The Language of Physics

Capacitor

Two conductors of any size or shape carrying equal and opposite charges are called a capacitor. The charge on the capacitor is directly proportional to the potential difference between the plates. The importance of the capacitor lies in the fact that energy can be stored in the electric field between the two bodies (p. 603).

Energy density

The energy per unit volume that is stored in the electric field (p. 607).

Capacitors in series

When capacitors are connected in series, each capacitor has the same charge on its plates. The reciprocal of the equivalent capacitance is equal to the sum of the reciprocals of each capacitor (p. 609).

Capacitors in parallel

When capacitors are connected in parallel there is a different amount of charge deposited on the plates of each capacitor, but the potential difference is the same across each of the parallel capacitors. The equivalent capacitance is equal to the sum of the individual capacitances (p. 611).

Dielectric constant

The dielectric constant is defined as the ratio of the capacitance with a dielectric between the plates to the capacitance with air or vacuum between the plates (p. 616).

Capacitors with a dielectric

Placing a dielectric, an insulator, between the plates of a capacitor increases the capacitance of that capacitor; decreases the electric field and the potential difference between the plates; decreases the amount of energy that can be stored in the capacitor; and increases the dielectric strength of the capacitor (p. 616).

Dielectric strength

The value of the potential difference per unit plate separation when the dielectric breaks down (p. 619).

Summary of Important Equations

Capacitance of parallel plate capacitor

$$C = \frac{\epsilon_0 A}{d} \quad \textbf{(21.3)}$$

Capacitance for coaxial cylindrical capacitor

$$C = \frac{2\pi\epsilon_0 l}{\ln(r_B/r_A)} \quad \textbf{(21.4)}$$

Capacitance for concentric spherical capacitor

$$C = \frac{4\pi\epsilon_0 r_A r_B}{r_B - r_A} \quad \textbf{(21.5)}$$

Charge on a capacitor

$$q = CV \quad \textbf{(21.6)}$$

Definition of capacitance

$$C = \frac{q}{V} \quad \textbf{(21.7)}$$

Energy stored in a capacitor

$$W = \tfrac{1}{2}qV \quad \textbf{(21.8)}$$

$$W = \tfrac{1}{2}CV^2 \quad \textbf{(21.9)}$$

$$W = \frac{1}{2}\frac{q^2}{C} \quad \textbf{(21.10)}$$

Energy density in electric field

$$u_E = \tfrac{1}{2}\epsilon_0 E^2 \quad \textbf{(21.12)}$$

Equivalent capacitance of capacitors in series

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \quad \textbf{(21.19)}$$

Equivalent capacitance of capacitors in parallel

$$C = C_1 + C_2 + C_3 + \cdots \quad \textbf{(21.23)}$$

Capacitance with dielectric

$$C_d = \frac{q}{V_d} \quad \textbf{(21.25)}$$

Definition of dielectric constant

$$\kappa = \frac{C_d}{C_0} \quad \textbf{(21.28)}$$

Capacitance of capacitor with a dielectric

$$C_d = \kappa C_0 \quad \textbf{(21.29)}$$

Permittivity of the dielectric medium

$$\epsilon = \kappa\epsilon_0 \quad \textbf{(21.31)}$$

The effect of a dielectric between the plates

$$\frac{C_d}{C_0} = \frac{V_0}{V_d} = \frac{E_0}{E_d} = \kappa \quad \textbf{(21.36)}$$

Potential in the dielectric

$$V_d = \frac{V_0}{\kappa} \quad \textbf{(21.37)}$$

Electric field within the dielectric

$$E_d = \frac{E_0}{\kappa} \quad \textbf{(21.38)}$$

Energy stored in dielectric capacitor

$$W_d = \frac{W_0}{\kappa} \quad \textbf{(21.41)}$$

Force on a particle in a dielectric medium

$$F_d = \frac{F}{\kappa} \quad \textbf{(21.45)}$$

Coulomb's law within a dielectric medium

$$F = \frac{1}{4\pi\epsilon}\frac{q_1 q_2}{r^2} \quad \textbf{(21.46)}$$

Questions for Chapter 21

1. Can you speak of the capacitance of an isolated conducting sphere?
†2. How does the direction signal device on your car make use of a capacitor?
3. If an ammeter is connected in series to the circuit of figure 21.3, what will it indicate? Why?
4. Discuss the statement, "If a capacitor contains equal amounts of positive and negative charge it should be neutral and have no effect whatsoever."
†5. In tuning in your favorite radio station, you adjust the tuning knob, which is connected to a variable capacitor. How does this variable capacitor work?
6. If the potential difference across a capacitor is doubled what does this do to the energy that the capacitor can store?
7. Is more energy stored in capacitors when they are connected in parallel or in series? Why?
8. If the applied potential difference across a capacitor exceeds the dielectric strength of the medium between the plates, what happens to the capacitor?
†9. Sketch what you think a plot of the charge on a capacitor versus time would look like, when it is charging and when it is discharging.
10. Can a coaxial cable have a capacitance associated with it?

Problems for Chapter 21

21.2 The Parallel Plate Capacitor

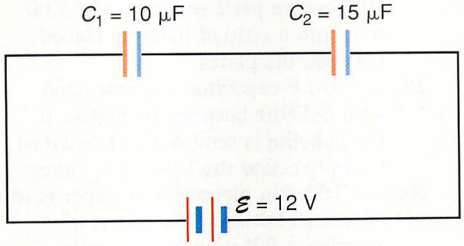
1. Find the charge on a 4.00-μF capacitor if it is connected to a 12.0-V battery.
2. If the charge on a 9.00-μF capacitor is 5.00×10^{-4} C, what is the potential across it?
3. How much charge must be removed from a capacitor such that the new potential across the plates is 1/2 of the original potential?
4. A charge of 6.00×10^{-4} C is found on a capacitor when a potential of 120 V is placed across it. What is the value of the capacitance of this capacitor?
5. A parallel plate capacitor has plates that are 6.00 cm by 4.00 cm and are separated by 8.00 mm. Find the capacitance of the capacitor.
6. If a 5.00-μF parallel plate capacitor has its plate separation doubled while its cross-sectional area is tripled, determine the new value of its capacitance.
7. You are asked to design your own parallel plate capacitor that is capable of holding a charge of 3.60×10^{-12} C when placed across a potential difference of 12.0 V. If the area of the plates is equal to 100 cm^2 what must the plate separation be? What is the value of the capacitance of this capacitor?
8. You are asked to design a parallel plate capacitor that can hold a charge of 9.87 pC when a potential difference of 24.0 V is applied across the plates. Find the ratio of the area of the plates to the plate separation, and then pick a reasonable set of values for them.
9. A cylindrical capacitance 10.0 cm long has an inner radius of 0.500 mm and an outside radius of 5.00 mm. Find the capacitance of the capacitor.
10. A cylindrical capacitor 1.00 m long has radii 20.0 cm and 50.0 cm. Find its capacitance.
11. A spherical capacitor has radii 20.0 cm and 50.0 cm. Find its capacitance.

21.3 Energy Stored in a Capacitor

12. A 7.00-μF capacitor is connected to a 400-V source. Find (a) the charge on the capacitor and (b) the energy stored in the capacitor.
13. How much energy can be stored in a capacitor of 9.45 μF when it is placed across a potential difference of 120 V? How much charge will be on this capacitor?
14. What value of capacitance is necessary to store 1.73×10^{-3} J of energy when it is placed across a potential difference of 24.0 V?

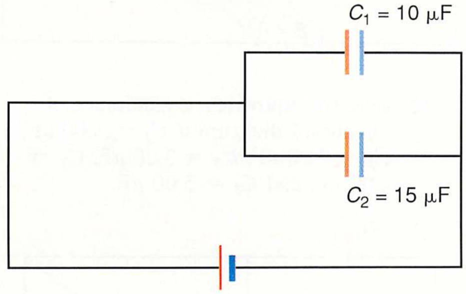
21.4 Capacitors in Series

15. Find the equivalent capacitance of 2.00-μF, 4.00-μF, and 8.00-μF capacitors connected in series.
16. Find (a) the equivalent capacitance, (b) the charge on each capacitor, (c) the voltage across each capacitor, and (d) the energy stored in each capacitor in the diagram.

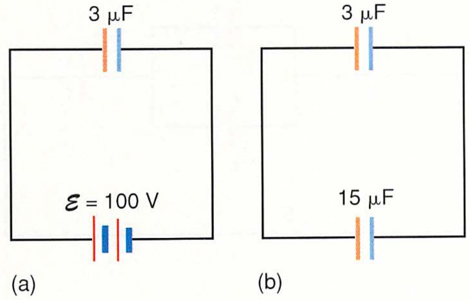


21.5 Capacitors in Parallel

17. Find the equivalent capacitance of 2.00-μF, 4.00-μF, and 8.00-μF capacitors connected in parallel.
18. Find the charge on each capacitor and the energy stored in each capacitor in the diagram.



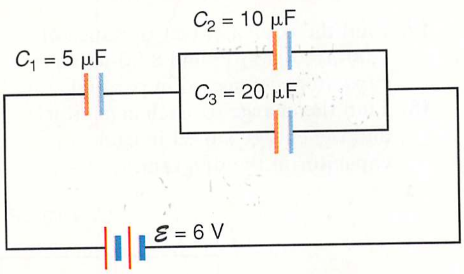
19. A 3.00-μF capacitor is initially connected to a battery and charged to 100 V. It is then removed and connected in parallel to a 15.0-μF capacitor that was uncharged. Find (a) the initial charge on the first capacitor, (b) the charge on each capacitor after being connected in parallel, (c) the initial energy stored in the first capacitor, and (d) the energy stored in each capacitor after they are connected in parallel.



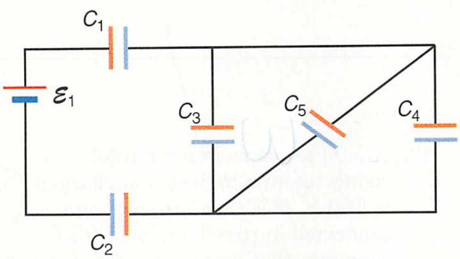
20. A 40.0-μF capacitor and an 80.0-μF capacitor are initially connected in parallel across a 10.0-V potential difference. The two capacitors are then disconnected from each other and from the potential source, and reconnected with plates of unlike sign connected together. Find the charge on each capacitor and the potential difference across each capacitor after the two capacitors are so connected.

21.6 Combinations of Capacitors in Series and Parallel

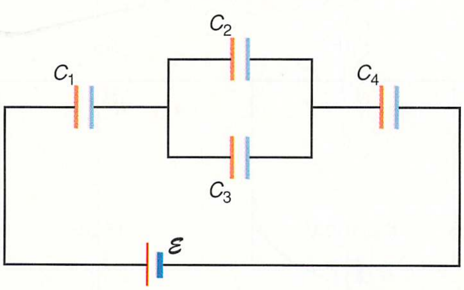
†21. Find (a) the equivalent capacitance of the circuit, (b) the total charge drawn from the battery, (c) the voltage across each capacitor, (d) the charge on each capacitor, and (e) the energy stored in each capacitor in the accompanying diagram.



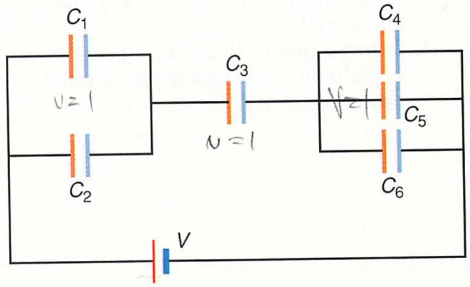
22. Find the equivalent capacitance of the circuit diagram if $C_1 = 1.00\ \mu$F, $C_2 = 2.00\ \mu$F, $C_3 = 3.00\ \mu$F, $C_4 = 4.00\ \mu$F, and $C_5 = 5.00\ \mu$F.



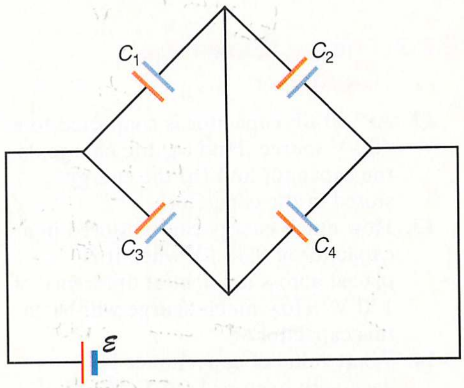
†23. In the accompanying diagram find (a) the equivalent capacitance, (b) the charge on capacitor C_2, and (c) the voltage drop across C_4, if $C_1 = 10.0\ \mu$F, $C_2 = 20.0\ \mu$F, $C_3 = 30.0\ \mu$F, $C_4 = 40.0\ \mu$F, and $\mathscr{E} = 12.0$ V.



24. The network of capacitors in the diagram is connected across a potential difference $V = 3.00$ V. If $C_1 = C_2 = 30.0\ \mu$F, $C_3 = 60.0\ \mu$F, and $C_4 = C_5 = C_6 = 20.0\ \mu$F, find (a) the equivalent capacitance of the network, (b) the charge on each capacitor, (c) the potential across each capacitor, and (d) the energy stored in each capacitor.



†25. Find the equivalent capacitance and the charge on each capacitor in the diagram if $C_1 = 5.00\ \mu$F, $C_2 = 15.00\ \mu$F, $C_3 = 8.00\ \mu$F, $C_4 = 9.00\ \mu$F, and $\mathscr{E} = 12.0$ V.



21.7 Capacitors with Dielectrics Placed between the Plates

26. A 5.00-μF parallel plate capacitor has air between the plates. When an insulating material is placed between the plates, the capacitance increases to 13.5 μF. Find the dielectric constant of the insulator.

27. If the dielectric constant of rubber is 2.94, what is its permittivity?

28. Find the capacitance of a parallel plate capacitor having an area of 75.0 cm^2, a plate separation of 3.00 mm, and a strip of bakelite placed between the plates.

29. A 350-μF capacitor is constructed with bakelite between its plates. If the bakelite is removed and replaced by amber, find the new capacitance.

30. A 0.100-mm piece of wax paper is to be placed between two pieces of aluminum foil to make a parallel plate capacitor of 5 μF. What must the area of the foil be?

31. A parallel plate capacitor has plates that are 5.00 cm by 6.00 cm and are separated by an air gap of 1.50 mm. Calculate the maximum voltage that can be applied to the capacitor before dielectric breakdown.

32. A 1.00-mm piece of wax paper is placed between two pieces of aluminum foil that are 10.0 cm by 10.0 cm. How much energy can be stored in this capacitor if it is connected to a 24.0 V battery?

33. If 1.73×10^{-3} J of energy can be stored in a capacitor with air between the plates, how much energy can be stored in the capacitor if a slab of mica is placed between the plates?

34. What is the maximum charge that can be placed on the plates of an air capacitor of 9.00 μF before breakdown if the plate separation is 1.00 mm? If a strip of mica is placed between the plates, what is the maximum charge?

35. A parallel plate capacitor has a value of 3.00 pF. A 1.00-mm piece of plexiglass is placed between the plates of the capacitor, which is then connected to a 12.0-V battery. Find (a) the electric field E_0 without the dielectric between the plates and (b) the induced electric field in the dielectric E_i.

36. A sample of NaCl is placed into a solution of ethyl alcohol. If the distance between the Na^+ and the Cl^- ions is 1.00×10^{-7} cm, find the force on the ions.

Additional Problems

†37. Show that if the radii of a spherical capacitor are very large, the equation for the capacitance of a spherical capacitor reduces to the equation for a parallel plate capacitor.

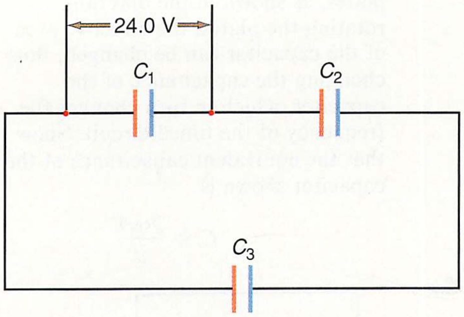
†38. Find the formula for the capacitance of an isolated sphere. (*Hint:* use the relation for a concentric spherical capacitor and let the radius of the outer sphere go to infinity.)

†39. Consider the earth to be an isolated sphere. Find the capacitance of the earth. If the electric field of the earth is measured to be 100 V/m, what charge must be on the surface of the earth? (Note that the fair weather electric field points downward, indicating an effective negative charge on the solid sphere.)

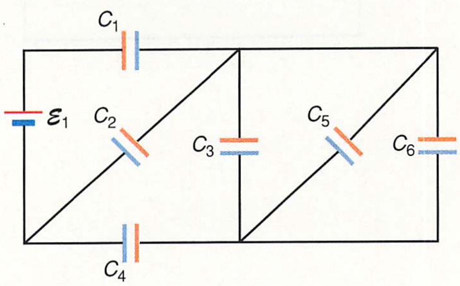
40. Determine all the values of capacitance that you can obtain from the three capacitors of 3.00 μF, 6.00 μF, and 9.00 μF.

41. (a) Find the equivalent capacitance of 50 identical 2.50-μF capacitors in series. (b) Find the equivalent capacitance of 50 identical 2.50-μF capacitors in parallel.

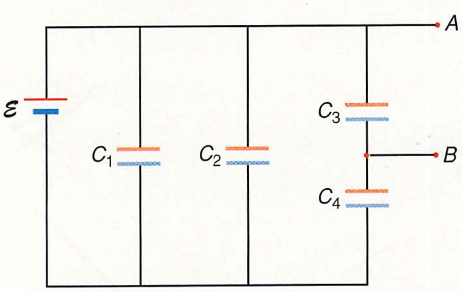
†42. A potential of 24.0 V is applied across capacitor $C_1 = 2.50\ \mu F$. Find the charge on capacitors C_2 and C_3 if $C_2 = 6.00\ \mu F$ and $C_3 = 4.00\ \mu F$.



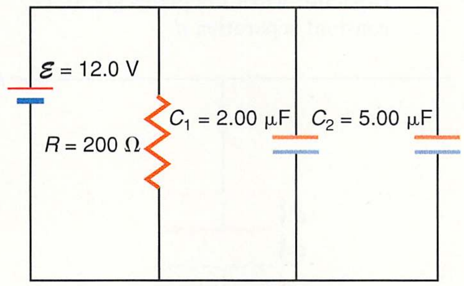
†43. In the accompanying diagram find (a) the equivalent capacitance, (b) the voltage drop across each capacitor, and (c) the charge on each capacitor if $C_1 = 3.00\ \mu F$, $C_2 = 6.00\ \mu F$, $C_3 = 9.00\ \mu F$, $C_4 = 12.00\ \mu F$, $C_5 = 15.00\ \mu F$, $C_6 = 18.00\ \mu F$, and $\mathscr{E}_1 = 12.0$ V.



†44. Find the potential drop across AB in the diagram if $C_1 = 3.00\ \mu F$, $C_2 = 6.00\ \mu F$, $C_3 = 9.00\ \mu F$, $C_4 = 12.00\ \mu F$, and $\mathscr{E} = 24.0$ V.



†45. In the accompanying circuit, find (a) the final value of current recorded by an ammeter that is first placed in series with the resistor R, then the capacitor C_1, and finally the capacitor C_2; (b) find the voltage drop recorded by a voltmeter when placed across the resistor R, the capacitor C_1, and the capacitor C_2; and (c) find the charge on C_1 and C_2.



46. A cylindrical capacitor is made by wrapping a sheet of aluminum foil around a hollow cardboard core 2.50 cm in diameter. A 1.00 mm thick piece of wax paper is then wrapped around the foil and then another sheet of aluminum foil is wrapped around the wax paper. The length of the core is 25.0 cm. Find the capacitance of this cylindrical capacitor.

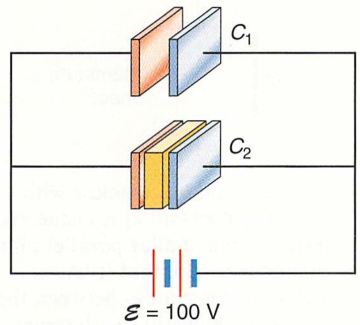
†47. Find (a) the capacitance of a parallel plate capacitor having an area of 40.0 cm² and separated by air 2.00 mm thick, (b) the capacitance if 2.00 mm of mica is placed between the plates, (c) if the capacitor is connected to a 6.00-V battery, find the charge on the capacitor with air and with mica between the plates, (d) the electric field between the plates, and (e) the energy stored in each capacitor.

†48. The electric field between the plates of a parallel plate capacitor filled with air is $E_0 = q/(\epsilon_0 A)$. When a dielectric is placed between the plates, the induced electric field within the dielectric is given by $E_i = q'/(\epsilon_0 A)$, where q' is the bound charge. (a) Show that the bound charge on the dielectric is given by

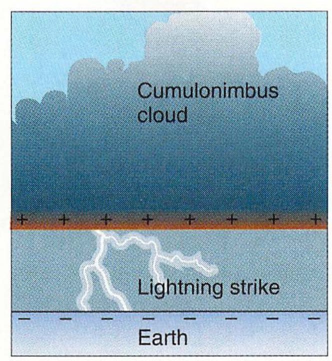
$$q' = q\left(1 - \frac{1}{\kappa}\right)$$

(b) If a piece of mica is placed between the plates of a capacitor of 6.00 μF and is then connected to the plates of a 24.0-V battery, find the charge q and q'.

49. Two identical parallel plate capacitors, one with air between the plates, and the other with mica, are connected to a 100-V battery, as shown in the diagram. If $C_1 = 3.00\ \mu F$ and the plate separation is 0.500 mm, find (a) the charge on each plate, (b) the electric field between the plates, and (c) the energy stored in each capacitor.

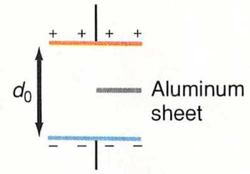


50. A cumulonimbus cloud is 5.00 km long by 5.00 km wide and has its base 1.00 km above the surface of the earth, as shown in the diagram. Consider the cloud and earth to be a parallel plate capacitor with air as the dielectric. (a) Find the capacitance of the cloud-earth combination. (b) Find the potential difference between the cloud and the earth when lightning occurs. (c) Calculate the charge that must be on the base of the cloud when lightning occurs.

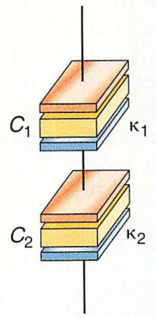


†51. A parallel plate capacitor with air between the plates has a plate separation d_0 and area A. A sheet of aluminum of negligible thickness is now placed midway between the parallel plates. Find the capacitance (a) before and (b) after the aluminum sheet is placed between the plates.

†52. A parallel plate capacitor with air between the plates has a plate separation d_0 and area A. A half of a sheet of aluminum of negligible thickness is now placed midway between the parallel plates as shown. Show what this combination is equivalent to and find the equivalent capacitance.



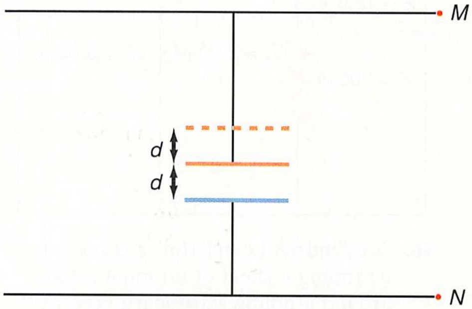
†53. A parallel plate capacitor with dielectric constant κ_1 is connected in series with a similar parallel plate capacitor except that it has a dielectric constant κ_2 between the plates, as shown in the diagram. (a) Find a single equation for the equivalent capacitor. (b) What is the value of an equivalent dielectric that could be placed in one of the original capacitors to give an equivalent capacitor?



†54. A parallel plate capacitor has a moveable top plate that executes simple harmonic motion at a frequency f. The displacement of the top plate varies such that the plate separation varies from d to $2d$. Show that the voltage across MN varies as

$$V = V_0(1 + \tfrac{1}{2}\cos 2\pi ft)$$

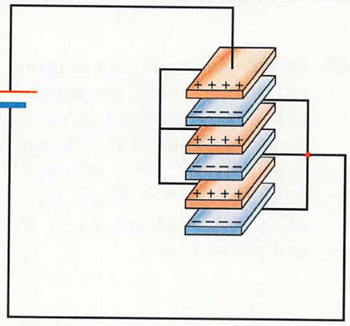
where V_0 is the voltage across the capacitor when the plates are at a constant separation d.



†55. One model of a red blood cell considers the cell to be a spherical capacitor. The radius of a blood cell is about 6.00×10^{-4} cm and the membrane wall of 0.100×10^{-4} cm is considered to be a dielectric material with a dielectric constant of 5.00. Find (a) the capacitance of the cell. (b) If a potential difference of 100 mV is measured across the blood cell, find the charge on a red blood cell.

†56. When you tune in your favorite radio station, you use a variable capacitor in the radio. A variable capacitor consists of a series of ganged metal plates, as shown in the diagram. By rotating the plates, the effective area of the capacitor can be changed, thus changing the capacitance of the capacitor, which in turn changes the frequency of the tuned circuit. Show that the equivalent capacitance of the capacitor shown is

$$C = \frac{5\epsilon_0 A}{d}$$



Interactive Tutorials

57. A parallel plate capacitor has plates that have an area $A = 83.5\ \text{cm}^2$ and a plate separation $d = 6.00$ mm. The space between the plates is filled with air. The capacitor is connected to a 12.0-V battery. Find (a) the capacitance C of the capacitor, (b) the charge q deposited on its plates, and (c) the energy stored in the capacitor.

58. Capacitors in series. Three capacitors, $C_1 = 2.55 \times 10^{-6}$ F, $C_2 = 5.35 \times 10^{-6}$ F, and $C_3 = 8.55 \times 10^{-6}$ F are connected in series to a 100-V battery. Find (a) the equivalent capacitance, (b) the charge on each capacitor, (c) the voltage drop across each capacitor, (d) the energy stored in each capacitor, and (e) the total energy stored in the circuit.

59. Capacitors in parallel. Three capacitors, $C_1 = 2.55 \times 10^{-6}$ F, $C_2 = 5.35 \times 10^{-6}$ F, and $C_3 = 8.55 \times 10^{-6}$ F are connected in parallel to a 100-V battery. Find (a) the equivalent capacitance, (b) the total charge on the equivalent capacitor, (c) the voltage drop across each capacitor, (d) the charge on each capacitor, (e) the energy stored in each capacitor, and (f) the total energy stored in the circuit.

60. Combination of capacitors in series and parallel. Capacitor $C_1 = 2.55 \times 10^{-6}$ F is in series with capacitors $C_2 = 5.35 \times 10^{-6}$ F and $C_3 = 8.55 \times 10^{-6}$ C, which are in parallel with each other. The entire combination is connected to a 100-V battery. A similar schematic is shown in figure 21.6. Find (a) the equivalent capacitance of the combination, (b) the total charge q on the equivalent capacitor, (c) the charge q_1 on capacitor C_1, (d) the voltage drop across each capacitor, (e) the charge on capacitors C_2 and C_3, (f) the energy stored in each capacitor, and (g) the total energy stored in the circuit.

61. A capacitor with a dielectric. A parallel plate capacitor, having a plate area $A = 3.8 \times 10^{-3}\ \text{m}^2$ and a plate separation $d = 1.75$ mm, is charged by a battery to $V_0 = 85.0$ V. The battery is then removed. Find (a) the capacitance of the capacitor and (b) the charge on the plates of the capacitor. A slab of mica is then placed between the plates of the capacitor. Find (c) the new value of the capacitance, (d) the potential difference across the capacitor with the dielectric, (e) the initial electric field between the plates, (f) the final electric field between the plates, (g) the initial energy stored in the capacitor, and (h) the final energy stored in the capacitor.