Solution

The change in volume, found from equation 10.20, is

Δ

$$V = -\frac{V_0}{B}p$$

= $-\frac{(0.500 \text{ m}^3)(3.00 \times 10^5 \text{ N/m}^2)}{14 \times 10^{10} \text{ N/m}^2}$
= $-1.1 \times 10^{-6} \text{ m}^3$

The minus sign indicates that the volume has decreased.

The Language of Physics

Elasticity

That property of a body by which it experiences a change in size or shape whenever a deforming force acts on the body. The elastic properties of matter are a manifestation of the molecular forces that hold solids together (p. 283).

Lattice structure of a solid

A regular, periodically repeated, threedimensional array of the atoms or molecules comprising the solid (p. 283).

Stress

For a body that can be either stretched or compressed, the stress is the ratio of the applied force acting on a body to the crosssectional area of the body (p. 285).

Strain

For a body that can be either stretched or compressed, the ratio of the change in length to the original length of the body is called the strain (p. 285).

Hooke's law

In an elastic body, the stress is directly proportional to the strain (p. 285).

Young's modulus of elasticity

The proportionality constant in Hooke's law. It is equal to the ratio of the stress to the strain (p. 285).

Elastic limit

The point where the stress on a body becomes so great that the atoms of the body are pulled permanently away from their equilibrium position in the lattice structure. When the stress exceeds the elastic limit, the material will not return to its original size or shape when the stress is removed. Hooke's law is no longer valid above the elastic limit (p. 286).

Shear

That elastic property of a body that causes the shape of the body to be changed when a stress is applied. When the stress is removed the body returns to its original shape (p. 288).

Shearing strain

The angle of shear, which is a measure of how much the body's shape has been deformed (p. 289).

Shearing stress

The ratio of the tangential force acting on the body to the area of the body over which the tangential force acts (p. 289).

Shear modulus

The constant of proportionality in Hooke's law for shear. It is equal to the ratio of the shearing stress to the shearing strain (p. 290).

Bulk modulus

The constant of proportionality in Hooke's law for volume elasticity. It is equal to the ratio of the compressional stress to the strain. The strain for this case is equal to the change in volume per unit volume (p. 291).

Elasticity of volume

Vo

When a uniform force is exerted on all sides of an object, each side of the object becomes compressed. Hence, the entire volume of the body decreases. When the force is removed the body returns to its original volume (p. 291).

Summary of Important Equations

Hooke's law in general stress ∝ strain	(10.5)	Hooke's law for a spring $F = kx$	(10.9)	Hooke's law for volume elasticity	
Hooke's law for stretching or compression $\frac{F}{A} = Y \frac{\Delta L}{L_0}$	(10.6)	Hooke's law for shear $\frac{F_t}{A} = S\phi$	(10.15)	$\frac{F}{A} = -B\frac{\Delta V}{V_0}$ Hooke's law for volume elasticity $p = -B\frac{\Delta V}{\Delta V}$	(10.19)

Questions for Chapter 10

- 1. Why is concrete often reinforced with steel?
- *2. An amorphous solid such as glass does not have the simple lattice structure shown in figure 10.1. What effect does this have on the elastic properties of glass?
- 3. Discuss the assumption that the diameter of a wire does not change when under stress.
- 4. Compare the elastic constants of a human bone with the elastic constants of other materials listed in table 10.1. From this standpoint discuss the bone as a structural element.

- 5. Why are there no Young's moduli for liquids or gases?
- 6. Describe the elastic properties of a cube of jello.
- 7. If you doubled the diameter of a human bone, what would happen to the maximum compressive force that the bone could withstand without breaking?
- ***8.** In the profession of Orthodontics, a dentist uses braces to realign teeth. Discuss this process from the point of view of stress and strain.
- **†9.** Discuss Hooke's law as it applies to the bending of a beam that is fixed at one end and has a load placed at the other end.



†10. How do the elastic properties of a material affect the vibration of that material?

Problems for Chapter 10

10.2 Hooke's Law—Stress and Strain

- 1. An aluminum wire has a diameter of 0.850 mm and is subjected to a force of 1000 N. Find the stress acting on the wire.
- 2. A copper wire experiences a stress of 5.00×10^3 N/m². If the diameter of the wire is 0.750 mm, find the force acting on the wire.
- **3.** A brass wire 0.750 cm long is stretched by 0.001 cm. Find the strain of the wire.
- **4.** A steel wire, 1.00 m long, has a diameter of 1.50 mm. If a mass of 3.00 kg is hung from the wire, by how much will it stretch?
- 5. A load of 50,000 lb is placed on an aluminum column 4.00 in. in diameter. If the column was originally 4.00 ft high find the amount that the column has shrunk.
- 6. A mass of 25,000 kg is placed on a steel column, 3.00 m high and 15.0 cm in diameter. Find the decrease in length of the column under this compression.



- 7. An aluminum wire, 1.50 m long, has a diameter of 0.750 mm. If a force of 60.0 N is suspended from the wire, find (a) the stress on the wire, (b) the elongation of the wire, and (c) the strain of the wire.
- 8. A copper wire, 1.00 m long, has a diameter of 0.750 mm. When an unknown weight is suspended from the wire it stretches 0.200 mm. What was the load placed on the wire?
- **9.** A steel wire is 1.00 m long and has a diameter of 0.75 mm. Find the maximum value of a mass that can be suspended from the wire before exceeding the elastic limit of the wire.
- **10.** A steel wire is 1.00 m long and has a 10.0-kg mass suspended from it. What is the minimum diameter of the wire such that the load will not exceed the elastic limit of the wire?
- 11. Find the maximum load that can be applied to a brass wire, 0.750 mm in diameter, without exceeding the elastic limit of the wire.
- 12. Find the maximum change in length of a 1.00-m brass wire, of 0.800 mm diameter, such that the elastic limit of the wire is not exceeded.
- 13. If the thigh bone is about 25.0 cm in length and about 4.00 cm in diameter determine the maximum compression of the bone before it will break. The ultimate compressive strength of bone is $1.70 \times 10^8 \text{ N/m}^2$.
- 14. If the ultimate tensile strength of glass is $7.00 \times 10^7 \text{ N/m}^2$, find the maximum weight that can be placed on a glass cylinder of 0.100 m² area, 25.0 cm long, if the glass is not to break.

- 15. A human bone is 2.00 cm in diameter. Find the maximum compression force the bone can withstand without fracture. The ultimate compressive strength of bone is $1.70 \times 10^8 \text{ N/m^2}$.
- **16.** A copper rod, 0.400 cm in diameter, supports a load of 150 kg suspended from one end. Will the rod return to its initial length when the load is removed or has this load exceeded the elastic limit of the rod?

10.3 Hooke's Law for a Spring

- 17. A coil spring stretches 4.00 cm when a mass of 500 g is suspended from it. What is the force constant of the spring?
- 18. A coil spring stretches by 2.00 cm when an unknown load is placed on the spring. If the spring has a force constant of 3.5 N/m, find the value of the unknown force.
- 19. A coil spring stretches by 2.50 cm when a mass of 750 g is suspended from it. (a) Find the force constant of the spring. (b) How much will the spring stretch if 800 g is suspended from it?
- **20.** A horizontal spring stretches 20.0 cm when a force of 10.0 N is applied to the spring. By how much will it stretch if a 30.0-N force is now applied to the spring? If the same spring is placed in the vertical and a weight of 10.0 N is hung from the spring, will the results change?
- **21.** A coil spring stretches by 4.50 cm when a mass of 250 g is suspended from it. What force is necessary to stretch the spring an additional 2.50 cm?

10.4 Elasticity of Shape—Shear

- **22.** A brass cube, 5.00 cm on a side, is subjected to a tangential force. If the angle of shear is measured in radians to be 0.010 rad, what is the magnitude of the tangential force?
- A copper block, 7.50 cm on a side, is subjected to a tangential force of 3.5 × 10³ N. Find the angle of shear.
- 24. A copper cylinder, 7.50 cm high, and 7.50 cm in diameter, is subjected to a tangential force of 3.5×10^3 N. Find the angle of shear. Compare this result with problem 23.



25. An annular copper cylinder, 7.50 cm high, inner radius of 2.00 cm and outer radius of 3.75 cm, is subjected to a tangential force of 3.5×10^3 N. Find the angle of shear. Compare this result with problems 23 and 24.



10.5 Elasticity of Volume

- 26. A cube of lead 15.0 cm on a side is subjected to a uniform pressure of $5.00 \times 10^5 \text{ N/m^2}$. By how much does the volume of the cube change?
- 27. A liter of glycerine contracts 0.21 cm³ when subject to a pressure of 9.8 × 10⁵ N/m². Calculate the bulk modulus of glycerine.
- 28. A pressure of $1.013 \times 10^7 \text{ N/m}^2$ is applied to a volume of 15.0 m^3 of water. If the bulk modulus of water is $0.020 \times 10^{10} \text{ N/m}^2$, by how much will the water be compressed?

- **29.** Repeat problem 28, only this time use glycerine that has a bulk modulus of 0.45×10^{10} N/m².
- **30.** Normal atmospheric pressure is 1.013×10^5 N/m². How many atmospheres of pressure must be applied to a volume of water to compress it to 1.00% of its original volume? The bulk modulus of water is 0.020×10^{10} N/m².
- 31. Find the ratio of the density of water at the bottom of a 50.0-m lake to the density of water at the surface of the lake. The pressure at the bottom of the lake is $4.90 \times 10^5 \text{ N/m}^2$. (*Hint:* the volume of the water will be decreased by the pressure of the water above it.) The bulk modulus for water is $0.21 \times 10^{10} \text{ N/m}^2$.

Additional Problems

- **32.** A lead block 50.0 cm long, 10.0 cm wide, and 10.0 cm thick, has a force of 200,000 N placed on it. Find the stress, the strain, and the change in length if (a) the block is standing upright, and (b) the block is lying flat.
- 33. An aluminum cylinder must support a load of 450,000 N. The cylinder is 5.00 m high. If the maximum allowable stress is 1.4×10^8 , what must be the minimum radius of the cylinder in order for the cylinder to support the load? What will be the length of the cylinder when under load?
- 34. This is essentially the same problem as 33, but now the cylinder is made of steel. Find the minimum radius of the steel cylinder that is necessary to support the load and compare it to the radius of the aluminum cylinder. The maximum allowable stress for steel is 2.4×10^{10} N/m².
- **35.** How many 1.00-kg masses may be hung from a 1.00-m steel wire, 0.750 mm in diameter, without exceeding the elastic limit of the wire?
- **36.** A solid copper cylinder 1.50 m long and 10.0 cm in diameter, has a mass of 5000 kg placed on its top. Find the compression of the cylinder.
- **37.** This is the same problem as 36, except that the cylinder is an annular cylinder with an inner radius of 3.50 cm and outer radius of 5.00 cm. Find the compression of the cylinder and compare with problem 36.

38. This is the same problem as problem 36 except the body is an I-beam with the dimensions shown in the diagram. Find the compression of the I-beam and compare to problems 36 and 37. The crossbar width is 2.00 cm.



†39. Two pieces of metal rod, 2.00 cm thick, are to be connected together by riveting a steel plate to them as shown in the diagram. Two rivets, each 1.00 cm in diameter, are used. What is the maximum force that can be applied to the metal rod without exceeding a shearing stress of $8.4 \times 10^8 \text{ N/m}^2$.



†40. A copper and steel wire are welded together at their ends as shown. The original length of each wire is 50.0 cm and each has a diameter of 0.780 mm. A mass of 10.0 kg is suspended from the combined wire. By how much will the combined wire stretch?



†41. A copper and steel wire each 50.0 cm in length and 0.780 mm in diameter are connected in parallel to a load of 98.0 N, as shown in the diagram. If the strain is the same for each wire, find (a) the force on wire 1, (b) the force on wire 2, and (c) the total displacement of the load.



†42. Repeat problem 41 with the diameter of wire 1 equal to 1.00 mm and the diameter of wire 2 equal to 1.50 mm.

***43.** Two steel wires of diameters 1.50 mm and 1.00 mm, and each 50.0 cm long, are welded together in series as shown in the diagram. If a weight of 98.0 N is suspended from the bottom of the combined wire, by how much will the combined wire stretch?



†44. Two springs are connected in parallel as shown in the diagram. The spring constants are $k_1 = 5.00$ N/m and $k_2 = 3.00$ N/m. A force of 10.0 N is applied as shown. If the strain is the same in each spring, find (a) the displacement of mass *m*, (b) the force on spring 1, and (c) the force on spring 2.



†45. Two springs are connected in series as shown in the diagram. The spring constants are $k_1 = 5.00$ N/m and k_2 = 3.00 N/m. A force of 10.0 N is applied as shown. Find (a) the displacement of mass *m*, (b) the displacement of spring 1, and (c) the displacement of spring 2.



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□ 46. Young's modulus for a wire is $Y = 2.10 \times 10^{11}$ N/m². The wire has an initial length of $L_0 = 0.700$ m and a diameter d = 0.310 mm. A force F = 1.00 N is applied in steps from 1.00 to 10.0 N. Calculate the wire's change in length ΔL with increasing load F, and graph the result.