acted on the car during impact? What is the impulse?
- Answer the same questions in problem 2 if the car hit a sand barrier in front of the tree and came to rest in 0.300 s.
- A 0.150-kg ball is thrown straight upward at an initial velocity of 30.0 m/s. Two seconds later the ball has a velocity of 10.4 m/s. Find (a) the initial momentum of the ball, (b) the momentum of the ball at 2 s, (c) the force acting on the ball, and (d) the weight of the ball.
- How long must a force of 5.00 N act on a block of 3.00-kg mass in order to give it a velocity of 4.00 m/s?
- A force of 25.0 N acts on a 10.0-kg mass in the positive x -direction, while another force of 13.5 N acts in the negative x -direction. If the mass is initially at rest, find (a) the time rate of change of momentum, (b) the change in momentum after 1.85 s, and (c) the velocity of the mass at the end of 1.85 s.

8.2 and 8.3 Conservation of Momentum

7. A 10.0-g bullet is fired from a 5.00-kg rifle with a velocity of 300 m/s. What is the recoil velocity of the rifle?
8. In an ice skating show, a 200-lb man at rest pushes a 100-lb woman away from him at a speed of 4.00 ft/s. What happens to the man?
9. A 5000-kg cannon fires a shell of 3.00-kg mass with a velocity of 250 m/s. What is the recoil velocity of the cannon?
10. A cannon of 3.50×10^3 kg fires a shell of 2.50 kg with a muzzle speed of 300 m/s. What is the recoil velocity of the cannon?
11. A 150-lb boy at rest on roller skates throws a 2.00-lb ball horizontally with a speed of 25.0 ft/s. With what speed does the boy recoil?
12. An 80.0-kg astronaut pushes herself away from a 1200-kg space capsule at a velocity of 3.00 m/s. Find the recoil velocity of the space capsule.
13. A 200-lb man is standing in a 500-lb boat. The man walks forward at 3.00 ft/s relative to the water. What is the final velocity of the boat? Neglect any resistive force of the water on the boat.
14. A 78.5-kg man is standing in a 275-kg boat. The man walks forward at 1.25 m/s relative to the water. What is the final velocity of the boat? Neglect any resistive force of the water on the boat.
15. A water hose sprays 2 kg of water against the side of a building in 1 s. If the velocity of the water is 15 m/s, what force is exerted on the wall by the water? (Assume that the water does not bounce off the wall of the building.)

8.4 Impulse

16. A boy kicks a football with an average force of 20.0 lb for a time of 0.200 s. (a) What is the impulse? (b) What is the change in momentum of the football? (c) If the football has a mass of 250 g, what is the velocity of the football as it leaves the kicker's foot?
17. A boy kicks a football with an average force of 66.8 N for a time of 0.185 s. (a) What is the impulse? (b) What is the change in momentum of the football? (c) If the football has mass of 250 g, what is the velocity of the football as it leaves the kicker's foot?

18. A baseball traveling at 150 km/hr is struck by a bat and goes straight back to the pitcher at the same speed. If the baseball has a mass of 200 g, find (a) the change in momentum of the baseball, (b) the impulse imparted to the ball, and (c) the average force acting if the bat was in contact with the ball for 0.100 s.
19. A 10.0-kg hammer strikes a nail at a velocity of 12.5 m/s and comes to rest in a time interval of 0.004 s. Find (a) the impulse imparted to the nail and (b) the average force imparted to the nail.
20. If a gas molecule of mass 5.30×10^{-26} kg and an average speed of 425 m/s collides perpendicularly with a wall of a room and rebounds at the same speed, what is its change of momentum? What impulse is imparted to the wall?

8.5 Collisions in One Dimension

21. Two gliders moving toward each other, one of mass 200 g and the other of 250 g, collide on a frictionless air track. If the first glider has an initial velocity of 25.0 cm/s toward the right and the second of -35.0 cm/s toward the left, find the velocities after the collision if the collision is perfectly elastic.
22. A 250-g glider overtakes and collides with a 200-g glider on an air track. If the 250-g glider is moving at 35.0 cm/s and the second glider at 25.0 cm/s, find the velocities after the collision if the collision is perfectly elastic.
- †23. A 200-g ball makes a perfectly elastic collision with an unknown mass that is at rest. If the first ball rebounds with a final speed of $v_{1f} = \frac{1}{2}v_{1i}$, (a) what is the unknown mass, and (b) what is the final velocity of the unknown mass?
24. A 30.0-g ball, m_1 , collides perfectly elastically with a 20.0-g ball, m_2 . If the initial velocities are $v_{1i} = 50.0$ cm/s to the right and $v_{2i} = -30.0$ cm/s to the left, find the final velocities v_{1f} and v_{2f} . Compute the initial and final momenta. Compute the initial and final kinetic energies.
25. A 150-g ball moving at a velocity of 25.0 cm/s to the right collides with a 250-g ball moving at a velocity of 18.5 cm/s to the left. The collision is imperfectly elastic with a coefficient of restitution of 0.65. Find (a) the velocity of each ball after the collision, (b) the kinetic energy before the collision, (c) the kinetic

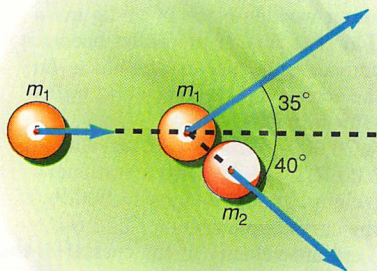
energy after the collision, and (d) the percentage of energy lost in the collision.

26. A 2000-lb car traveling at 60.0 mph collides "head-on" with a 20,000-lb truck traveling toward the car at 30.0 mph. The car becomes stuck to the truck during the collision. What is the final velocity of the car and truck?
27. A 1150-kg car traveling at 110 km/hr collides "head-on" with a 9500-kg truck traveling toward the car at 40.0 km/hr. The car becomes stuck to the truck during the collision. What is the final velocity of the car and truck?
28. A 3.00-g bullet is fired at 200 m/s into a wooden block of 10-kg mass that is at rest. If the bullet becomes embedded in the wooden block, find the velocity of the block and bullet after impact.
29. A 20,000-lb freight car traveling at 3.00 ft/s collides with a stationary freight car that weighs 15,000 lb. If the cars couple together find the resultant velocity of the cars after the collision.
30. A 9500-kg freight car traveling at 5.50 km/hr collides with an 8000-kg stationary freight car. If the cars couple together, find the resultant velocity of the cars after the collision.
31. Two gliders are moving toward each other on a frictionless air track. Glider 1 has a mass of 200 g and glider 2 of 250 g. The first glider has an initial speed of 25.0 cm/s while the second has a speed of 35.0 cm/s. If the collision is perfectly inelastic, find (a) the final velocity of the gliders, (b) the kinetic energy before the collision, and (c) the kinetic energy after the collision. (d) How much energy is lost, and where did it go?

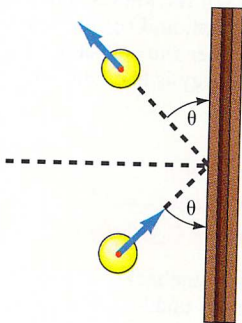
8.6 Collisions in Two Dimensions—Glancing Collisions

32. A 230-lb linebacker moving due east at 25 mph tackles a 175-lb halfback moving south at 40 mph. The two stick together during the collision. What is the resultant velocity of the two of them?
33. A 10,000-kg truck enters an intersection heading north at 45 km/hr when it makes a perfectly inelastic collision with a 1000-kg car traveling at 90 km/hr due east. What is the final velocity of the car and truck?

- †34. Billiard ball 2 is at rest when it is hit with a glancing collision by ball 1 moving at a velocity of 50.0 cm/s toward the right. After the collision ball 1 moves off at an angle of 35.0° from the original direction while ball 2 moves at an angle of 40.0°, as shown in the diagram. The mass of each billiard ball is 0.017 kg. Find the final velocity of each ball after the collision. Find the kinetic energy before and after the collision. Is the collision elastic?



35. A 0.150-kg ball, moving at a speed of 25.0 m/s, makes an elastic collision with a wall at an angle of 40.0°, and rebounds at an angle of 40.0°. Find (a) the change in momentum of the ball and (b) the magnitude and direction of the momentum imparted to the wall. The diagram is a view from the top.

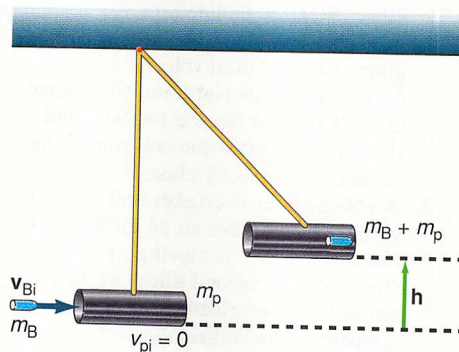


Additional Problems

- †36. A 0.250-kg ball is dropped from a height of 1.00 m. It rebounds to a height of 0.750 m. If the ground exerts a force of 300 N on the ball, find the time the ball is in contact with the ground.

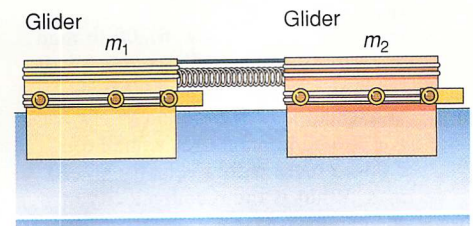
37. A 200-g ball is dropped from the top of a building. If the speed of the ball before impact is 40.0 m/s, and right after impact it is 25.0 cm/s, find (a) the momentum of the ball before impact, (b) the momentum of the ball after impact, (c) the kinetic energy of the ball before impact, (d) the kinetic energy of the ball after impact, and (e) the coefficient of restitution of the ball.

- †38. A ball is dropped from a height of 1.00 m and rebounds to a height of 0.920 m. Approximately how many bounces will the ball make before losing 90% of its energy?
39. A 60.0-g tennis ball is dropped from a height of 1.00 m. If it rebounds to a height of 0.560 m, (a) what is the coefficient of restitution of the tennis ball and the floor, and (b) how much energy is lost in the collision?
- †40. A 25.0-g bullet strikes a 5.00-kg ballistic pendulum that is initially at rest. The pendulum rises to a height of 14.0 cm. What is the initial speed of the bullet?



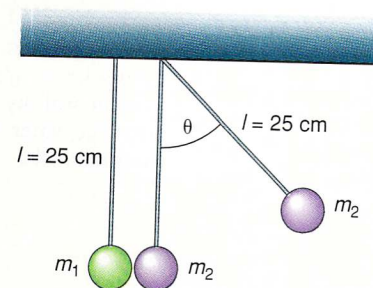
41. A 25.0-g bullet with an initial speed of 400 m/s strikes a 5-kg ballistic pendulum that is initially at rest. (a) What is the speed of the combined bullet-pendulum after the collision? (b) How high will the pendulum rise?
42. An 80-kg caveman, standing on a branch of a tree 5 m high, swings on a vine and catches a 60-kg cavegirl at the bottom of the swing. How high will both of them rise?
- †43. A hunter fires an automatic rifle at an attacking lion that weighs 300 lb. If the lion is moving toward the hunter at 10.0 ft/s, and the rifle bullets weigh 2.00 oz each and have a muzzle velocity of 2500 ft/s, how many bullets must the man fire at the lion in order to stop the lion in his tracks?
- †44. Two gliders on an air track are connected by a compressed spring and a piece of thread as shown; $m_1 = 300$ g and m_2 is unknown. If the

connecting string is cut, the gliders separate. Glider 1 experiences the velocity $v_1 = 10.0$ cm/s, and glider 2 experiences the velocity $v_2 = 20.0$ cm/s, what is the unknown mass?



Air track

- †45. Two gliders on an air track are connected by a compressed spring and a piece of thread as shown. The masses of the gliders are $m_1 = 300$ g and $m_2 = 250$ g. The connecting string is cut and the compressed spring causes the two gliders to separate from each other. If glider 1 has moved 35.0 cm from its starting point, where is glider 2 located?
- †46. Two balls, $m_1 = 100$ g and $m_2 = 200$ g, are suspended near each other as shown. The two balls are initially in contact. Ball 2 is then pulled away so that it makes a 45.0° angle with the vertical and is then released. (a) Find the velocity of ball 2 just before impact and the velocity of each ball after the perfectly elastic impact. (b) How high will each ball rise?



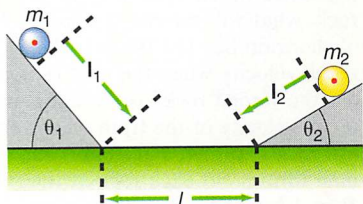
- †47. Two swimmers simultaneously dive off opposite ends of a 200-lb boat. If the first swimmer has a weight $w_1 = 200$ lb and a velocity of 5.00 ft/s toward the right, while the second swimmer has a weight $w_2 = 150$ lb and a velocity of -3.00 ft/s toward the left, what is the final velocity of the boat?
48. Two swimmers simultaneously dive off opposite ends of a 110-kg boat. If the first swimmer has a mass $m_1 = 66.7$ kg and a velocity of 1.98 m/s toward the right, while the second swimmer has a mass $m_2 = 77.8$ kg and a velocity of -7.63 m/s toward the left, what is the final velocity of the boat?

†49. Show that the kinetic energy of a moving body can be expressed in terms of the linear momentum as $KE = p^2/2m$.

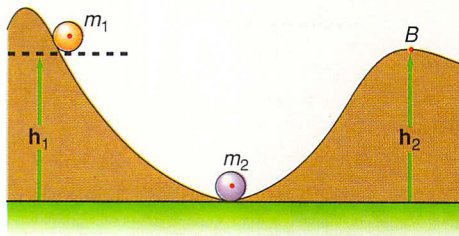
†50. A machine gun is mounted on a small train car and fires 100 bullets per minute backward. If the mass of each bullet is 10.0 g and the speed of each bullet as it leaves the gun is 900 m/s, find the average force exerted on the gun. If the mass of the car and machine gun is 225 kg, what is the acceleration of the train car while the gun is firing?

†51. An open toy railroad car of mass 250 g is moving at a constant speed of 30 cm/s when a wooden block of 50 g is dropped into the open car. What is the final speed of the car and block?

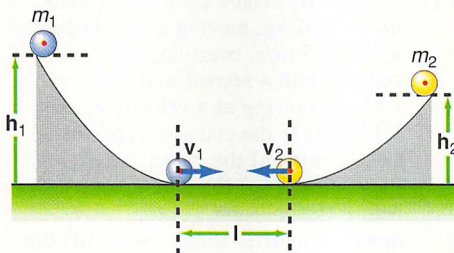
†52. Masses m_1 and m_2 are located on the top of the two frictionless inclined planes as shown in the diagram. It is given that $m_1 = 30.0$ g, $m_2 = 50.0$ g, $l_1 = 50.0$ cm, $l_2 = 20.0$ cm, $l = 100$ cm, $\theta_1 = 50.0^\circ$, and $\theta_2 = 25.0^\circ$. Find (a) the speeds v_1 and v_2 at the bottom of each inclined plane, note that ball 1 reaches the bottom of the plane before ball 2; (b) the position between the planes where the masses will collide elastically; (c) the speeds of the two masses after the collision; and (d) the final locations l'_1 and l'_2 where the two masses will rise up the plane after the collision.



†53. The mass $m_1 = 40.0$ g is initially located at a height $h_1 = 1.00$ m on the frictionless surface shown in the diagram. It is then released from rest and collides with the mass $m_2 = 70.0$ g, which is at rest at the bottom of the surface. After the collision, will the mass m_2 make it over the top of the hill at position B, which is at a height of 0.500 m?



†54. Two balls of mass m_1 and m_2 are placed on a frictionless surface as shown in the diagram. Mass $m_1 = 30.0$ g is at a height $h_1 = 50.0$ cm above the bottom of the bowl, while mass $m_2 = 60.0$ g is at a height of $3/4 h_1$. The distance $l = 100$ cm. Assuming that both balls reach the bottom at the same time, find (a) the speed of each ball at the bottom of each surface, (b) the position where the two balls collide, (c) the speed of each ball after the collision, and (d) the height that each ball will rise to after the collision.



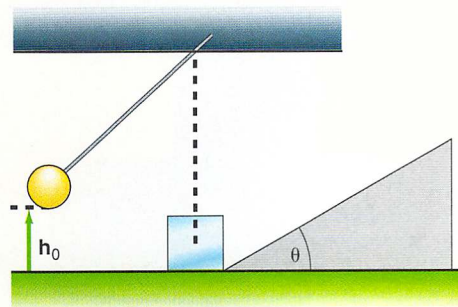
†55. A person is in a small train car that weighs 500 lb and contains 500 lb of rocks. The train is initially at rest. The person starts to throw large rocks, each weighing 100 lb, from the rear of the train at a speed of 5.00 ft/s. (a) If the person throws out 1 rock what will the recoil velocity of the train be? The person then throws out another rock at the same speed. (b) What is the recoil velocity now? (c) The person continues to throw out the rest of the rocks one at a time. What is the final velocity of the train when all the rocks have been thrown out?

†56. A bullet of mass 20.0 g is fired into a block of mass 5.00 kg that is initially at rest. The combined block and bullet moves a distance of 5.00 m over a rough surface of coefficient of kinetic friction of 0.500, before coming to rest. Find the initial velocity of the bullet.

†57. A bullet of mass 20.0 g is fired at an initial velocity of 200 m/s into a 15.0-kg block that is initially at rest. The combined bullet and block move over a rough surface of coefficient of kinetic friction of 0.500. How far will the combined bullet and block move before coming to rest?

58. A 15.0-kg bullet moving at a speed of 250 m/s hits a 2.00-kg block of wood, which is initially at rest. The bullet emerges from the block of wood at 150 m/s. Find (a) the final velocity of the block of wood and (b) the amount of energy lost in the collision.

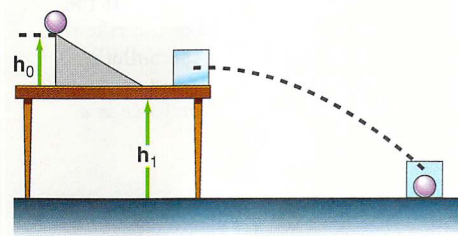
†59. A 5-kg pendulum bob, at a height of 0.750 m above the floor, swings down to the ground where it hits a 2.15 kg block that is initially at rest. The block then slides up a 30.0° incline. Find how far up the incline the block will slide if (a) the plane is frictionless and (b) if the plane is rough with a value of $\mu_k = 0.450$.



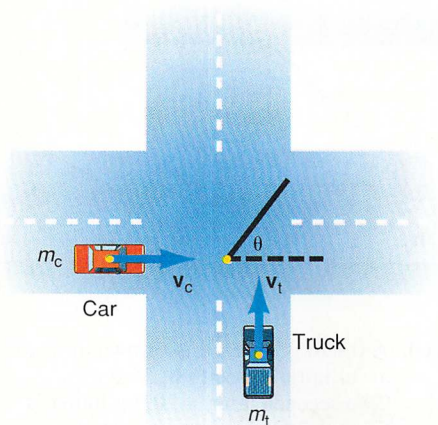
†60. A 0.15-kg baseball is thrown upward at an initial velocity of 35.0 m/s. Two seconds later, a 20.0-g bullet is fired at 250 m/s into the rising baseball. How high will the combined bullet and baseball rise?

†61. A 25-g ball slides down a smooth inclined plane, 0.850 m high, that makes an angle of 35.0° with the horizontal. The ball slides into an open box of 200-g mass and the ball and box slide on a rough surface of $\mu_k = 0.450$. How far will the combined ball and box move before coming to rest?

†62. A 25-g ball slides down a smooth inclined plane, 0.850 m high, that makes an angle of 35.0° with the horizontal. The ball slides into an open box of 200-g mass and the ball and box slide off the end of a table 1.00 m high. How far from the base of the table will the combined ball and box hit the ground?



- †63. A 1300-kg car collides with a 15,000-kg truck at an intersection and they couple together and move off as one leaving a skid mark 5 m long that makes an angle of 30.0° with the original direction of the car. If $\mu_k = 0.700$, find the initial velocities of the car and truck before the collision.



64. A bomb of mass $M = 2.50$ kg, moving in the x -direction at a speed of 10.5 m/s, explodes into three pieces. One fragment, $m_1 = 0.850$ kg, flies off at a velocity of 3.5 m/s at an angle of 30.0° above the x -axis. Fragment $m_2 = 0.750$ kg, flies off at an angle of 43.5° below the positive x -axis, and the third fragment flies off at an angle of 150° with respect to the positive x -axis. Find the velocities of m_2 and m_3 .

Interactive Tutorials

65. Recoil velocity of a gun. A bullet of mass $m_b = 10.0$ g is fired at a velocity $v_b = 300$ m/s from a rifle of mass $m_r = 5.00$ kg. Calculate the recoil velocity v_r of the rifle. If the bullet is in the barrel of the rifle for $t = 0.004$ s, what is the bullet's acceleration and what force acted on the bullet? Assume the force is a constant.

66. An inelastic collision. A car of mass $m_1 = 1000$ kg is moving at a velocity $v_1 = 50.0$ m/s and collides inelastically with a car of mass $m_2 = 750$ kg moving in the same direction at a velocity of $v_2 = 20.0$ m/s. Calculate (a) the final velocity v_f of both vehicles; (b) the initial momentum p_i ; (c) the final momentum p_f ; (d) the initial kinetic energy KE_i ; (e) the final kinetic energy KE_f of the system; (f) the energy lost in the collision ΔE ; and (g) the percentage of the original energy lost in the collision, $\%E_{\text{lost}}$.
67. A perfectly elastic collision. A mass, $m_1 = 3.57$ kg, moving at a velocity, $v_1 = 2.55$ m/s, overtakes and collides with a second mass, $m_2 = 1.95$ kg, moving at a velocity $v_2 = 1.35$ m/s. If the collision is perfectly elastic, find (a) the velocities after the collision, (b) the momentum before the collision, (c) the momentum after the collision, (d) the kinetic energy before the collision, and (e) the kinetic energy after the collision.
68. An imperfectly elastic collision. A mass, $m = 2.84$ kg, is dropped from a height $h_0 = 3.42$ m and hits a wooden floor. The mass rebounds to a height $h = 2.34$ m. If the collision is imperfectly elastic, find (a) the velocity of the mass as it hits the floor, v_{1i} ; (b) the velocity of the mass after it rebounds from the floor, v_{1f} ; (c) the coefficient of restitution, e ; (d) the kinetic energy, KE_A , just as the mass approached the floor; (e) the kinetic energy, KE_S , after the separation of the mass from the floor; (f) the actual energy lost in the collision; (g) the percentage of energy lost in the collision; (h) the momentum before the collision; and (i) the momentum after the collision.

69. An imperfectly elastic collision—the bouncing ball. A ball of mass, $m = 1.53$ kg, is dropped from a height $h_0 = 1.50$ m and hits a wooden floor. The collision with the floor is imperfectly elastic and the ball only rebounds to a height $h = 1.12$ m for the first bounce. Find (a) the initial velocity of the ball, v_i , as it hits the floor on its first bounce; (b) the velocity of the ball v_r , after it rebounds from the floor on its first bounce; (c) the coefficient of restitution, e ; (d) the initial kinetic energy, KE_i , just as the ball approaches the floor; (e) the final kinetic energy, KE_f , of the ball after the bounce from the floor; (f) the actual energy lost in the bounce, $E_{\text{lost/bounce}}$; and (g) the percentage of the initial kinetic energy lost by the ball in the bounce, $\%E_{\text{lost/bounce}}$. The ball continues to bounce until it loses all its energy. (h) Find the cumulative total percentage energy lost, $\% \text{ Energy lost}$, for all the bounces. (i) Plot a graph of the $\%$ of Total Energy lost as a function of the number of bounces.
70. A variable mass system. A train car of mass $m_T = 1500$ kg, contains 35 rocks each of mass $m_r = 30$ kg. The train is initially at rest. A man throws out each rock from the rear of the train at a speed $v_r = 8.50$ m/s. (a) When the man throws out one rock, what will the recoil velocity, V_T , of the train be? (b) What is the recoil velocity when the man throws out the second rock? (c) What is the recoil velocity of the train when the n th rock is thrown out? (d) If the man throws out each rock at the rate $R = 1.5$ rocks/s, find the change in the velocity of the train and its acceleration. (e) Draw a graph of the velocity of the train as a function of the number of rocks thrown out of the train. (f) Draw a graph of the acceleration of the train as a function of time.